Transfers and alliance break-up*

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Abstract

In this paper we analyzed the role of transfers for peace to hold in equilibrium in a conflict between a single party and an alliance of two parties. The conflict concerns the resources that belong to the single party and the war is destructive. Moreover, the ability of single party to transfer resources differs among the members of the alliance. We analyzed three different cases. In the first case we assumed that the transfers were binding, and found that the peace holds in equilibrium if the ability of the first party to transfer resources is sufficiently large. For the rest two cases, we analyzed the conflict with non-cooperative and cooperative allies, respectively, in an environment where no binding agreement exists. It is revealed that the peace holds under many parameter values with a relatively low amount of transfer. However, the amount of transfer required for peace to hold in equilibrium is larger for the case of cooperative allies.

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1 Introduction

There has been assiduous interest in how rational agents would decide to engage in war, which is a wasteful use of resources, e.g. Garfinkel and Skaperdas (2007), Hirshleifer (1991), Skaperdas (1998). On contrary with the conventional economic analyses, the possibility of conflict has profound implications for the distribution of resources and the costly arming decisions taken by opposing parties.

Our main interest in this paper is to analyze a conflict between an alliance and and a single party. The conflict takes place over the resources that belong to a single party and an alliance is interested in claiming these resources. Hence, in this paper we consider that the alliance is the attacker and the single party is the defender. We are mainly interested in the ability of transfers from the single party to the alliance to persuade the parties to settle under peace. We also will assume that the ability of transferring resources of the single party differs with respect to the members of the alliance due to possible ethnical, religious or geographical reasons. Therefore we would be able to interpret the importance of the intimacy between the subjects in a conflict for a possible war decision. To our knowledge Beviá and Corchón (2009) were the first and the only scholars to analyze the importance of pre-war peace agrements, i.e. transfers, for settling under peace. They showed in this paper that war could be avoided in many circumstances under resource constraints in a conflict between two parties. Despite being in line with the main motivation of Beviá and Corchón (2009), we will assume that the parties are not constrained in their resources. Hence, we would be able to understand the circumstances and specefic characteristics of the conflict under which peace could hold in equilibrium when parties have plenty of resources to be spent in warfare. Moreover, the type of conflict we are interested in is in line with many instances of disputes in the history of wars. Over numerous occasions, the history has witnessed the alliance of many nations against a single party. One can instantly give the example of the Crusades, which began taking place in the late 11th century and happened to be a war composed of ten large attacks waged by Catholic European states against the sole Muslim Seljuk Turks (Particularly the First Crusade) (Riley-Smith (1999)); or the Balkan Wars, in which four Orthodox Balkan states raged war against Ottoman Turks, i.e. the first Balkan War, however after victory due to conflict on the territory conquered, this alliance has fallen apart resulting in the second Balkan War. As Ferguson (2006) states the dispute had profoundly impactful consequences that could even be attributed to the First World War.

The examples are not just restricted for the case of war, but also political contest. In modern democracies, it is common to observe alliances of parties with similar political views to announce a single candidate in order to stand stronger to opposing political parties. However, we choose to restrict our attention for the cases of war.

Garfinkel and Skaperdas (2007) reports the main framework for the analyses of conflict of alliances in their extended survey of conflict theory. In our analysis we will closely follow the model they presented. Alliances are considered as beneficial for the parties involved, as Garfinkel (2004) also states, while in an alliance the members are able to summon up their war efforts, resources and stand stronger to the opposing party. However, there is also room for free riding unless the alliance acts cooperatively.

The formation and stability of alliances also received notable interest in conflict theory. For instance, Garfinkel (2004) analyzes the stability of alliances under farsighted stability, which is essentially a refinement of Nash equilibrium and implies that the parties inside of an alliance could foresee that the defection of one member will result in further defections and consequently may result in one-party alliances. On the other hand, Sánchez-Pagés (2007) considered the stability of alliances under a gamma game in which the defection of one party results in the breaking apart of the alliance and a delta game in which even though a single party defects, the remaining parties are able to stick together. In contrast, we exclude the formation of the alliance and treat it as exogenous. While, allowing for transfers between the alliances would lead to complex calculations, and even the existence of an equilibrium would not be guaranteed. Hence, it could be stated that our approach is more in line with the model by Olson Jr and Zeckhauser (1966).

We will analyze this conflict for three different cases. As a benchmark, we will consider that once a transfer is received, none of the members of the alliance is able to declare war, i.e. the agreement is binding. Secondly, we will relax this assumption in order to be more in line with the ancient conflicts in which no credible regulatory or enforcing mechanism exists. Lastly we will consider that the alliance acts cooperatively in the war against the single party to see the consequences of closely related allies in a conflict situation.

The paper is organized as follows. In the second part, we present the model. In the third part the analysis with a binding peace agreement, in the fourth part the analysis with a non-binding peace agreement, and in the fifth part the analysis with cooperative parties, will be presented. In the sixth chapter we present the conclusions and possible extensions.

2 The Model

The conflict concerns the members of a set $I = \{1, 2, 3\}$. There is an alliance whose members are given by the set $A = \{1, 2\} \subset I$. Each $i \in I$ is in possession of a consumable resource $V_i \in \Re_+$. The parties are competing for a resource, which belongs to the single party, denoted by $\widetilde{R}$. So, the third party has one safe resource, V_3, and one under attack. Ergo, the total resources of the third party, in the first stage, are given by $V_3 + \widetilde{R}$.

The outcome of the conflict between the alliance and the single party is determined by the contest success function (CSF), or technology of conflict as called by Hirshleifer (1989), in ratio form. This form was first used by Tullock (1980) and axiomatized by Skaperdas (1996). The ratio form is stated as follows:

$$p_i(g_i, g_{-i}) = \frac{g_i}{\sum_{i \in I} g_i} \tag{1}$$

where g_i is the costly war effort put by party $i \in I$, and $p_i\left(.\right)$ is the probability of victory for party $i \in I$ if the war is waged. However, because of the nature of the conflict we study, the total war efforts put by the members of the alliance should be summed up. Let $G_A = g_1 + g_2$ be the total war effort put by the alliance, and $G_S = g_3$ be the war effort put by the single party. Therefore (1) can be written as follows for $j \in \{A, I/A\}$:

$$p_j\left(G_A, G_S\right) = \frac{G_j}{G_A + G_S} \tag{2}$$

We adapt the ratio form not only for its simplicity, but also for its customary usage in the literature. With this CSF form, the resources spent for war efforts determines the probability of victory as a proportion of individual war effort in the aggregate war effort put by all of the parties the conflict binds. Jackson and Morelli (2007) uses the same CSF form, however in contrast they assume that the resources of the parties directly determine the probability of victory in war. We further assume that the parties are risk neutral. This assumption is also customary in the literature. Finally we assume, following Garfinkel and Skaperdas (2007), the war is destructive, i.e. the occurrence of war destroys $\varphi \in (0, 1)$ ratio of the resources under attack, $\widetilde{R}$. Thus, if the war is waged, the victor claims $(1 - \varphi)\,\widetilde{R}$. Notice that, for a large destruction parameter φ, the parties would be reluctant to engage in war.

