1 Introduction

Elimination tournaments are frequently used for incentive provision and to select the most appropriate candidates for promotion. In real world applications, tournaments typically involve heterogeneous agents that compete on several stages, for instance in settings where it is impracticable or undesirable to have all participants compete with each other at once. While elimination tournaments with multiple stages and a homogenous pool of participants, have been analyzed extensively in the literature, fairly little is known about the properties of different settings of multistage tournaments when participants are heterogeneous. For example, is it better in terms of incentive provision to have unequal agents compete early on in the tournament? Or should a principal design the tournament to have equal workers compete with each other in the early stages? And what are the consequences for selection, i.e., for the probability that the best participant wins the entire tournament? Addressing these questions has direct implications for practical applications and provides valuable information about the optimal design of tournament structures for different purposes.

This paper addresses these questions and thereby contributes to the literature in several ways. The theoretical investigation of the incentive and selection properties of multi-stage tournaments with homogeneous and heterogeneous agents extends the seminal multi-stage pairwise elimination tournament structure investigated by Rosen (1986) and generalizes the results with respect to the consideration of heterogeneous agents. In terms of heterogeneity, we consider the simplest case with four agents that differ with respect to their effort effectiveness, prize valuations or effort costs, where two agents are strong and two are weak. To address the question of optimal tournament design, we investigate the properties of a two-stage pairwise elimination tournament, where the "semi-finals" can either be such that equal types compete with each other, or where mixed couples compete to reach the final stage that grants a prize for the overall winner. We find that in terms of incentive provision, a tournament among homogeneous agents delivers the highest total effort exertion. In heterogeneous settings, having equal types compete on the first stage leads to higher effort provision than having heterogeneous types already competing on the first stage. In terms of the probability that a strong agent (i.e., the "best worker") wins the overall tournament, however, the mixed first stage setting dominates the setting with equal contestants. This implies that there is a fundamental trade-off between selection and incentive provision from the perspective of a tournament designer. The second main contribution of the paper is a systematic assessment of the empirical relevance of the theoretical results using experimental methods. We find that the theoretical predictions are confirmed by the behavioral outcomes in anonymous incentivized interactions in a computerized lab experiment.

The results have implications for many different applications of tournaments and contests. Many real life interactions between different persons, such as elections, rent-seeking games, promotion tournaments, or sports, can best be described and analyzed by use of a tournament (or contest) model. Most tournaments involve multiple stages where the number of participating agents is narrowed in successive stages until a winner is finally chosen. The most prominent among the models which share this feature is the sequential pairwise elimination tournament, which is commonly known from playoff rounds in sport competitions, but which is also a good description of many corporate tournaments where employees from low hierarchy levels compete for promotions to higher hierarchy levels. This tournament structure can also be encountered in politics, where, candidates run compete in localized contests and where the winners then often compete against each other on higher levels.[1]

Our paper contributes to an extensive theoretical and empirical literature that considers multi-stage pairwise elimination tournaments. The theoretical literature, starting with Rosen (1986), has mostly concentrated on settings with homogeneous agents. In his article, Rosen considers also heterogeneous settings, but does not derive analytical results. Among the recent contributions in this area, Stein and Rapoport (2005) investigate the effect of budget constraints in homogeneous two-stage pairwise elimination contests, while Fu and Lu (2009) determine the optimal structure of multi-stage contests in terms of maximizing incentive provision for effort exertion. Theoretical works that analyze the behavior of asymmetric agents in two-stage contests include Stein and Rapoport (2004), who consider the difference between 'semi-final' settings and 'between group' settings, in which heterogeneous teams that consist of homogeneous players within the team compete for a prize either by first eliminating all teams but the best performing one and then determining the winner of the prize within the winning team, or by determining the winning team member that then competes with the winners from all other teams on the final stage, respectively. Their paper thus only considers different orderings of within and between group competitions and thereby avoids the major complication that arises in the case of multi-stage competition between heterogeneous contestants and that the model below addresses explicitly, namely the the relevance of endogenously determined continuation values. Klumpp and Polborn (2006) also consider heterogeneous contestants in a multi-stage competition, but their contest structure considers repeated interaction among the same agents, rather than elimination of agents on each stage. Harbaugh and Klumpp (2005) use a similar contest structure as we do, but make use of the simplifying assumption that a fixed endowment, which is without intrinsic value to agents, is split across the two-stages of the contest game. Our paper also complements the findings of Orrison,

[1]The competition for U.S. presidency exhibits a related structure, where candidates first compete against their rival from their own party in primaries (which should therefore be rather homogeneous) and only meet their competitor from the other party in the second stage.

Schotter, and Weigelt (2004), who consider tournaments with homogeneous players but study the effects of different tournament designs in terms of handicapping, prize structure and tournament size. They also provide experimental evidence that is in line with their theoretical results. Also related is the work by Groh, Moldovanu, Sela, and Sunde (2010), who consider the case of heterogeneous optimizing agents in a pairwise elimination tournament in the specific case of an all-pay auction, i.e., the case of a perfectly discriminating contest success function, and discuss various optimality criteria for different tournament designs. We consider the complementary case of a tournament with a standard Tullock Contest-Success-Function (CSF) with discriminatory power one, which we also implement in the experimental setting. As a consequence, the equilibrium of our model is in pure-strategies, which facilitates the experimental testing of our results.

In terms of the empirical literature, Eriksson (1999), Bognanno (2001) as well as Sunde (2009) consider tournaments: While both Eriksson (1999) and Bognanno (2001) use firm data to analyze the tournament aspect of promotions, Sunde (2009) employs data from professional tennis tournaments. Even though the tournaments described in those papers are dynamic multi-stage interactions, their empirical analysis focuses on investigating the behavior in prototype static one time interaction settings due to data constraints. Specific aspects of multi-stage elimination tournaments are not considered. Controlled laboratory experiments allow for systematic ceteris paribus variations and therefore lend themselves better for an investigation of the empirical relevance of theoretical results. Considering multi-stage contests, Parco, Rapoport, and Amaldoss (2005) and Amegashie, Cadsby, and Song (2007) provide experimental evidence for the role of budget constraints. Closest to our paper are the contributions by Altmann, Falk, and Wibral (2008) and Sheremeta (2010) that investigate how the behavior of agents in laboratory experiments compares to the theoretical predictions from two-stage tournament models with homogeneous agents. Their findings indicate over-provision of effort in both stages, but in particular in stage 1. Our analysis extends theirs by considering the (arguably more relevant) case of heterogeneous agents.

The remainder is structured as follows. Section 2 presents the theoretical results. We describe our experimental procedures in section 3. Section 4 presents the experimental results, and 5 provides a summary and some concluding remarks.

2 Theoretical Results

We consider a two-stage pairwise elimination tournament with four risk-neutral agents. In this tournament, there are three pairwise interactions: Two interactions of two players each in stage 1, and one interaction of two players in stage 2. The winner of each interaction is determined by a random draw, where the winning probability of a given player is given by a standard Tullock contest success function (CSF) with discriminatory power one. This means, the probability that player i wins against player j is given by $p_i = \frac{z_i}{z_i+z_j}$, where z_i (z_j) denotes the effort of player i (j). We assume that the agents compete for a single prize P that is awarded to the winner of the stage 2 interaction, i.e., there is no reward for winning stage 1 except for participation in stage 2 which implies the chance to win the prize P in the stage 2 interaction.[2]

We consider agents of two different types: Agents can be either "weak" or "strong". Heterogeneity between the two types is modeled in terms of different effort costs:[3] While the costs per unit of effort are equal to $c_S = 1$ for strong agents, they amount to $c_W \geq c_S = 1$ for weak agents, such that weak agents have a strategic disadvantage as compared to strong ones. It is assumed throughout that all agents are perfectly informed about both their own type and the type of their co-players.

In our baseline specification (short A: `SSSS`), all four agents are of the "strong" type (`S`). Apart from that, we consider the case where two agents are "strong" (`S`) and two are "weak" (`W`). Note that two different tournament designs arise in this case: either both stage 1 interactions are homogeneous (B: `SSWW`), i.e., the two weak agents and the two strong agents, respectively, interact with each other on stage 1. Or both stage 1 interactions are heterogeneous, which implies that each strong agent meets a weak agent (C: `SWSW`). As will become clear below, the two settings in which agents are heterogeneous have quite different properties with respect to several criteria.

2.1 Solving the Model

The equilibrium concept for the two-stage tournaments is subgame perfect Nash, independent of the setting under consideration. The two-stage games are solved via backwards induction, i.e., we first solve all potential situations that can occur in stage 2, and subsequently solve stage 1, taking as given the optimal actions in stage 2. Recall that the only reward for winning stage 1 is the participation in stage 2. To derive the equilibrium, we need the expected equilibrium payoffs for both agent types and all potential stage 2 interactions, since they determine the continuation values (and thus the optimal strategies) in stage 1.

[2] This assumption is without consequence for the theoretical implications.

[3] Heterogeneity between agents can either be modeled in terms of effort cost, productivity of effort, or valuations of the prize. The theoretical results presented below hold independent of the source of heterogeneity. Proofs are available from the authors upon request.

2.1.1 Solution for Stage 2

With four players of two types, there are three potential stage 2 games, namely (1) SS, (2) WW, or (3) SW, each of which is considered below. Independent of the specific interaction, x_i denotes the stage 2 effort by agent i. Equilibrium efforts are marked with an asterisk.

(1) SS: If two arbitrary "strong" agents l and k compete with each other in stage 2, they both face the same maximization problem. Without loss of generality, we consider the optimization by agent l, who maximizes his expected payoff $\pi_l(\texttt{SS})$ by choosing an optimal level of effort x_l. Formally, the maximization problem reads

$$\max_{x_l \geq 0} \pi_l(\texttt{SS}) = \frac{x_l}{x_l + x_k} P - c_S x_l .$$

Using the normalization $c_S = 1$ for the cost of effort parameter, the first-order condition for agent l ($x_k P - (x_l + x_k)^2 = 0$) and a symmetry condition ($x_k^* = x_l^*$) delivers $x_l^* = x_k^* = \frac{P}{4}$. Inserting optimal actions in the objective function gives the payoff that a "strong" agent can expect if he meets another "strong" agent in stage 2.[4] This yields

$$\pi^*(\texttt{SS}) = \frac{P}{4} \, . \tag{1}$$

(2) WW: The symmetry argument with respect to the maximization problem does also hold if two arbitrary "weak" agents l and k compete with each other in stage 2. As before, we consider the optimization by player l without loss of generality: $\max_{x_l \geq 0} \pi_l(\texttt{WW}) = \frac{x_l}{x_l + x_k} P - c_W x_l$. Going through the same steps as previously, we derive equilibrium efforts $x_l^* = x_k^* = \frac{P}{4 c_W}$. Inserting equilibrium efforts in the objective function above yields the expected equilibrium payoff for a "weak" player in a stage 2 interaction WW, which amounts to:

$$\pi^*(\texttt{WW}) = \frac{P}{4} \, . \tag{2}$$

(3) SW: Finally, consider the situation where an arbitrary "strong" agent s meets an arbitrary "weak" agent w in stage 2. The agents solve the following optimization problems:

$$\begin{aligned} \max_{x_s \geq 0} \pi_s(\texttt{SW|S}) &= \frac{x_s}{x_s + x_w} P - x_s \, , \\ \max_{x_w \geq 0} \pi_w(\texttt{SW|W}) &= \frac{x_w}{x_s + x_w} P - c_W x_w \, . \end{aligned}$$

The combination of first-order conditions gives equilibrium efforts $x_s^* = \frac{c_W}{(1+c_W)^2} P$ for the strong agent and $x_w^* = \frac{1}{(1+c_W)^2} P$ for the weak agent, respectively. Inserting optimal actions in the two objective

[4] Note that the expected payoff is the same for player l and player k, therefore the indices can be dropped.

functions gives the expected payoffs for strong and weak agents in a stage 2 interaction $\texttt{SW}$:

$$\pi_s^*(\texttt{SW}|\texttt{S}) = \frac{c_W^2}{(1+c_W)^2} P\,, \quad (3)$$

$$\pi_w^*(\texttt{SW}|\texttt{W}) = \frac{1}{(1+c_W)^2} P\,. \quad (4)$$

2.1.2 Solution for Stage 1

With all potential stage 2 interactions solved, we proceed to compute the equilibrium in stage 1 of the tournament for the three specifications A: $\texttt{SSSS}$, B: $\texttt{SSWW}$ (homogeneous stage 1 interactions), and C: $\texttt{SWSW}$ (heterogeneous stage 1 interactions). We first consider the simplest setting $\texttt{SSSS}$, then the slightly more complicated situation $\texttt{SSWW}$, and finally setting $\texttt{SWSW}$ for which optimal behavior is somewhat harder to characterize. We denote the stage 1 effort by agent i ($i = 1, 2, 3, 4$) in setting n ($n = A, B, C$) by y_i^n; as before, equilibrium efforts are marked with an asterisk.

Setting A: $\texttt{SSSS}$ A solution for a two-stage tournament with four agents of the same type has already been presented and discussed several times due to its simplicity.[5] Note that the only potential outcome in stage 2 is the interaction $\texttt{SS}$. Consequently, all agents have the same continuation value in stage 1, which amounts to $\pi^*(\texttt{SS}) = \frac{P}{4}$ (see (1)). All agents face exactly the same strategic situation and make identical decisions in the unique symmetric equilibrium, such that it suffices to consider the optimization problem of one representative agent. We assume that agent 1 interacts with agent 2 in stage 1, which immediately implies that agents 3 and 4 compete in the second stage 1 interaction. We explicitly consider agent 1 who maximizes his expected payoff Π_1^A by choosing an optimal level of effort y_1^A in stage 1. Formally, his optimization problem can be described as follows:

$$\max_{y_1^A \geq 0} \Pi_1^A = \frac{y_1^A}{y_1^A + y_2^A} \pi^*(\texttt{SS}) - \quad y_1^A$$

Combination of the first-order condition and the symmetry condition $y_1^{A*} = y_2^{A*} = y_3^{A*} = y_4^{A*}$ leaves us with

$$y_1^{A*} = y_2^{A*} = y_3^{A*} = y_4^{A*} = \frac{P}{16} \quad (5)$$

Setting B: $\texttt{SSWW}$ Two-stage tournaments with this structure have also been analyzed in the literature before; in fact, setting B: $\texttt{SSWW}$ corresponds to what Stein and Rapoport (2004) refer to as "semifinals"

[5]A recent example is Sheremeta (2010).

model in their paper. This setting is straightforward to solve since one strong and one weak agent reach stage 2 with certainty, independent of stage 1 actions. Consequently, SW is the only possible constellation in stage 2, which implies that continuation values are easy to derive: Both strong agents know that, conditional on reaching stage 2, they will meet a weak agent there, while weak agents know that they interact with a strong agent in stage 2, if they win in stage 1.[6] We assume without loss of generality that agents 1 and 2 are strong, while agents 3 and 4 are weak. Then, the value of participation in stage 2 is given by (3) for agents 1 and 2, while the continuation value for agents 3 and 4 is defined by (4). Note that due to symmetry of the optimization problems, it suffices to solve the optimization problem of one strong agent (1 or 2) and one weak agent (3 or 4). Without loss of generality, we consider agents 1 and 3 and obtain:

$$\begin{aligned}\max_{y_1^B \geq 0} \Pi_1^B &= \frac{y_1^B}{y_1^B + y_2^B}\pi^*(\texttt{SW}|\texttt{S}) - \quad y_1^B \\ \max_{y_3^B \geq 0} \Pi_3^B &= \frac{y_3^B}{y_3^B + y_4^B}\pi^*(\texttt{SW}|\texttt{W}) - c_W y_3^B\end{aligned}$$

First-order and symmetry conditions give us the following stage 1 equilibrium efforts:

$$y_1^{B*} = y_2^{B*} = \frac{c_W^2}{4(1+c_W)^2}P \tag{6}$$

$$y_3^{B*} = y_4^{B*} = \frac{1}{4c_W(1+c_W)^2}P. \tag{7}$$

Setting C: SWSW Finally, we present the solution for setting SWSW. To the best of our knowledge, a closed-form solution for this setting in the context of a Tullock contest success function has not been presented in the literature.[7] As will become clear below, continuation values are endogenously determined in this setting which complicates the solution. Nevertheless, properties of this setting were previously discussed by Rosen (1986) who claimed (without proof) that certain properties which he derived from numerical simulations do hold in general.