We call the alliance the attacker and the single party the defender, while the contest takes place over the resources of the single party. [1]

The game entailed is composed of four stages. In the first stage the transfers are made from the defending party to the members of the alliance. In the second stage the decision to engage in war is made. In the third stage the game ends if war is not waged. Otherwise, the parties make arming decisions and outcome of war is determined. The game ends in the fourth stage if the single party is the victor, otherwise the members of the alliance engage in further conflict against each other. We will use subgame perfection as the equilibrium concept to solve the game.

Apart from the assumptions stated above, we make some more specific assumptions as follows.

Assumption 1: *The ability of single party to transfer resources differs between the members of the alliance, i.e., whenever any transfer T_j is sent from the single party to any ally $j \in A$, the ally would receive $\alpha_j T_j$, where $\alpha_j \in (0,1]$.*

The assumption above implies that whenever the single party transfers some resources, only some part of this transfer is received by the member of the alliance who received it. The reason for this assumption could be given as follows: In most cases one member or more members of the alliance could be in some sense "closer" to the defender. This marginal intimacy could be interpreted as evolving from geographical, ethnical or religious reasons.

Assumption 2: *The declaration of war by both members of the alliance is necessary and sufficient for the war to take place.*

In fact there is another rule one can use which states that the declaration of war by one member of the alliance is sufficient for war to take place. However, we do not analyze this complicated situation here and leave it as a possible extension.

[1] There's also a functional form that was used by Grossman and Kim (1996) in order to distinguish between attack and defense, which could be stated as follows for a two party contest. (See Clark and Riis (1998) for the axiomatization of this form):

$$p_1(G_1, G_2) = \frac{\phi f(G_1)}{\phi f(G_1) + (1-\phi) f(G_2)} \tag{3}$$

. where, $\phi \in (0,1)$. As evident, ϕ could be interpreted as the advantage parameter. If $\phi > 1/2$ then the first party is more advantageous in the war, while his efforts are more efficient in composing into military capabilities. But, for the sake of simplicity, for our paper we will use the CSF given by (2).

Notice that if the alliance is the victor in the war against the single party, the resource left from the single party should be shared according to some rule, or otherwise parties may engage in a further conflict. Following Garfinkel and Skaperdas (2007), we assume that this resource is shared according to the sharing rule σ as follows. Let s_j denote the arming decision made by $j \in A$, then the share, σ_j, $j \in A$ would get, could be stated as follows:

$$\sigma_j(s_1, s_2) = \frac{s_j}{s_1 + s_2} \tag{4}$$

Garfinkel and Skaperdas (2007) uses this sharing rule mostly for its simplicity and to avoid the undesirability of the second war for the alliance. Notice that if there would be another waged war the resource left would be destroyed once more, and the desirability of the resource for the members of the alliance will vanish significantly even with a rather low destruction parameter, φ. Realize also that according to this sharing rule the members of the alliance spend on arming in the fourth stage, however the war does not take place. So this sharing rule has the same characteristics with a cold war. The reason we allow the members of the alliance to make arming decisions is the fact that we want to be as close as the historical examples we have presented in the introduction part.

3 Equilibrium under a binding peace agreement

In this section we will first solve the game without the existence of transfers, and then determine the conditions in which the peace holds in equilibrium in the existence of binding transfers. Realize that, with assumption one, one can determine the sufficient transfer parameters, α_1 and α_2, for peace to hold in equilibrium under a binding peace agreement. In other words, the analysis we are going to present now, would be useful to determine how "close" the parties are supposed to be for peace to hold in equilibrium with an external enforcing mechanism.

Before we start the analysis, one needs to decide on a rule for the payoff of the single party under victory. Since this rule needs to be altered for the different cases to be considered, we choose to state it whenever the case requires us to do so. Below we state this assumption.

Assumption 3: *If the single party is the victor he reclaims his resource under attack minus the part destroyed in warfare.*

With this assumption the single party could only prevail occupying its own resources left after warfare. Moreover, the defeat of the alliance will only result for the members of it the loss of arming they made for war. This could be attributed to the absence of a mechanism of settlement after war.

This assumption could be altered for different post-war settlement rules. However, our main motive in this section is to understand if war could occur in the direst case for the single party in an environment with binding peace agreements. Realize also, that this assumption guarantees that war is totally undesirable for the defender, i.e., the single party.

Let us now formulate the expected overall payoff of the conflict for each $i \in I$. Using the assumptions stated in part two, and assumption three; the expected overall payoff, W_i, for each $i \in I$ is given as follows:

$$W_1 = V_1 + p_A(G_A, G_S)\left\{\sigma_1(s_1, s_2)(1-\varphi)\widetilde{R} - s_1\right\} - g_1 \tag{5}$$

$$W_2 = V_2 + p_A(G_A, G_S)\left\{\sigma_2(s_1, s_2)(1-\varphi)\widetilde{R} - s_2\right\} - g_2 \tag{6}$$

$$W_3 = V_3 + p_S(G_A, G_S)(1-\varphi)\widetilde{R} - g_3 \tag{7}$$

We begin by analyzing the last stage. Realize that if the third party is defeated in the third stage he loses the resource under attack; otherwise the war ends in that stage. Ergo, independent from the outcome of the war waged, the third party provides no effort after third stage. Also, recall that we assumed, in the case of a victory for the alliance, the resources left from the single party are shared according to the ratio of the war efforts made in the last stage, or by the sharing rule given by equation (4). Thus, in the last stage, given that the alliance is the victor, the payoff for a member of the alliance depends on the resource claimed from the single party minus the destructed part of this resource, $(1-\varphi)\widetilde{R}$, the further war efforts to be spent at stage four, s_j, the resources left from his own war efforts in the third stage, $V_j - g_j$, and to the sharing rule, σ. Therefore, the payoff in fourth stage, w_j, for any $j \in A$ is given as follows.

$$w_j = V_j - g_j + \sigma_j (s_1, s_2) (1-\varphi) \widetilde{R} - s_j \tag{8}$$

Each $j \in A$ in the fourth stage maximizes (8) subject to (4), where the war efforts made in the third stage, g_j, are given. Assuming an interior solution, which implies that none of the members of the alliance is constrained in the last stage, and using the optimality condition $\frac{\partial w_j}{\partial s_j} = 0$, henceforth; the war effort of each member of the alliance, given the war effort of the other member of the alliance, is stated by the following reaction functions.[2] The plot of these reaction functions are also given in Figure 1 for parameters $\varphi = 0.3$ and $\widetilde{R} = 100$.