As for setting SSWW, we again define that agents 1 and 2 are strong, whereas agents 3 and 4 are weak. Further, we assume that agents 1 and 3 as well as agents 2 and 4 interact in stage 1, i.e. each strong agents meets a weak agent, and vice versa. Note that the two stage 1 interactions are identical, i.e., it suffices to analyze one of the two.[8] Without loss of generality, we consider the interactions between

[6]Strong agents do not care which of the two weak agents they meet in stage 2, since any agent of type W chooses the same equilibrium strategy. An analogous argument holds for weak agents.

[7]The only paper that deals with endogenously determined continuation values is Groh, Moldovanu, Sela, and Sunde (2010), but they consider the limit case of an all-pay auction, i.e., where the contest-success function is perfectly discriminating.

[8]The unique equilibrium is symmetric and exists independent of the degree of heterogeneity. A proof is available upon

agents 1 and 3, who face the following maximization problems:

$$\max_{y_1^C} \Pi_1 = \frac{y_1^C}{y_1^C + y_3^C} \underbrace{\left[\frac{y_2^C}{y_2^C + y_4^C}\pi^*(\texttt{SS}) + \frac{y_4^C}{y_2^C + y_4^C}\pi^*(\texttt{SW|S})\right]}_{\equiv P_1(y_2^C, y_4^C)} - \ y_1^C,$$

$$\max_{y_3^C} \Pi_3 = \frac{y_3^C}{y_1^C + y_3^C} \underbrace{\left[\frac{y_2^C}{y_2^C + y_4^C}\pi^*(\texttt{SW|W}) + \frac{y_4^C}{y_2^C + y_4^C}\pi^*(\texttt{WW})\right]}_{\equiv P_3(y_2^C, y_4^C)} - c_W y_3^C \ .$$

Note that $P_1(y_2^C, y_4^C)$ and $P_3(y_2^C, y_4^C)$ denote the continuation values for agents 1 and 3, respectively, which depend on the behavior of agents 2 and 4 in the other stage 1 interaction, i.e. the continuation values are endogenously determined. The reason for this complication is that the tournament structure allows for three different stage 2 interactions, namely SS, WW, and SW, which are of different value to agents of both types and occur with certain probabilities.

Independent of this complication, the following two first-order conditions are still necessary equilibrium conditions:

$$\begin{aligned} y_3^C P_1(y_2, y_4) - \ (y_1^C + y_3^C)^2 &= 0 \\ y_1^C P_3(y_2, y_4) - c_W(y_1^C + y_3^C)^2 &= 0 \end{aligned}$$

If we combine those conditions, we obtain another necessary equilibrium condition which defines a relation between equilibrium actions of agents 1 and 3:

$$\frac{y_1^{C*}}{y_3^{C*}} = c_W \frac{P_1(y_2^C, y_4^C)}{P_3(y_2^C, y_4^C)} = \frac{(1 + c_W)^2 c_W y_2^C + 4c_W^3 y_4^C}{4y_2^C + (1 + c_W)^2 y_4^C} \ . \tag{8}$$

Recall that the two stage 1 interactions are identical. This implies that the conditions $y_1^{C*} = y_2^{C*}$ and $y_3^{C*} = y_4^{C*}$ do hold in the (unique symmetric) equilibrium. This gives the following quadratic equation in y_1^{C*} and y_3^{C*}:

$$\begin{aligned} \frac{y_1^{C*}}{y_3^{C*}} &= \frac{(1 + c_W)^2 c_W y_1^{C*} + 4c_W^3 y_3^{C*}}{4y_1^{C*} + (1 + c_W)^2 y_3^{C*}} \\ \Leftrightarrow \ 0 &= 4[y_1^{C*}]^2 + (1 - c_W)(1 + c_W)^2 y_1^{C*} y_3^{C*} - 4c_W^3 [y_3^{C*}]^2 \\ \Leftrightarrow \ 0 &= [y_1^{C*}]^2 + \frac{(1 - c_W)(1 + c_W)^2}{4} y_1^{C*} y_3^{C*} - c_W^3 [y_3^{C*}]^2 \\ \Leftrightarrow \ y_1^{C*} &= F^*(c_W) \times y_3^{C*}, \end{aligned}$$

request.

where

$$F^*(c_W) = \frac{(c_W - 1)(1 + c_W)^2 + \sqrt{64c_W^3 + (1 - c_W)^2(1 + c_W)^4}}{8}. \tag{9}$$

The above expression allows for an analytical solution of the game. Essentially, $F(c_W)$ is a measures for the additional heterogeneity between strong and weak agents in stage 1 that is due to differences in continuation values.[9] Thus, the two interdependent stage 1 interactions can be disentangled in equilibrium. The continuation values satisfy

$$\begin{aligned}
P_1(y_2^{C*}, y_4^{C*}) = P_2(y_1^{C*}, y_3^{C*}) &= \frac{(1 + c_W)^2 F^*(c_W) + 4c_W^2}{4(1 + c_W)^2[1 + F^*(c_W)]} P \\
P_3(y_2^{C*}, y_4^{C*}) = P_4(y_1^{C*}, y_3^{C*}) &= \frac{4F^*(c_W) + (1 + c_W)^2}{4(1 + c_W)^2[1 + F^*(c_W)]} P\,,
\end{aligned}$$

since, due to symmetry, $P_1(y_2^{C*}, y_4^{C*}) = P_2(y_1^{C*}, y_3^{C*})$ and $P_3(y_2^{C*}, y_4^{C*}) = P_4(y_1^{C*}, y_3^{C*})$. Given these continuation values, one can determine equilibrium efforts as

$$y_1^{C*} = y_2^{C*} = \frac{(1 + c_W)^2[F^*(c_W)]^2 + 4c_W^2 F^*(c_W)}{4(1 + c_W)^2[1 + F^*(c_W)]^3} P \tag{10}$$

$$y_3^{C*} = y_4^{C*} = \frac{(1 + c_W)^2 F^*(c_W) + 4c_W^2}{4(1 + c_W)^2[1 + F^*(c_W)]^3} P\,. \tag{11}$$

2.2 Properties of the Equilibrium Solutions

With the closed form solutions for all settings, we are in a position to compare their equilibrium properties. We do this by defining two performance measures that capture the properties of the tournament that appear to be most relevant for a tournament-designing principal, and compare settings A: `SSSS`, B: `SSWW` and C: `SWSW` in terms of their equilibrium properties in those performance dimensions.

Incentives: Elimination tournaments have been analyzed mostly as a means to provide participants with incentives for effort provision. As shown by Lazear and Rosen (1981), under certain conditions tournaments are optimal labor contracts, e.g., where they can help solve moral hazard problems. As a measure for the provision of incentives, we use total expected equilibrium effort provision, i.e., the sum of equilibrium efforts in stage 1 and (expected) equilibrium efforts in stage 2, which we denote by

[9] The continuation value for strong agents is always higher than the continuation value for weak agents, as the first-order conditions above clearly show.

$\mathcal{E}$. For setting A: SSSS, this incentive measure, is given by

$$\mathcal{E}_{\texttt{SSSS}} = \underbrace{\frac{P}{4}}_{\text{stage 1 effort}} + \underbrace{\frac{P}{2}}_{\text{stage 2 effort}} . \tag{12}$$

Similarly, we obtain $\mathcal{E}_{\texttt{SSWW}}$, the incentive measure for setting B: SSWW:

$$\mathcal{E}_{\texttt{SSWW}} = \underbrace{\frac{1+c_W^3}{2c_W(1+c_W)^2}}_{\text{stage 1 effort}} P + \underbrace{\frac{1}{1+c_W}}_{\text{stage 2 effort}} P \tag{13}$$

It is somewhat more tedious to compute the corresponding measure for setting C: SWSW, since three different interactions can occur in stage 2 with certain probabilities. Therefore, $\mathcal{E}_{\texttt{SWSW}}$ really is an *expected* value which has a positive variance from an ex-ante perspective. *Realized* ex-post equilibrium effort provision varies and depends on the structure of the stage 2 interaction; it is highest in case of the interaction SS which is observed with probability $[F^*(c_W)]^2/[1+F^*(c_W)]^2$, lowest in situation WW which realizes with probability $1/(1+F^*(c_W))^2$, and intermediate in pairing SW which occurs with probability $2F^*(c_W)/[1+F^*(c_W)]^2$. This does not hold for $\mathcal{E}_{\texttt{SSSS}}$ and $\mathcal{E}_{\texttt{SSWW}}$, where SS, or SW, respectively, are always observed with certainty in stage 2. After some simplifications, we obtain:

$$\mathcal{E}_{\texttt{SWSW}} = \underbrace{\frac{(1+c_W)^2F^*(c_W)+4c_W^2}{2(1+c_W)^2[1+F^*(c_W)]^2}P}_{\text{stage 1 effort}} + \underbrace{\frac{(1+c_w)[c_wF^*(c_W)^2+1]+4c_WF^*(c_W)}{2c_W(1+c_W)[1+F^*(c_W)]^2}P}_{\text{stage 2 effort}} \tag{14}$$

A comparison of $\mathcal{E}_{\texttt{SWSW}}$, $\mathcal{E}_{\texttt{SSWW}}$, and $\mathcal{E}_{\texttt{SWSW}}$ delivers the following relation between the three settings in terms of total effort:

Proposition 1 (Total Effort)**.** *When the cost of effort is strictly higher for weak than for strong agents ($c^W > c^S$), total effort provision is always highest in the homogeneous setting A: SSSS with strong agents only, intermediate in the heterogeneous setting B: SSWW, and lowest in setting C: SWSW; formally, this implies that $\mathcal{E}_{SSSS} > \mathcal{E}_{SSWW} > \mathcal{E}_{SWSW}$ for all $c_W > c_S$.*

Proof. See Appendix. □

This result shows that the commonly known feature of standard one-shot tournaments that heterogeneity between agents reduces incentive provision carries over to multi-stage elimination tournaments. Apart from that, the Proposition implies that heterogeneity between agents has different effects on incentive provision in settings B: SSWW than in setting C: SWSW. Intuitively, the distortion that weak agents invest less effort in the tournament is weaker in setting B: SSWW, where weak agents compete among themselves in stage 1 and do not have to interact with strong agents. In contrast, weak agents meet a strong agent for sure in stage 1 of setting C: SWSW, and with a high probability also in stage 2 (if they reach stage 2). Consequently, they reduce their effort provision even more. One can show that the difference between those two settings is most pronounced for intermediate degrees of heterogeneity, since total effort provision in both settings converges towards $\mathcal{E}_{\texttt{SSSS}}$ for $c_W \to 1$, while this measure approaches $\frac{P}{2}$ if $c_W \to \infty$. However, even in situations with very low or very high degrees of heterogeneity where the difference in total effort expenditures is rather small in equilibrium, the difference becomes increasingly relevant if one allows for multiple prizes, which are likely to be common practice rather than an exception in reality. If some share of the prize P is moved to previous stages, total effort provision will decrease in both specifications, but especially so in setting SWSW where the stage 1 interactions are heterogeneous. Contrary, both stage 1 interactions are homogeneous in setting SSWW, such that the difference in total effort expenditures between the two settings increases. In this sense, the chosen theoretical specification is likely to underestimate the difference between settings SSWW and SWSW.[10]

Selection: A second important application of elimination tournaments is the selection of the best-performing participant for promotion. According to Rosen (1986, p.701), the

> *" [...] inherent logic of [...] [sequential elimination tournaments] is to determine the best contestants and promote survival of the fittest."*

Therefore, apart from providing incentives for effort provision, tournaments are often used to deal with adverse selection problems. This holds especially for multi-stage elimination contests, which closely resemble promotions tournaments within firms. Suppose it is in the interest of a principal to promote an agent of type S, then a suitable measure for the selection property of a tournament is the probability that one of the two strong agents in the field of participants wins. We refer to this selection measure as $\mathcal{S}$. By definition, this measure $\mathcal{S}_{\texttt{SSSS}}$ is equal to one in setting A: SSSS, where a strong agent wins with certainty. This is not the case, however, in settings B: SSWW and C: SWSW, where a weak agent wins with strictly positive probability. The respective selection measures $\mathcal{S}_{\texttt{SSWW}}$ and $\mathcal{S}_{\texttt{SWSW}}$ for settings B: SSWW and C: SWSW are given by

$$\mathcal{S}_{\texttt{SSWW}} = \frac{c_W}{1+c_W} \tag{15}$$

$$\mathcal{S}_{\texttt{SWSW}} = \frac{(1+c_W)F^*(c_W)^2 + 2c_W F^*(c_W)}{(1+c_W)[1+F^*(c_W)]^2}. \tag{16}$$

[10]Note that the solution technique presented above can be easily extended to the consideration multiple prizes. Formal proofs for the verbal arguments above are available from the authors upon request.

As with the incentive measure $\mathcal{E}$, we compare settings A: SSSS, B: SSWW, and C: SWSW in terms of their selection properties.

Proposition 2 (Selection)**.** *Independent of the degree of heterogeneity between agents, the ex-ante probability that a strong agent wins the tournament is highest in the homogenous setting A: SSSS (by construction), intermediate in the heterogenous specification C: SWSW, and lowest in setting B: SSWW, i.e.* $\mathcal{S}_{SSSS} > \mathcal{S}_{SWSW} > \mathcal{S}_{SSWW}$ *for all* $c_W > c_S$.

Proof. See Appendix. □

The result that the homogenous setting A: SSSS exhibits the best performance in terms of maximizing the probability that a strong agent wins is trivial. The finding that setting C: SWSW performs better in terms of selecting the most able agent than does setting B: SSWW is also intuitive. There is selection in both stages of the tournament in setting C: SWSW, while a weak agent reaches stage 2 with certainty in setting B: SSWW. Consequently, stage 1 is "lost" in terms of selection in setting B: SSWW.

2.3 Further Implications

The main results of the analysis of the equilibrium properties so far can be summarized as follows: First, the homogeneous specification A: SSSS performs best in both performance dimensions, total effort provision and selection. Second, even though heterogeneity between agents is detrimental for total effort and selection in both specifications with heterogeneous agents, it has different implications for those specifications. In particular, total effort provision is higher in setting B: SSWW, while setting C: SSWW has better selection properties.

One caveat with respect to the selection aspect of the different tournament designs is that the analysis so far has implicitly assumed that a principal has a choice whether to implement setting B: SSWW or setting C: SWSW. This requires some prior knowledge by part of the principal about the agent's types in order to be able to seed them accordingly in stage 1. If the principal knows the type of the agents, however, in principle there is no need for a tournament for selection purposes, unless there are institutional reasons for holding a tournament, or the selection criterion achieves different goals.[11] Without a priori knowledge about the agents' types, the principal may end up in either of the two specifications with a certain probability. In the specification of the tournament analyzed so far, there are two agents of each type in the pool of participants. If the principal sequentially draws two agents

[11]For example, even with prior knowledge about agent types, tendering procedures for awarding contracts, e.g., in the context of construction for the public sector, might require a tournament. Likewise, maximizing the probability of the best participant winning the tournament might be one of the objectives of the organizer of a sports contest, due to, e.g., fairness concerns.

from the pool of four for the first stage-1 interaction, the probability that this interaction is between two players of different types is equal to $\frac{2}{4} \times \frac{2}{3} = \frac{1}{3}$; similarly, the probability that both agents are of the same type is given by $\frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$. For each situation, one has to account for two different orders of drawing, which implies the result that the principal ends up with a setting B: `SSWW` with probability $\frac{1}{3}$, while the probability for setting C: `SWSW` is twice as high, i.e. equal to $\frac{2}{3}$.