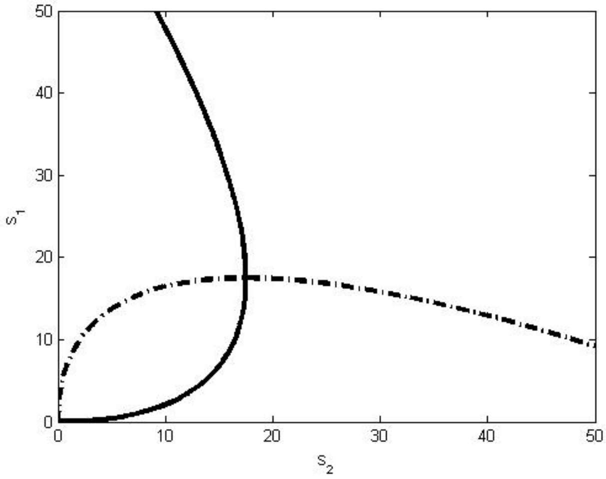
$$s_1(s_2) = \sqrt{s_2(1-\varphi)\widetilde{R}} - s_2 \tag{9}$$

$$s_2(s_1) = \sqrt{s_1(1-\varphi)\widetilde{R}} - s_1 \tag{10}$$

While no $j \in A$ is constrained in his resources, we are able to use the symmetry of the reaction functions. Hence, using (9) and (10), we can reach the following equilibrium war efforts by each $j \in A$, and their equilibrium payoff in the last stage using the equilibrium

[2] To guarantee the existence of a maximum it is important that we also present the second order conditions. The second order conditions for this maximization problem for each $j \in A$ given as follows: $\frac{\partial w_j^2}{\partial^2 s_j} = \frac{-2s_{-j}(1-\varphi)\widetilde{R}}{\left(\sum_{j\in A} s_j\right)^3}$, which is negative, guaranteeing a maximum.

Figure 1: Reaction functions (9) and (10)



war efforts and equation (8).

$$s_1^* = s_2^* = s^* = \frac{(1-\varphi)\,\widetilde{R}}{4} \tag{11}$$

$$w_j^* = \frac{(1-\varphi)\,\widetilde{R}}{4} + V_j - g_j \tag{12}$$

Now, obtaining the equilibrium war efforts put and the expected payoffs of the members of the alliance in the last stage, we are ready to determine the war efforts put in the third stage by all parties. First of all, using equations (11) and (12), we are able to restate equations (5) and (6) as follows:

$$W_1 = V_1 - g_1 + p_A(G_A, G_S)\frac{(1-\varphi)\widetilde{R}}{4} \tag{13}$$

$$W_2 = V_2 - g_2 + p_A(G_A, G_S)\frac{(1-\varphi)\widetilde{R}}{4} \tag{14}$$

In the third stage, given that war is declared at stage two, each $i \in I$ maximizes (7), (13) and (14), respectively, with respect to g_i, and subject to (2). Assuming that none of the parties are constrained in this stage either; we can use the optimality condition $\frac{\partial W_i}{\partial g_i} = 0$. The first order conditions of this maximization problem are given as follows:

$$\frac{\partial W_1}{\partial g_1} = \frac{g_3}{\left(\sum_{i \in I} g_i\right)^2}\frac{(1-\varphi)\,\widetilde{R}}{4} - 1 = 0 \tag{15}$$

$$\frac{\partial W_2}{\partial g_2} = \frac{g_3}{\left(\sum_{i \in I} g_i\right)^2} \frac{(1-\varphi)\widetilde{R}}{4} - 1 = 0 \tag{16}$$

$$\frac{\partial W_3}{\partial g_3} = \frac{g_1 + g_2}{\left(\sum_{i \in I} g_i\right)^2} (1-\varphi)\widetilde{R} - 1 = 0 \tag{17}$$

Using the first order conditions stated above, we are able to determine the reaction of each player as a function of the rest of the players' war efforts.[3] The reaction functions implied by the equations (15) to (17) are given as follows.

$$g_1(g_2, g_3) = \sqrt{\frac{(1-\varphi)\widetilde{R}}{4} g_3} - (g_2 + g_3) \tag{18}$$

$$g_2(g_1, g_3) = \sqrt{\frac{(1-\varphi)\widetilde{R}}{4} g_3} - (g_1 + g_3) \tag{19}$$

$$g_3(g_1, g_2) = \sqrt{(1-\varphi)\widetilde{R}(g_1 + g_2)} - (g_1 + g_2) \tag{20}$$

In Figures 2 and 3, the plots of the reaction functions of the first member of the alliance and the single party, respectively, are presented. First of all, realize that the reaction functions of the members of the alliance are identical. (18) and (19) together imply the following function of the total effort put by the alliance.

Realize that, any amount of effort put by the members of the alliance that satisfies equation (21) would be an equilibrium outcome. Therefore we have infinite number of equilibria. At this point we prefer to occupy a refinement. As Garfinkel and Skaperdas (2007) also states, because of the fact that the members of the alliance are identical, i.e., they value the resource equally; it would be reasonable to assume that the efforts put would be the same in equilibrium. That is also the reason for figure 2 to be drawn in two dimensions.

$$g_1 + g_2 = \sqrt{\frac{(1-\varphi)\widetilde{R}}{4} g_3} - g_3 \tag{21}$$

[3] The second order conditions for each player are given by $\frac{\partial^2 W_1}{\partial g_1^2} = \frac{-g_3(1-\varphi)\widetilde{R}}{2\left(\sum_{i \in I} g_i\right)^3}$, $\frac{\partial^2 W_2}{\partial g_2^2} = \frac{g_3(1-\varphi)\widetilde{R}}{2\left(\sum_{i \in I} g_i\right)^3}$, $\frac{\partial^2 W_3}{\partial g_3^2} = \frac{-2(g_1+g_2)(1-\varphi)\widetilde{R}}{\left(\sum_{i \in I} g_i\right)^3}$. All three expressions are negative, hence a maximum is guaranteed.

Figure 2: Reaction function (20)

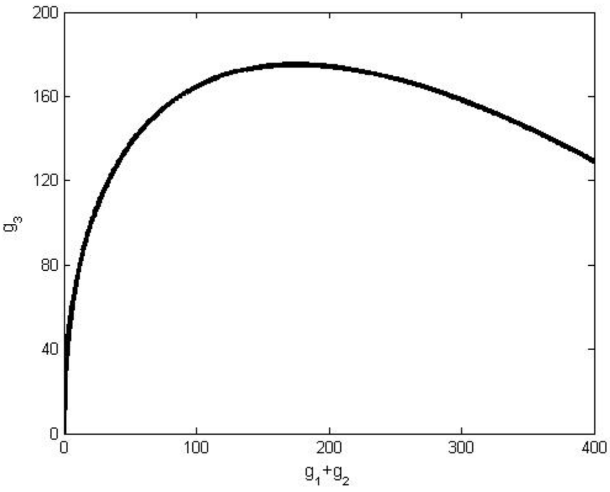
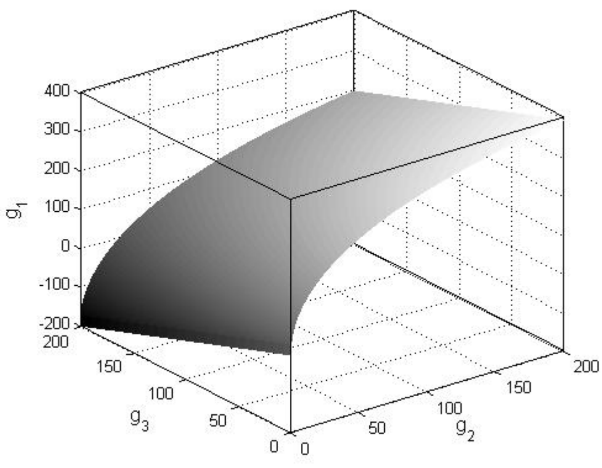