With these probabilities one can compute the theoretical benchmarks for total effort provision and selection from an ex-ante perspective, where types are unknown to the principal. This situation is defined as setting D: `2S2W`. The expected performance measures for effort provision and selection in this setting are then given by

$$\mathcal{E}_{\texttt{2S2W}} = \frac{\mathcal{E}_{\texttt{SSWW}} + 2 \cdot \mathcal{E}_{\texttt{SWSW}}}{3} \tag{17}$$

$$\mathcal{S}_{\texttt{2S2W}} = \frac{\mathcal{S}_{\texttt{SSWW}} + 2 \cdot \mathcal{S}_{\texttt{SWSW}}}{3}. \tag{18}$$

Since the resulting expressions are rather involved, we abstain from providing them in more detail. Instead, we summarize their properties in the following Proposition.

Proposition 3 (Setting D: 2S2W)**.** *If cost of effort is lower for strong than for weak agents, i.e. if $c^W \geq c^S$, it holds that:*

(a) Total effort provision $\mathcal{E}_{\texttt{2S2W}}$ equals $\mathcal{E}_{\texttt{SSSS}} = \frac{3}{4c_S}P$ for $c^W = c^S$, and approaches a value of $\frac{P}{2c_S}$ in the limit as $c_W \to \infty$.

(b) The probability that a strong agent wins, $\mathcal{S}_{\texttt{2S2W}}$, is equal to 0.5 for $c_W = c_S$, and approaches $\mathcal{S}_{\texttt{SSSS}} = 1$ in the limit as $c_W \to \infty$.

Proof. See Appendix. □

An immediate implication of these considerations is that a priori knowledge about the participants of the tournament has a value for the principal, since it allows him to seed the tournament by allocating participants in a way that is optimal in light of the principal's objectives. Of course, this value depends on the respective objective of the principal, but it is positive and measurable in terms of the difference between the optimal outcome (e.g., $\mathcal{E}_{\texttt{SSWW}} - \mathcal{E}_{\texttt{2S2W}}$ if the principal aims at maximizing effort provision, and $\mathcal{S}_{\texttt{SWSW}} - \mathcal{S}_{\texttt{2S2W}}$ if the principal aims at maximizing the winning probability of a strong type). This implication is important as it opens up the possibility for the principal to make a decision about investing in screening of tournament participants in order to have the possibility to seed them.[12]

Returning to the open questions in the introduction, the results indicate that heterogeneity of agents affects performance in multi-stage elimination tournaments. Tournaments that involve competition among heterogeneous agents already in early stages have better selection properties, but exhibit a worse performance in terms of incentive provision. To test the empirical validity of these theoretical predictions, we continue by providing experimental evidence for the propositions derived before.

3 Experimental Design

This section describes the design of the experiment that was conducted to test the theoretical predictions derived before. In the experiment, four subjects competed in a two-stage elimination tournament for a unique prize P of 240 Taler (the currency of the experiment), which the winner of the stage 2 interaction receives; 200 Taler equal 1.00 Euro. The prize had the same value for all participants. Heterogeneity between agents was modeled in terms of different cost of effort: *strong* agents had cost of effort equal to $c_S = 1.00$ Taler in all treatments, while *weak* agents had costs of 1.50 Taler. We modeled heterogeneity in terms of cost of effort rather than productivities or valuations, since this form of heterogeneity is particularly easy to understand for experimental participants. To test the predictions about the effect of increased heterogeneity, we also conducted treatments in which the costs for *weak* agents were set to 2.50 Taler (*high heterogeneity*). Effort provision was implemented in terms of investments in a lottery: Participants were told that they can buy a discrete number of balls in the stage 1 interaction.[13] The balls purchased by the subjects as well as those purchased by their respective opponents in the stage 1 interaction were then said to be placed in the same urn, of which one ball was randomly drawn. The agent who had purchased this ball proceeded to the stage 2 interaction. This setting closely reflects the theoretical counterpart of the Tullock CSF with discriminatory power 1. The two stage-1 interactions are independent from one another, i.e., subjects knew that there were two separate urns in stage 1. When agents made their decision in stage 1, they did not know whom they would encounter in stage 2; they only knew the types of those players who competed in the other stage-1 interaction. In stage 2, the two agents who won the stage 1 interaction met. Again, both players could buy a certain number of balls, which were placed in a third urn; the player whose ball was randomly drawn received the prize of 240 Taler. The two players who did not proceed to stage 2 saw a waiting screen until the decision

[12]For instance, with a given screening technology, i.e., the possibility to find out the agents' types with a some precision at a given cost, the principal faces a trade-off between the expected gain from being able to seed the tournament and the expected cost for screening.

[13]With the given parameterization, there is a unique equilibrium in pure strategies and the discrete grid has no consequences for the equilibrium strategies.

round had been completed. Note that players had to buy and pay for a certain number of balls *before* they knew whether or not they won the prize in a given game. To avoid limited liability problems, each participant received an endowment of 240 Taler which he could use to buy a certain number of balls in stage 1; if he reached stage 2, he could use whatever remained of his endowment to buy balls in stage 2; the part of the endowment which a participant did not use to buy balls was added to the payoffs for that round. Since the endowment was as high as the prize which could be won, agents were not budget-constrained at any time.

The experiment has three different baseline treatments: A: `SSSS`, B: `SSWW`, and C: `SWSW`, with weak players in treatments `SSWW`, and `SWSW` exhibiting *low heterogeneity* in terms of $c_W = 1.5$. Additional treatments extend this by varying the degree of heterogeneity by setting $c_W = 2.5$. The framing was identical across all treatments. The only difference across treatments concerned the information participants received about their own cost parameter and the cost parameters of their co-players. For more details, see the set of instructions provided in Appendix **??**. We adopted a between-subject design, such that each participant encountered only one of the five treatments. Each participant played the same tournament 30 times. Note that the endowment could only be used in a given decision round, it could not be transferred. Therefore, the strategic interaction was the same in each of the 30 decision rounds. Random matching in each round ensured that the same participants did not interact repeatedly. After each game, participants were informed about their own decision, the aggregate decision of their three opponents, and about their own payoff. This allows for an investigation whether players learn when completing the task repeatedly. To avoid income effects, however, the participants were told that only four decision rounds (out of 30) would be chosen randomly and paid out at the end of the experiment.

The procedures in an experimental session were as follows for all treatments: First, the participants received some general information about the experimental session. Then, instructions for part one of the experimental session, the two-stage tournament with four players as described above, were distributed. After each participant confirmed that he had understood the instructions on the computer screen, participants were informed about their individual cost parameter. Subsequently, participants had to answer a set of control questions correctly to ensure that they had fully understood the instructions as well as the implication of their personal cost parameter. Only then did the first decision round start. After the experiment, individual information (including socio-economic characteristics as well as risk preferences and distributional preferences) was elicited. Only thereafter were participants informed about their overall payoff from the different parts of the experimental session. We ran a total of six computerized sessions with 20 participants each using the software z-Tree (Fischbacher 2007). All

120 participants were students from the university of Innsbruck, which were recruited with ORSEE (Greiner 2004). Each session lasted approximately 1.5 hours, and participants earned between 13-21 Euro (approximately 16 Euro on average).

4 Experimental Results

4.1 Main Results

This section presents experimental evidence as to whether the decisions by experimental subjects are in line with the theoretical predictions. We consider the outcomes for a measure of aggregate tournament outlays to test the predictions of Proposition 1 and present evidence for the selection measure in terms of the probability of a strong agent winning the tournament to test the predictions of 2.

Table 1 presents results for the homogeneous setting A: SSSS, as well as the heterogeneous setting that emerges if the principal has no prior knowledge about agent types (setting D: 2S2W). In terms of the measure of aggregate effort provision, $\mathcal{E}$, the upper half of the table presents the theoretical predictions of Propositions 1 and 3(a) in the parametrization of the experiment. Theory predicts that equilibrium aggregate effort exertion is highest in the homogeneous setting and is decreasing in the degree of heterogeneity ($\mathcal{E}_{\texttt{SSSS}} = 180 > \mathcal{E}_{\texttt{2S2W-low}} = 149.16 > \mathcal{E}_{\texttt{2S2W-high}} = 128.36$). Compared to this theoretical benchmark, we find substantial overprovision of effort by the subjects in the experiment, with average total effort levels of around 250 or above throughout all settings. The substantial overprovision of 60-90% is consistent with similar findings in the literature; see, for example, Bull, Schotter, and Weigelt (1987) and Orrison, Schotter, and Weigelt (2004) for one-stage tournaments with heterogeneous agents, or Sheremeta (2010) and Altmann, Falk, and Wibral (2008) for two-stage elimination tournaments with homogeneous agents. With respect to the theoretical results, however, the predicted pattern that effort provision is highest in the homogeneous setting and decreasing with heterogeneity is consistent with the findings in the data. There a significant difference between incentive provision in homogeneous and heterogeneous settings with a low and with a high degree of heterogeneity.[14] A Jonkheere-Terpstra test of the null of equal effort provision in all three treatments against the ordered alternative predicted by the theoretical results rejects the null with a p-value of 0.000.