Figure 3: Reaction function (18)



Now, using equation (20) and (21), we can determine the equilibrium war effort of the third party. Using equation (21) and substituting it to the reactipn function of the single party results in the following expression.

$$g_3 = \sqrt{(1-\varphi)\,\widetilde{R}\left(\sqrt{\frac{(1-\varphi)\,\widetilde{R}}{4}g_3} - g_3\right)} - \sqrt{\frac{(1-\varphi)\,\widetilde{R}}{4}} + g_3 \tag{22}$$

Simplifying the equation above and solving for g_3 provides the following equilibrium war effort put forward by the single party in the third stage.

$$g_3^* = \frac{4(1-\varphi)\widetilde{R}}{25} \tag{23}$$

Using equation (23) and substituting this expression into equation (21) invokes the following total equilibrium war efforts put by the alliance.

$$(g_1 + g_2)^* = \frac{(1-\varphi)\widetilde{R}}{25} \tag{24}$$

Because of the refinement we made, the members of the alliance would put the same amount of effort to satisfy the equation (21). Hence, the resources spent for the by each member of the alliance is given as follows.

$$g_1^* = g_2^* = \frac{(1-\varphi)\widetilde{R}}{50} \tag{25}$$

Now; using the equations (23) and (25), and substituting the equilibrium war efforts spent by each $i \in I$ into (5)-(7), we are able to state the overall expected payoff of the conflict for each party as follows:

$$W_1^e = V_1 + \frac{3(1-\varphi)\widetilde{R}}{100} \tag{26}$$

$$W_2^e = V_2 + \frac{3(1-\varphi)\widetilde{R}}{100} \tag{27}$$

$$W_3^e = V_3 + \frac{16(1-\varphi)\widetilde{R}}{25} \tag{28}$$

It is obvious from equations (26)-(28) that without the existence of a binding peace agreement, both members of the alliance has incentives to declare war in the second stage, while $V_1 < V_1 + \frac{3(1-\varphi)\widetilde{R}}{100}$ and $V_2 < V_2 + \frac{3(1-\varphi)\widetilde{R}}{100}$. On the other hand, the single party obviously prefers to settle in peace because $V_3 + \frac{16(1-\varphi)\widetilde{R}}{25} < V_3 + \widetilde{R}$. These results imply that, if the transfers are not possible, the war will take place in the third stage.

Since it is certain that the war will be waged, some resources would be destroyed in the warfare and some resources would be spent unproductively on war efforts. It is illuminative to determine the welfare effects of warfare. Using the results of this analysis, one can also compare the welfare effects of different settlement rules and peace agreements. Denoting the total welfare as W, the expected welfare effects of the war could be stated as follows:

$$\Delta W = V_1 + V_2 + V_3 + \widetilde{R} - (V_1 + V_2 + V_3 + \frac{6(1-\varphi)\widetilde{R}}{100} + \frac{16(1-\varphi)\widetilde{R}}{25}) = \frac{(3+7\varphi)}{10}\widetilde{R} \quad (29)$$

Realize that even if the destruction parameter is zero, i.e., no resource is destroyed in war, the parties spend an amount equal to 30% of the resource under attack in arming.

Let us consider now the case where transfer from the single party to the members of the alliance is possible. Realize that in this situation, the availability of transfers from the members of the alliance to the single party will not matter, as equations (26) and (27) imply. The settlement in peace is possible if the following set of inequalities hold.

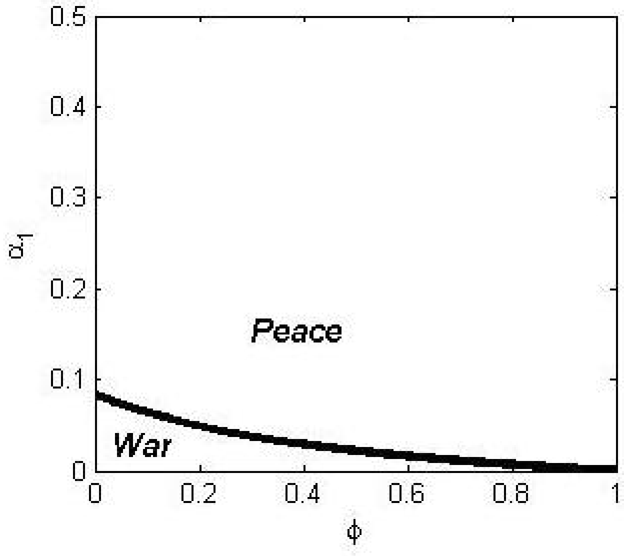
$$\alpha_1 T_1 \geq \frac{3(1-\varphi)\widetilde{R}}{100} \quad (30)$$

$$\alpha_2 T_2 \geq \frac{3(1-\varphi)\widetilde{R}}{100} \quad (31)$$

$$\frac{(9+16\varphi)\widetilde{R}}{25} \geq T_1 + T_2 \quad (32)$$

The first two expressions are the conditions of the first and the second members of the alliance to settle in peace, respectively. The last inequality is the condition for the single party to consider settling in peace instead of responding to the war declared on him. As we assumed that the decision of both parties in the allience to declare war is necessary for war to be waged, the single party gains the advantage of making transfers only to one party; namely, to the party that he can transfer his resources more easily. Let us assume wlog, $\alpha_1 > \alpha_2$. Then, this would imply that $T_2 = 0$. Hence, we are able to reduce (30)-(32)

Figure 4: Condition for peace to hold in equilibrium



to the inequality given below.

$$\frac{(9+16\varphi)\widetilde{R}}{25} \geq T_1 \geq \frac{3(1-\varphi)\widetilde{R}}{100\alpha_1} \tag{33}$$

(33) implies the condition $\alpha_1 \geq \frac{3(1-\varphi)}{4(9+16\varphi)}$. This is the condition for the peace to hold in equilibrium under a binding peace agreement. The single party would not commit to excess spending, then $T_1 = \frac{3(1-\varphi)\widetilde{R}}{100\alpha_1}$, given that α_1 satisfies the condition. Below, we plot this relation as a function of the destruction parameter, φ. As one can realize from Figure 4, for peace to hold in equilibrium in that case, a relatively low value of α_1 is sufficient. Now, in the existence of a transfer mechanism, we are able to determine the probability of war by assuming some properties about the distribution of α_1 and α_2. First of all, let us assume that these transfer parameters are independently identically distributed according to a beta distribution with parameters γ and β.[4] The reason we choose to use a beta distribution is, the domain of the cumulative distribution function of the beta distribution[5] is exactly the same with the domain of the transfer parameters, α_1 and α_2. Moreover, one can obtain a variety distribution properties with just switching through the parameters γ and β. For comparison we also use a continuous uniform distribution with

[4]To avoid confusion with the transfer parameters we chose γ instead of α, which is the customary one used in literature for the first parameter of beta distribution.