The bottom half of Table 1 presents empirical results concerning the theoretical predictions of Propositions 2 and 3(b), according to which the probability of a strong agent winning the tournament increases in the degree of heterogeneity. With the parameters of the experiment, the probability of

[14] The p-value of a two-sided t-test of the null that $\mathcal{E}_{\texttt{SSSS}} = \mathcal{E}_{\texttt{2S2W-low}}$ is 0.233; the p-value of a respective ranksum test 0.055. The corresponding p-values for the null that $\mathcal{E}_{\texttt{SSSS}} = \mathcal{E}_{\texttt{2S2W-high}}$ are 0.005 and 0.000.

a type S player winning the tournament is 0.674 in setting 2S2W with low heterogeneity, and 0.816 in setting 2S2W with high heterogeneity. To evaluate this hypothesis empirically, we use the realized outcomes in terms of the winner of the tournament being of type S as the respective measure.[15] The results indicate that the theoretical prediction is in line with the data, with the empirical probabilities of 0.576 and 0.775 in the treatments with low and high heterogeneity, respectively. We can reject the null of equality of the respective winning probability in the two settings (p-value is 0.000 for two-sided t-test or ranksum test). The null of equality against the alternative of an ordered alternative is rejected at the 0.01 level using a Jonckheere-Terpstra test. Interestingly, the overprovision of effort found before appears to translate into a higher degree of competitiveness of the tournaments, as reflected by the finding that the empirical probabilities of an agent of type S winning the tournament are lower than the one predicted by theory in the heterogeneous settings 2S2W. This implies that the overprovision of effort was more pronounced among players of type W. We return to this issue below.

In order to understand the role of different settings of heterogeneous tournaments in terms of mixed versus homogeneous first-round tournaments for this findings, Table 2 presents results for settings B: SSWW and C: SWSW separately. From Proposition 1, theory predicts that incentive provision is highest in homogeneous tournaments, SSSS, but also that heterogeneous tournaments with homogeneous first stages, SSWW, elicit higher aggregate effort than heterogeneous tournaments with heterogeneous contestants meeting already on the first stage, SWSW, for the same degree of heterogeneity. In the parametrization of the experiment with a low degree of heterogeneity (with effort costs of strong agents of $c_S = 1.0$ and of weak agents of $c_W = 1.5$), this difference is 152 versus 146.32 units of effort. The experimental results, depicted in columns (2) and (3), reveal an interesting pattern: the modest difference between homogeneous and heterogeneous tournaments in terms of incentives is driven by a substantial overprovision compared to the theoretical prediction in the SSWW-treatment (318 units), which is even higher than effort provision in the homogeneous treatment SSSS (282.3). Overprovision is much more modest in the SWSW-treatment (249.5 units), which is lower than effort provision in the homogeneous treatment. Equality between effort provision between homogeneous tournaments (282.3 units) and the different heterogeneous tournaments can be rejected in terms of pairwise two-sided tests.[16] In order to test the more precise theoretical prediction of an ordering of the different settings with respect to the aggregate effort they elicit, we apply again a Jonckheere-Terpstra test for the null of equal effort in all

[15]The results are virtually identical when we compute the predicted probability of the tournament winner being of type S from the experimental data, by using the effort of all players in a given tournament and imputing the corresponding probability using the linear Tullock CSF that determined the winning probabilities in the experimental implementation. Details are available upon request.

[16]The p-values for the null of equality of effort in treatments SSSS and SSWW are 0.003 (t-test) and 0.001 (ranksum). The p-values for the null of equality of effort in treatments SSSS and SWSW are 0.001 (t-test) and 0.001 (ranksum).

three settings against the ordered alternative of effort being highest in the SSSS treatment, intermediate in the SSWW treatment, and lowest in the SWSW treatment. The test rejects the null against the alternative (p-value 0.035), which provides evidence that is consistent with the theoretical prediction of Proposition 1.

Columns (4) and (5) present results for the parametrization with a high degree of heterogeneity (effort costs of $c_S = 1.0$ and $c_W = 2.5$). In the heterogeneous treatments with high heterogeneity, we still find substantial overprovision of effort compared to the theoretical benchmark, but the data are consistent with the ordering predicted by theory. The difference in total effort provision between the homogeneous treatment SSSS (282.3) and treatment SSWW (275.4) are too small to be significant, however, while effort provision in treatment SWSW is significantly lower than in SSSS.[17] Again, a Jonckheere-Terpstra test rejects the null of equal effort in all three settings against the ordered alternative of effort being highest in the SSSS treatment (p-value 0.006). By and large, the experimental results therefore appear to be qualitatively in line with the theoretical predictions in terms of incentive provision and selection properties. The findings concerning the difference in incentive provision between homogeneous and heterogeneous tournament settings are only fully consistent with theory in terms of the overall picture (e.g., in light of the test of the null of equality against the ordered alternative predicted by the theory) in the treatments with a high degree of heterogeneity, whereas the findings contradict the theoretical predictions in terms of incentive provision in setting SSSS and SSWW when the degree of heterogeneity is low.

The bottom half of Table 2 presents evidence on the predictions concerning the selection properties of the different settings of heterogeneous tournaments as stated in Proposition 2. In line with the results reported in Table 1, the results indicate that the systematic overprovision of effort compared to the theoretical benchmark implies a systematically lower probability of a participant of type S winning the tournament. Also consistent with the earlier results and the theoretical predictions, the findings indicate that settings with homogeneous stage-1 interactions (SSWW) have lower selection performance than settings with heterogeneous stage-1 interactions (SWSW).[18] Finally, the results also indicate that higher heterogeneity increases the selectivity of heterogeneous tournaments.[19]

[17] The p-values for the null of equality of effort in treatments SSSS and SSWW are 0.427 (t-test) and 0.83 (ranksum). The p-values for the null of equality of effort in treatments SSSS and SWSW are 0.001 (t-test and ranksum).

[18] The null of equality of the selection criterion in treatments SSWW and SWSW is rejected at p-values 0.01 for low and high degrees of heterogeneity.

[19] The null of equality of the selection criterion in treatments SSWW with low and high heterogeneity is rejected with a p-value of 0.011. The null of equality of the selection criterion in treatments SWSW with low and high heterogeneity is rejected with a p-value of 0.01.

4.2 Robustness: Results by Rounds, Stages and Types

A commonly found pattern in the literature is that subjects in experiments adapt their behavior as the decisions are repeated, and often converge towards the theoretical benchmark (see, e.g., Bull et al., 1987, Orrison et al., 2004, or Sheremeta, 2010b). Figure 1 presents information on the total effort provision over the different rounds of the experiment.[20] Throughout treatments, there is a strong downward trend in effort provision, implying that the systematic overprovision of effort compared to the theoretical benchmark was particularly strong during the early stages, but became weaker over the course of the experiment. Interestingly, the figure also shows that the convergence towards the theoretical benchmark was strongest in the homogeneous treatment `SSSS`, where total effort provision was only about 20% above the theoretical prediction of 180 during the last rounds of the experiment. Overprovision was more pronounced throughout in the heterogeneous treatments `SSWW` and `SWSW`, and there in particular in the high heterogeneity treatments.

These findings are complemented by Table 3, which presents the corresponding average effort levels when disaggregating into rounds 1-10, 11-20, and 21-30. Effort in the homogeneous setting `SSSS` exhibits a substantial reduction of overprovision of effort during the experiment, reflected in the pronounced downward trend of aggregate effort along the experimental rounds. Compared to that, the reduction in effort is more moderate in the heterogeneous settings, particularly in the setting `SSWW`. Again, throughout we find overprovision of effort compared to the value predicted by theory. The comparison between the homogeneous treatment `SSSS` and treatment `SSWW` with a low degree of heterogeneity reveals a higher effort provision in the heterogeneous setting, which contradicts the theoretical predictions throughout all rounds when heterogeneity is low.[21] The treatment with a high degree of heterogeneity provides evidence that overall is in line with the theoretical prediction in Proposition 1 in that effort is slightly lower in the heterogeneous tournament `SSWW` with 275.44 units compared to 282.3 in the homogeneous treatment `SSSS` (see Table 2). The picture is more subtle when considering the results for the different rounds. The difference is more pronounced and significant considering rounds 1-10 of the experiment.[22] However, over the course of the experiment, the overprovision of effort drops more in the homogeneous setting `SSSS` than in the heterogeneous setting `SSWW`, which implies that effort provision during rounds 11-20 is fairly similar, and in fact somewhat higher during rounds 21-30 in the heterogeneous setting.[23] When comparing the settings `SSSS` and `SWSW` we find similar patterns. With a

[20] In order to visualize the pattern, the figure presents polynomial smoothed regressions with the corresponding 95% confidence intervals.

[21] The difference is insignificant during rounds 1-10, but significant with p-value 0.01 (t-test and ranksum) for rounds 11-20 and 21-30.

[22] The p-values are 0.01 for t-test and ranksum.

[23] This difference is significant with p-values of 0.029 (t-test) and 0.019 (ranksum), respectively.

low or high degree of heterogeneity, we find that effort provision is typically lower in the heterogeneous setting, even though the difference is only significant during the first 20 rounds of the experiment.[24] Jonckheere-Terpstra tests reject the null of equal efforts against the order predicted by theory for rounds 11-20 for a low degree of heterogeneity (p-value 0.01), and for rounds 1-10 and 11-20 for a high degree of heterogeneity (p-value 0.01 and 0.05, respectively).

The pattern of selection in terms of winning probabilities of an `S`-type agent over the course of the experiment can be seen from the results for the selection criterion in Table 4. While winning probabilities are substantially too low in setting `SSWW` compared to the theoretical predictions in the treatment with a low degree of heterogeneity, there is not clear trend over the different rounds. In the treatment with high heterogeneity, in contrast, the winning probability of an `S`-type agent is also too low during the first rounds but converges to the theoretically predicted level during rounds 21-30. Compared to that, the selection properties of setting `SWSW` in the low heterogeneity treatment are fairly close to the benchmark during rounds 1-10 in the treatment with low heterogeneity, and decline thereafter. In the treatment with high heterogeneity, there is a similar but less pronounced trend. Overall, we find significant differences between the selection properties of settings `SSWW` and `SWSW` in the direction predicted by theory. In the low heterogeneity treatment, the difference is only significant in rounds 1-10 (p-value 0.01), whereas it is significant throughout all rounds in the high heterogeneity treatment (p-value 0.01 for rounds 1-10 and 11-20, and p-value 0.1 for rounds 21-30).

An in depth analysis of the experimental data shows that the substantial overprovision of effort on the aggregate level is the result of overprovision on both stages of the experiments, throughout all rounds and for both types of players, regardless of the setting of the tournament, homogeneous or heterogeneous.[25] There are some striking differences, however. First, overprovision is more pronounced on the first stage of the tournament than in the second. This finding is consistent with the results for homogeneous 2-stage tournaments reported by Altmann, Falk, and Wibral (2008) and Sheremeta (2010). Second, and complementing the earlier results in the literature, we find that overprovision is more pronounced in heterogeneous 2-stage tournaments than in homogeneous 2-stage tournaments. Third, overprovision is particularly strong for weak types, `W`, as compared to strong types, `S`. This overall pattern is qualitatively identical in the treatments with a low and with a high degree of heterogeneity. Despite the greater disparity in effort levels between strong and weak types in the treatment with a high degree of heterogeneity, we still find that overprovision is more pronounced in heterogeneous tournaments, that overprovision is stronger for weak types (relative to the theoretical level and

[24]This difference is significant with p-value 0.02 for the t-test and 0.05 for the ranksum test during rounds 1-20.
[25]Details are available from the authors upon request.

compared to the extent of overprovision of strong types), and that overprovision is stronger on the first stage. Nevertheless, in particular the latter result is somewhat weaker, as we see substantially more overprovision on the second stage when the degree of heterogeneity is high compared to the baseline (i.e., comparing the stage 2 outcomes).

5 Conclusion

This paper has theoretically and empirically investigated the properties of multi-stage elimination tournaments with heterogeneous agents in terms of incentive provision and selection properties. First, our analytical results predict that different settings of multi-stage elimination tournaments with heterogeneous agents in terms of the matching of contestants on the first stage of the tournament have distinct incentive and selection properties: Tournaments in which similar agents compete on the first stage (setting B: `SSWW`) generally have better incentive properties in terms of eliciting higher aggregate effort than tournaments with mixed first stages (setting C: `SWSW`); in contrast, tournaments with mixed first stages have better selection properties in terms of the probability of the best contestant winning the tournament. This theoretical result suggests that there is a fundamental trade-off between Incentive provision and selection in tournaments from the point of view of a tournament designing principal. Empirical evidence from laboratory experiments gives strong support to this theoretical prediction. While we find substantial overprovision of effort in the experiments compared to the quantitative predictions of the theory, which is in line with earlier findings in the literature, we find all qualitative results of the theory confirmed. Second, we find that heterogeneity is detrimental to incentives for effort provision in multi-stage tournaments, a result that is well known from the literature on one-stage tournaments. Empirical evidence is broadly in line with this theoretical prediction, in particular for a high degree of heterogeneity of the contestants.

An interesting topic for future research is the question as to whether the trade-off between incentive provision and selection performance of tournaments with heterogeneous agents does hold in general, i.e. independent of the specification of the contest success function. If this trade-off is really robust, as we believe, this suggests there is no tournament format which is optimal in both of our optimality dimensions, namely incentive provision and selection. Said differently, the trade-off should not only exist when different settings of the same tournament format are compared, as in this paper, but it should also be at work when different tournament formats are compared, e.g. simulatenous and sequential tournaments.

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A Proofs

Lemma 1. *Assume without loss of generality* $c_W \geq c_S = 1$ *and define* $f(c_W) = \frac{5c_W^3+2c_W^2+c_W}{c_W^2+2c_W+5}$. *Then, the relation* $F^*(c_W) > f(c_W)$ *does hold for all* $c_W > 1$*, where* $F^*(c_W)$ *is defined in* (9)*. Further, if* $c_W = 1$*, it holds that* $F^*(c_W) = f(c_W)$*.*

Proof. From equation (8), we know that $\frac{y_1^{C*}}{y_3^{C*}} = c_W \frac{P_1(y_2^C, y_4^C)}{P_3(y_2^C, y_4^C)}$. Further, equation (9) tells us that $\frac{y_1^{C*}}{y_3^{C*}} = F^*(c_W)$. Consequently, it must hold that

$$F^*(c_W) = c_W \frac{P_1(y_2^C, y_4^C)}{P_3(y_2^C, y_4^C)} = \frac{4c_W^3 + c_W(1+c_W)^2 \times \frac{y_2^C}{y_4^C}}{(1+c_W)^2 + 4 \times \frac{y_2^C}{y_4^C}}.$$

Note that

$$\frac{\partial F^*(c_W)}{\partial \frac{y_2^C}{y_4^C}} = \frac{(1+c_W)^4 - 16c_W^2}{[(1+c_W)^2 + 4 \times \frac{y_2^C}{y_4^C}]^2} > 0$$

if $c_W > 1$. Further, recall that player 4 has both higher cost ($c_W > 1$) and a lower continuation value ($P_2 > P_4$), such that $y_2^C > y_4^C$ does hold. Therefore, assuming $y_2^C = y_4^C$ underestimates $F^*(c_W)$. Since

$$f(c_W) = \frac{5c_W^3 + 2c_W^2 + c_W}{c_W^2 + 2c_W + 5}$$

is the expression we derive from $F^*(c_W)$ under this assumption, we have proven $F^*(c_W) > f(c_W)$. If we assume $c_W = 1$, all players are perfectly symmetric, such that $y_2^C = y_4^C$ does hold. Consequently, the relation $F^*(c_W) = f(c_W)$ does hold for $c_W = 1$. □