[5] $F(x;\gamma,\beta) = \frac{\int_0^x t^{\gamma-1}(1-t)^{\beta-1}dt}{\int_0^1 t^{\gamma-1}(1-t)^{\beta-1}dt}$

parameters $a = 0$ and $b = 1$.

With the result obtained, it is clear that the war would be waged if and only if $\alpha_1, \alpha_2 \leq \frac{3(1-\varphi)}{4(9+16\varphi)}$. Since we assumed that α_1 and α_2 are IID variables, the probability of war is given by the following expression.

$$Pr(War) = Pr\left(\alpha_1 \leq \frac{3\,(1-\varphi)}{4\,(9+16\varphi)}\right) Pr\left(\alpha_2 \leq \frac{3\,(1-\varphi)}{4\,(9+16\varphi)}\right) \tag{34}$$

In the Table 1, we calculated the probability of war under five different destruction parameters, φ, and for four different probability distributions. Realize that even with a highly right skewed distribution and with a very low destruction parameter, e.g. beta distribution with parameters $\gamma = 1$, $\beta = 10$ and $\varphi = 0.05$, the probability of war never exceeds 0.28. Hence, it is easy to realize the inexorability of war in the absence of a binding transfer mechanism drops down dramatically to a small probability when the transfers are allowed.

Distribution	Parameters	φ	α threshold	Probability of war
$\alpha_1, \alpha_2 \sim U[a, b]$	$a = 0, b = 1$	0.05	0.073	0.005
		0.35	0.033	0.001
		0.50	0.022	0.000
		0.65	0.014	0.000
		0.95	0.002	0.000
$\alpha_1, \alpha_2 \sim B(\gamma, \beta)$	$\gamma = 1, \beta = 5$	0.05	0.073	0.100
		0.35	0.033	0.024
		0.50	0.022	0.011
		0.65	0.014	0.005
		0.95	0.002	0.000
$\alpha_1, \alpha_2 \sim B(\gamma, \beta)$	$\gamma = 0.5, \beta = 0.5$	0.05	0.073	0.030
		0.35	0.033	0.014
		0.50	0.022	0.009
		0.65	0.014	0.006
		0.95	0.002	0.000
$\alpha_1, \alpha_2 \sim B(\gamma, \beta)$	$\gamma = 1, \beta = 10$	0.05	0.073	0.282
		0.35	0.033	0.081
		0.50	0.022	0.040
		0.65	0.014	0.018
		0.95	0.002	0.000

Table 1: Probability of war under different destruction and transfer parameters

4 Equilibrium under a non-binding peace agreement

The results obtained from the benchmark analysis reveals the fact that the war is unavoidable when the peace agreement obtained by transfers is non-binding, because of the fact that any member of the alliance who received the transfer would still have -even larger-incentives to declare war to the single party. Beviá and Corchón (2009) used resource constraints to enable the rich party in enforcing the poor party to settle in peace without the necessity of a binding agreement. However, the non-binding peace agreements in our model could still hold with a slight modification in assumption 3, which we state below.

Assumption 3': *If the single party is the victor he reclaims his resource under attack minus the part destroyed in warfare, and his transfer back multiplied by a per unit tax* $r \in [0, 1]$.

Realize that, Assumption 3' is not a modification, but an addition to Assumption 3 instead. In the presence of a binding agreement, once the single party transfers some resources to any ally, the ally is unable to declare war on him. However, Assumption 3 in the absence of a binding agreement is incomplete. Therefore Assumption 3' consummates the incomplete arguement in Assumption 3 when the agreement between parties is not binding. Moreover, with this addition the strength of the single party to enforce peace to the alliance changes structure. In the first part, the enforcement tool was the binding agreement, now with this addition the strength changes shape to the ability of claiming back the transfer sent, in case of victory. This assumption could be interpreted as a post-war settlement rule. Throughout history once an attack was repelled, it has been a common practice for the victors to claim back the war efforts or war destruction costs from the defeated parties. This modification could be attributed to a similar practice. We assume $r \in [0, 1]$, to allow for any imperfection concealed in this practice.

With this modification in Assumption 3, we begin solving the game starting from the last stage, given that a transaction of transfers took place. In this section we denote the undestroyed part of the resource, $(1 - \varphi)\widetilde{R}$, by ω for simplifying expressions. Because of the fact that the single party would transfer resources to only one member of the alliance, the last stage payoffs for each member of the alliance could be stated as follows.

$$w_1 = V_1 - g_1 - s_1 + \alpha_1 T_1 + \sigma_1(s_1, s_2)\,\omega \tag{35}$$

$$w_2 = V_2 - g_2 - s_2 + \sigma_2(s_1, s_2)\,\omega \tag{36}$$

Each $j \in A$ maximizes (35) and (36), respectively, subject to (4). Assuming that none of the parties are constrained in their resources, we are able to use the optimality condition $\frac{\partial w_j}{\partial s_j} = 0$ for each $j \in A$. The first order conditions for the problem are presented below.

$$\frac{\partial w_1}{\partial s_1} = \frac{s_2}{\left(\sum_{j \in A} s_j\right)^2}\omega - 1 = 0 \tag{37}$$

$$\frac{\partial w_2}{\partial s_2} = \frac{s_1}{\left(\sum_{j \in A} s_j\right)^2}\omega - 1 = 0 \tag{38}$$

Using the first order conditions presented above, one may reach the following reaction functions.

$$s_1 = \sqrt{s_2\omega} - s_2 \tag{39}$$

$$s_2 = \sqrt{s_1\omega} - s_1 \tag{40}$$

With the assumption of unconstrained parties one could reach the following equilibrium war efforts put by the members of the alliance in the fourth stage.

$$s_1^* = s_2^* = \frac{\omega}{4} \tag{41}$$

Using equations (35) and (36), and the equilibrium war efforts in the fourth stage, equilibrium payoff in the last stage for each $j \in A$ is given as follows.

$$w_1^* = V_1 - g_1 + \alpha_1 T_1 + \frac{\omega}{4} \tag{42}$$

$$w_2^* = V_2 - g_2 + \frac{\omega}{4} \tag{43}$$

Determining the last stage payoffs, now we are able to state the overall payoff of the conflict for each $i \in I$. Given our assumption, any member of the alliance that received the transfer has to pay it back in the case of a defeat. Hence, we can state the overall payoff of parties as follows.

$$W_1 = p_A\left(V_1 - g_1 + \alpha_1 T_1 + \frac{\omega}{4}\right) + (1 - p_A)\left(V_1 - g_1 + \alpha_1 T_1 - rT_1\right)$$

$$W_2 = p_A\left(V_2 - g_2 + \frac{\omega}{4}\right) + (1-p_A)\left(V_2 - g_2\right)$$

$$W_3 = p_S\left(V_3 - g_3 + (r-1+\omega)T_1\right) + (1-p_S)\left(V_3 - g_3 - T_1\right)$$

Working through the equations above and using the CSF, equation (2), one reaches the overall payoff of the conflict for each $i \in I$, which are stated below.

$$W_1 = V_1 - g_1 + (\alpha_1 - r)\,T_1 + \frac{g_1+g_2}{\sum_{i\in I} g_i}\left(\frac{\omega}{4} + rT_1\right) \tag{44}$$

$$W_2 = V_2 - g_2 + \frac{g_1+g_2}{\sum_{i\in I} g_i}\left(\frac{\omega}{4}\right) \tag{45}$$

$$W_3 = V_3 - g_3 - T_1 + \frac{g_3}{\sum_{i\in I} g_i}(\omega + rT_1) \tag{46}$$

Each $i \in I$ maximizes (44)-(46). Since none of the parties are constrained in their resources, the first order conditions for this optimization problem are given as follows.

$$\frac{\partial W_1}{\partial g_1} = \frac{g_3}{\left(\sum_{i\in I} g_i\right)^2}\left\{\frac{\omega}{4} + rT_1\right\} - 1 \leq 0 \tag{47}$$

$$\frac{\partial W_2}{\partial g_2} = \frac{g_3}{\left(\sum_{i\in I} g_i\right)^2}\left\{\frac{\omega}{4}\right\} - 1 \leq 0 \tag{48}$$

$$\frac{\partial W_3}{\partial g_3} = \frac{g_1+g_2}{\left(\sum_{i\in I} g_i\right)^2}\left\{\omega + rT_1\right\} - 1 \leq 0 \tag{49}$$

Even though none of the parties are constrained in their resources, because of the possibility of free riding in the alliance, it is safer to use first order conditions with an inequality for that case. The equations below imply the following equilibrium war effort relations.

$$\sqrt{g_3\left\{\frac{\omega}{4} + rT_1\right\}} - g_3 \leq g_1 + g_2 \tag{50}$$

$$\sqrt{g_3\left\{\frac{\omega}{4}\right\}} - g_3 \leq g_1 + g_2 \tag{51}$$

$$\sqrt{(g_1+g_2)\left\{\omega + rT_1\right\}} - (g_1+g_2) \leq g_3 \tag{52}$$

Notice that first two equations imply that the player two of the alliance free rides in that stage, i.e., $g_2^* = 0$. Moreover since none of the parties are constrained in their resources equations (50) and (52) hold with equality. Thus, we can reduce the equations above as follows.

$$g_1 = \sqrt{g_3\left(\frac{\omega}{4} + rT_1\right)} - g_3 \tag{53}$$

$$g_3 = \sqrt{g_1\left(\omega + rT_1\right)} - g_1 \tag{54}$$

Using the equations above we find the relation between equilibrium war efforts put by the first and the third party in the third stage.

$$g_3 = \frac{4\omega + 4rT_1}{\omega + 4rT_1} g_1 \tag{55}$$

Substituting this relation into the reaction functions of player three, and player one we reach the following equilibrium war efforts put by the first and the third parties.

$$g_1^* = \frac{\left(\omega + 4rT_1\right)^2\left(\omega + rT_1\right)}{\left(5\omega + 8rT_1\right)^2} \tag{56}$$

$$g_3^* = \frac{4\left(\omega + 4rT_1\right)^2\left(\omega + rT_1\right)}{\left(5\omega + 8rT_1\right)^2} \tag{57}$$

Finally, substituting the expressions above into the overall payoff for the party one and party three, one finds the overall equilibrium payoffs of the conflict, W_k^c for each $k \in \{1, 3\}$.

$$W_1^c = V_1 + \left(\alpha_1 - r\right)T_1 - \frac{\omega^3 + 9\omega^2 rT_1 + 24\omega r^2 T_1^2 + 16r^3 T_1^3}{25\omega^2 + 80\omega rT_1 + 64r^2 T_1^2} + \frac{\omega^4 + 13\omega^3 rT_1 + 60\omega^2 r^2 T_1^2 + 40\omega r^3 T_1^3 + 64r^4 T_1^4}{20\omega^3 + 144\omega^2 rT_1 + 240\omega r^2 T_1^2 + 128r^3 T_1^3} \tag{58}$$

$$W_3^c = V_3 - T_1 - \frac{4\omega^3 + 24\omega^2 rT_1 + 36\omega r^2 T_1^2 + 16r^3 T_1^3}{25\omega^2 + 80\omega rT_1 + 64r^2 T_1^2} + \frac{4\omega^4 + 28\omega^3 rT_1 + 60\omega^2 r^2 T_1^2 + 52\omega r^3 T_1^3 + 16r^4 T_1^4}{5\omega^3 + 36\omega^2 rT_1 + 60\omega r^2 T_1^2 + 32r^3 T_1^3} \tag{59}$$

Now, we know that the war depends on the choice of the first player. Receiving no amount of transfers and providing zero effort in the third stage, the second member of the alliance is always willing to engage in warfare. Below, the payoffs under settlement in peace are presented for the first and third player.

$$W_1^p = V_1 + \alpha_1 T_1 \tag{60}$$

$$W_3^p = V_3 + \widetilde{R} - T_1 \tag{61}$$

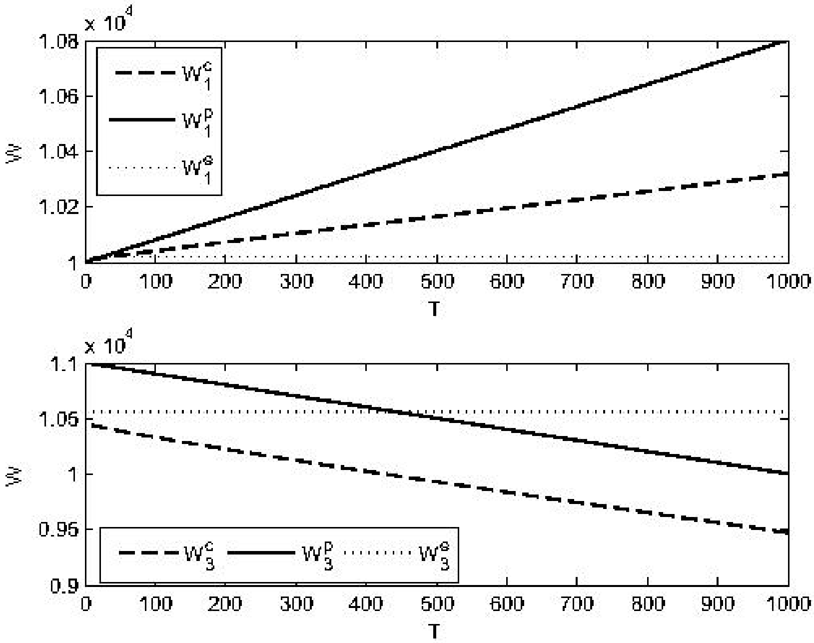
Given the parameters $\widetilde{R}$, φ, r; and any positive amount of transfer transaction, T_1, took place in the first stage; T_1 should be such that, for each $k \in \{1, 3\}$, settling in peace is at least as good as open warfare. Moreover, for peace to hold in equilibrium, it should also be guaranteed that, T_1 maximizes the payoff of the third party under peace. Hence, peace holds in equilibrium iff:

$$\begin{gathered} \exists T_1^* \in argmax_{T_1 \in \Re_+} W_3^p \\ s.t \\ W_1^p \geq W_1^c \\ W_1^p \geq W_1^e \\ W_3^p \geq W_3^e \end{gathered}$$

In Figure 11, we plot each of the functions stated above for $\widetilde{R} = 1000$, and for parameter values $r = 0.5$, $\varphi = 0.3$, and $\alpha_1 = 0.8$. One can observe that the payoffs of the first party under peace and conflict are decreasing functions of the transfers, i.e. the less the resources transferred, the more better off the single party is. Moreover, settling under peace is well above his payoff under conflict with transfers and under conflict without transfers. On the other side, the member of the alliance who took the transfer is also better of settling in peace whenever a small amount of transfer is sufficed. Therefore, we are able to state that peace holds in equilibrium under these specific parameter values. The table presented in the appendix[6] summarizes whether the peace holds in equilibrium, and how much transfer is transacted between parties, as a percentage of the resource under attack, $\widetilde{R}$. The table reveals the fact that peace holds in equilibrium unless r or α_1 is very low. The reason for the easiness for peace to hold in equilibrium with a small amount of transfer relies under the fact that, further conflict inside the alliance in the sixth stage discourages any member of the alliance put huge war efforts for the conflict with the single party. On the other hand, the single party is discouraged not only by the possibility of losing a territory and the transfers at the same time but also by the destruction that his land will suffer even under victory. Hence, one can instantly realize that each party has weak incentives for war to be waged.

[6] The values are chosen randomly, assuming that $\alpha_1 \in [0, 1]$, $r \in [0, 1]$, $\varphi \in [0, 1]$ are distributed identically and independently according to a continuous uniform distribution with parameters $a = 0$, and $b = 1$

Figure 5: Payoffs under peace and conflict



5 Equilibrium with cooperative allies

In this section we assume that the members of the alliance acts cooperatively in the war waged against the single party. However, given that the alliance is the victor, the members of the alliance engage in a conflict against each other, in which they share the resources according to the sharing rule, σ. While none of the members of the alliance are constrained in their resources, we assume that they would share the war efforts equally. The game is modified as follows.

If the alliance receives the transfer and declares war; the members of the alliance would gather the resource under attack minus the destroyed part of it, $(1-\varphi)\widetilde{R}$ or, ω, if they are victorious. Otherwise, they would pay the transfer back, multiplied with a per unit tax, $r \in [0,1]$. To be consistent with this assumption we made, we suppose that the transfers received and paid back in case of defiance of the alliance are shared equally among members. One could also assume that the cooperative allies share the transfers and war costs according to the amount of resources they own; however, as far as we are concerned, since parties do not have resource constraints this sharing rule will not be crucial for the analysis. We denote the total transfer received by the alliance with T, and the transfer parameter with α.

With this modification of the game, the CSFs for the alliance and the single party, denoting G as the total war effort put by the alliance, and g as the war effort put by the single party, are stated as follows.

$$p_A = \frac{G}{G+g} \tag{62}$$

$$p_S = \frac{g}{G+g} \tag{63}$$

Since we assumed that the conflict inside the alliance, in case the alliance is victorious against the single party, has the same features with the preceding sections, the fourth stage analysis results in the same last stage equilibrium arming and last stage equilibrium payoffs in the fourth stage. Then the overall payoffs of each $i \in I$ are given as follows.

$$W_1 = p_A\left\{V_1 - \frac{G}{2} + \frac{\alpha T}{2} + \frac{\omega}{4}\right\} + (1-p_A)\left\{V_1 - \frac{G}{2} + \frac{\alpha T}{2} - \frac{rT}{2}\right\}$$

$$W_2 = p_A \left\{ V_2 - \frac{G}{2} + \frac{\alpha T}{2} + \frac{\omega}{4} \right\} + (1 - p_A) \left\{ V_2 - \frac{G}{2} + \frac{\alpha T}{2} - \frac{rT}{2} \right\}$$

$$W_3 = p_S \{V_3 - g + (r-1)T + \omega\} + (1 - p_S)\{V_3 - g - T\}$$

Reducing the expressions above and using the CSFs (64) and (65), we reach the following overall payoff functions.

$$W_1 = V_1 - \frac{G}{2} + \frac{(\alpha - r)T}{2} + \frac{G}{G+g} \left\{ \frac{\omega}{4} + \frac{rT}{2} \right\} \tag{64}$$

$$W_2 = V_2 - \frac{G}{2} + \frac{(\alpha - r)T}{2} + \frac{G}{G+g} \left\{ \frac{\omega}{4} + \frac{rT}{2} \right\} \tag{65}$$

$$W_3 = V_3 - g - T + \frac{g}{G+g} \{\omega + rT\} \tag{66}$$

Each $i \in I$ maximizes W_i. While none of the parties are constrained in their resources, and we have already substituted the CSFs into the maximization problem, the problem turns into an unconstrained maximization problem. The first order conditions for this optimization problem are given as follows.

$$\frac{\partial W_1}{\partial G} = -\frac{1}{2} + \frac{g}{(G+g)^2} \left\{ \frac{\omega}{4} + \frac{rT}{2} \right\} = 0 \tag{67}$$

$$\frac{\partial W_2}{\partial G} = -\frac{1}{2} + \frac{g}{(G+g)^2} \left\{ \frac{\omega}{4} + \frac{rT}{2} \right\} = 0 \tag{68}$$

$$\frac{\partial W_3}{\partial g} = -1 + \frac{g}{(G+g)^2} \left\{ \frac{\omega}{4} + \frac{rT}{2} \right\} = 0 \tag{69}$$

Using either one of the equations (69) and (70) we find the following reaction function for the alliance:

$$\sqrt{2g \left\{ \frac{\omega}{4} + \frac{rT}{2} \right\}} - g = G \tag{70}$$

Moreover, using the first order condition for the third party:

$$\sqrt{G\{\omega + rT\}} - G = g \tag{71}$$

Using these two reaction functions (72) and (73), one finds the relation between the war efforts put by the alliance and the single party.

$$G = \left(\frac{\frac{\omega}{2} + rT}{\omega + rT} \right) g \tag{72}$$

Now, using this result and substituting it into the equations (72) and (73), one can find the equilibrium war efforts of the alliance and the single party, respectively, as follows.

$$G^* = \frac{\left(\frac{\omega}{2} + rT\right)^2 (\omega + rT)}{\left(\frac{3\omega}{2} + 2rT\right)^2} \tag{73}$$

$$g^* = \frac{\left(\frac{\omega}{2} + rT\right) (\omega + rT)^2}{\left(\frac{3\omega}{2} + 2rT\right)^2} \tag{74}$$

Now, substituting the expressions above into overall payoffs; one can find the overall payoff of the conflict for each $i \in I$.

$$W_1^c = V_1 + (\alpha - r)\frac{T}{2} + \frac{\omega^3 + 6\omega^2 rT + 12\omega r^2T^2 + 8r^3T^3}{36\omega^2 + 96\omega rT + 64r^2T^2} \tag{75}$$

$$W_2^c = V_2 + (\alpha - r)\frac{T}{2} + \frac{\omega^3 + 6\omega^2 rT + 12\omega r^2T^2 + 8r^3T^3}{36\omega^2 + 96\omega rT + 64r^2T^2} \tag{76}$$

$$W_3^c = V_3 - T + \frac{4\omega^3 + 12\omega^2 rT + 12\omega r^2T^2 + 8r^3T^3}{9\omega^2 + 24\omega rT + 16r^2T^2} \tag{77}$$

For peace to hold in equilibrium we need to state the payoffs of the parties under settlement in peace:

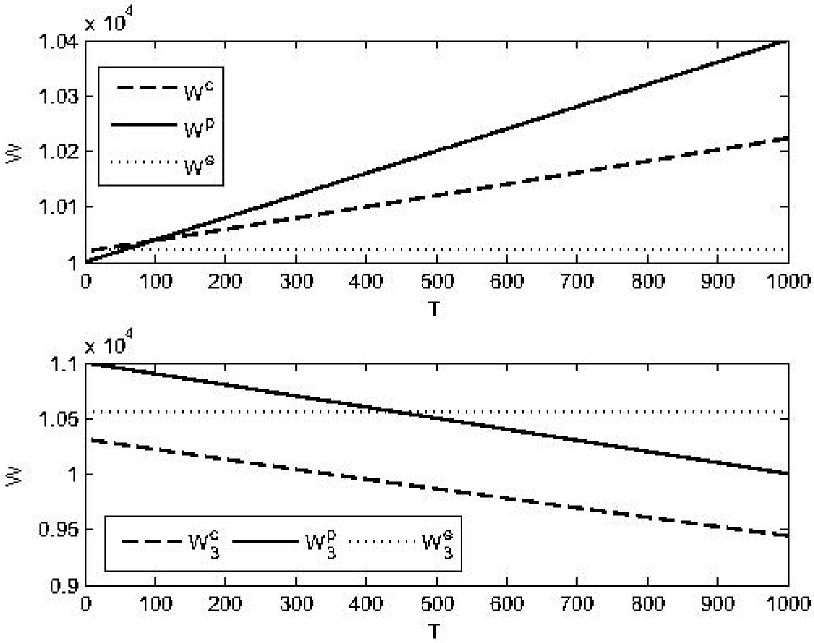
$$W_1^p = V_1 + \frac{\alpha T}{2} \tag{78}$$

$$W_2^p = V_2 + \frac{\alpha T}{2} \tag{79}$$

$$W_3^p = V_3 + \widetilde{R} - T \tag{80}$$

For peace to hold in equilibrium, the same set of conditions stated in section four should be satisfied again for each $i \in I$. However, realize that the relations for members of the alliance are identical. Thus, the conditions reduce into two. In figure 18, the payoffs for any member of the alliance and the single party are presented as a function of the transfers received for $\widetilde{R} = 1000$, and for parameter values $r = 0.5$, $\varphi = 0.3$, and $\alpha_1 = 0.8$. Combined with figure 18, the table 3 in the appendix shows that peace again holds in equilibrium unless, r, α are not insufficiently low. However, the amount of the transfer required to enforce the peace are always larger than the one in the case with non-cooperative allies. That mainly evolves from the removal of the free-riding problem inside the alliance, which consequently results in a firmer standing alliance against the single party in a possible war waged.

Figure 6: Payoffs under peace and conflict



6 Conclusions

Throughout history war has been a last resort for settling conflicts down. However, the investments and human resources spent in warfare are irrecoverable, that is the reason why it has become a tempting issue to understand the rational explanations of war. However, surprisingly, the interest in avoidance from war was not studied in depth.

Our motivation in this paper was to find out if transfers could be used to sustain peace and avoid excessive resources spent in warfare in a rather specific model of conflict. Concordantly, one may ask the question whether or not the assumptions and rules of the game are far too restrictive. However, as far as I am concerned, it is illuminating to investigate the conditions for war to be eliminated and henceforth the certain vital resources of the nations are not spent unproductively in a simplistic and primitive model. Moreover, as historical examples shows the observation of various multiple numbered allies to wage war against a single party. Therefore, the analysis we considered could be applied to analyze the disputes of that kind.

We find in this primitive game that the peace would hold with a binding peace agreement in equilibrium as long as the largest transfer parameter is not very close to zero. In other words for the case that it is really hard for the single party to transfer its resources to the members of the alliance, this could be attributed to geographical or ethnical differences between the opposing parties. On the other hand, whenever the larger transfer parameter gets sufficiently large, the war could almost always be avoided.

To be in line with the historical examples we have given, we also wanted to analyze a situation in which there exists a resource under attack which belong to a single party and it is threatened by an alliance. We have also found out that due to further conflict inside the alliance, single parties could easily enforce the members of the alliance to settle in peace, even without the necessity of a binding peace agreement and constrained parties, by a relatively small amount of transfer.

Recall, we stated that one can consider a different post-war settlement rule, i.e. peace agreements, to analyze the same situation. Actually, I consider this extension as part of my further research. As a short proposal, by examining the peace agreements throughout the history of conflicts, one would be able to classify the peace agreements, and analyze conflicts with different classes of post-war settlement rules. As far as I am concerned this would help the audience to grasp the decisions taken by the opposing parties in wars fully and would provide a worthy contribution to conflict management.

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7 Appendix

α	r	φ	Peace Non-Cooperative A	Peace Cooperative A	Transfer ($\widetilde{R}\%$) Non-Cooperative A	Transfer ($\widetilde{R}\%$) Cooperative A
0.95	0.62	0.06	Yes	Yes	3.0	10.4
0.23	0.79	0.35	Yes	Yes	9.0	17.0
0.61	0.92	0.81	Yes	Yes	1.0	1.9
0.49	0.74	0.01	Yes	Yes	6.1	12.2
0.89	0.18	0.14	Yes	Yes	5.1	32.8
0.76	0.41	0.20	Yes	Yes	3.2	13.4
0.46	0.94	0.20	Yes	Yes	5.3	10.5
0.02	0.92	0.60	Yes	No	60.0	-
0.82	0.41	0.27	Yes	Yes	2.7	12.2
0.44	0.89	0.20	Yes	Yes	5.5	10.9
0.01	0.42	0.83	Yes	No	51.0	-
0.75	0.85	0.02	Yes	Yes	4.0	7.9
0.45	0.53	0.68	Yes	Yes	2.2	4.3
0.93	0.20	0.38	Yes	Yes	3.3	21.3
0.01	0.04	0.23	No	No	-	-

Table 2: Settlement in peace and amount of transfers transacted