Lemma 2. *We assume* $c_W \geq c_S = 1$ *without loss of generality and define* $f_{\text{low}}(c_W) = 2c_W - 1$*. Then, the relation* $f(c_W) > f_{\text{low}}(c_W)$ *does hold for all* $c_W > 1$*. Further, if* $c_W = 1$*, it holds that* $f(c_W) = f_{\text{low}}(c_W)$*.*

Proof. We start with the relation that we want to prove, namely:

$$\begin{aligned} f(c_W) &> f_{\text{low}}(c_W) \\ \Leftrightarrow 5c_W^3 + 2c_W^2 + c_W &> (2c_W - 1)(c_W^2 + 2c_W + 5) \\ \Leftrightarrow 3c_W^3 - c_W^2 - 7c_W + 5 &> 0 \end{aligned}$$

We now have to prove that $\phi(c_W) \equiv 3c_W^3 - c_W^2 - 7c_W + 5 > 0$ does always hold for $c_W > 1$. Note that $\phi(c_W)$ has a local minimum at $c_W = 1$, and a local maximum at $c_W = -7/9$. Further, $\phi(1) = 0$. Therefore, it must be that $\phi(c_W) > 0$ for all $c_W > 1$. □

Proof of Proposition 1: To prove the relation $\mathcal{E}_{\texttt{SSSS}} > \mathcal{E}_{\texttt{SSWW}} > \mathcal{E}_{\texttt{SWSW}}$ for all $c^W > c_S$, we proceed in two steps. First, we show that $\mathcal{E}_{\texttt{SSSS}} > \mathcal{E}_{\texttt{SSWW}}$ for all $c^W > c_S$ does hold in part (a), and subsequently we establish $\mathcal{E}_{\texttt{SSWW}} > \mathcal{E}_{\texttt{SWSW}}$ for all $c^W > c_S$ in part (b).

(a) We assume without loss of generality that $c_W > c_S = 1$. Under this assumption, we obtain the expressions in equations (12) and (13) for $\mathcal{E}_{\texttt{SSSS}}$ and $\mathcal{E}_{\texttt{SSWW}}$, respectively. Therefore:

$$\begin{aligned}
\mathcal{E}_{\texttt{SSSS}} &> \mathcal{E}_{\texttt{SSWW}} \\
\Leftrightarrow 6c_W(1+c_W)^2 &> 4c_W^3 + 8c_W(1+c_W) + 4 \\
\Leftrightarrow c_W^3 + 2c_W^2 - c_W - 2 &> 0 \\
\Leftrightarrow (c_W+2)(c_W+1)(c_W-1) &> 0 \\
\Leftrightarrow c_W > 1 &\vee \; (-2 < c_W < -1),
\end{aligned}$$

which proves the statement $\mathcal{E}_{\texttt{SSSS}} > \mathcal{E}_{\texttt{SSWW}}$ for all $c_W > c_S$.

(b) As previously, we again make the innocuous assumption $c_W > c_S = 1$, which allows us to use of the expressions in (13) and (14) for $\mathcal{E}_{\texttt{SSWW}}$ and $\mathcal{E}_{\texttt{SWSW}}$, respectively. In the proof, we will proceed in two steps. First, we derive a necessary and sufficient condition in terms of the function $F^*(c_W)$ for the relation $\mathcal{E}_{\texttt{SSWW}} > \mathcal{E}_{\texttt{SWSW}}$ to hold. Second, we proof that the equilibrium function $F^*(c_W)$ which was derived in (9) indeed satisfies this condition. We start with the relation which we want to prove:

$$\begin{aligned}
\mathcal{E}_{\texttt{SSWW}} &> \mathcal{E}_{\texttt{SWSW}} \\
\Leftrightarrow \frac{c_W^3 + 2c_W(1+c_W)+1}{2c_W(1+c_W)^2} &> \frac{(1+c_W)^2[1+[1+F^*(c_W)]c_W F^*(c_W)] + 4c_W[c_W^2 + (1+c_w)F^*(c_W)]}{2c_W(1+c_W)^2[1+F^*(c_W)]^2}
\end{aligned}$$

Multiplying both sides by $2c_W(1+c_W)^2[1+F^*(c_W)]^2$ and rearranging gives

$$F^*(c_W)^2 + \frac{c_W^3 - 2c_W^2 - c_W + 2}{c_W+1} F^*(c_W) - \frac{3c_W^3 - c_W^2}{c_W+1} \quad > \quad 0$$

Solving for $F^*(c_W)$ gives us two conditions:

$$F^*(c_W) < \frac{-c_W^3 + 2c_W^2 + c_W - 2 - R(c_W)}{2c_W+2} \quad \vee \quad F^*(c_W) > Z(c_W) \equiv \frac{-c_W^3 + 2c_W^2 + c_W - 2 + R(c_W)}{2c_W+2},$$

where

$$R(c_W) = \sqrt{c_W^6 - 4c_W^5 + 14c_W^4 + 16c_W^3 - 11c_W^2 - 4c_W + 4}.$$

We do only have to consider the second relation, since the first one is below one for some values of c_W, while $F^*(c_W) \geq 1$ for all $c_W \geq 1$.[26] This completes the first part of the proof. We now have to prove that

$$F^*(c_W) > Z(c_W) \equiv \frac{-c_W^3 + 2c_W^2 + c_W - 2 + R(c_W)}{2c_W+2} \tag{19}$$

for all $c_W > 1$. From Lemmata 1 and 2 we know that $F^*(c_W) > f_{\text{low}}(c_W)$. Consequently, a sufficient condition for (19) is given by $f_{\text{low}}(c_W) > Z(c_W)$. Rearranging this condition gives

$$c_W^3 + 2c_W^2 + c_W > R(c_W).$$

[26]Note that $F^*(1) = 1$; also, we know from Lemma 1 that $\frac{\partial F^*(c_W)}{\partial c_W} > 0$. Therefore, $F^*(c_W) \geq 1$ for all $c_W \geq 1$.

Squaring both sides leaves us with[27]

$$\begin{aligned} 2c_W^5 - 2c_W^4 - 3c_W^3 + 3c_W^2 + c_W - 1 &> 0 \\ \Leftrightarrow 2(c_W - 1)^2(c_W + 1)(c_W - \frac{1}{\sqrt{2}})(c_W + \frac{1}{\sqrt{2}}) &> 0. \end{aligned}$$

This relation is always satisfied if $c_W > 1$, which completes the proof.

Proof of Proposition 2: In this proof, we first derive a necessary and sufficient condition which assures that the relation $\mathcal{S}_{\texttt{SSWW}} < \mathcal{S}_{\texttt{SWSW}}$ does hold in terms of the function $F(c_W)$. Then, we prove that the equilibrium function $F^*(c_W)$ satisfies this condition.
(1) As previously, we assume that $c_W > c_S = 1$ does hold without loss of generality. Consequently, we can use the expressions in equations (15) and (16) in what follows. We start with the relation which we want to prove:

$$\begin{aligned} \mathcal{S}_{\texttt{SWSW}} &> \mathcal{S}_{\texttt{SSWW}} \\ \Leftrightarrow (1 + c_W)F^*(c_W)^2 + 2c_W F^*(c_W) &> c_W F^*(c_W)^2 + 2c_W F^*(c_W) + c_W \\ \Leftrightarrow F^*(c_W)^2 &> c_W \\ \Leftrightarrow F^*(c_W) < -\sqrt{c_W} &\vee \quad F^*(c_W) > \sqrt{c_W} \end{aligned}$$

Note that it is sufficient to show that $F^*(c_W) > c_W$, since $c_W > \sqrt{c_W}$ for $c_W > 1$.
(2) From Lemma 1, we know that $F^*(c_W) > f(c_W)$. We will now proof that $f(c_W) > c_W$ for $c_W > 1$ to complete the proof. $f(c_W) > c_W$ implies that

$$\frac{5c_W^3 + 2c_W^2 + c_W}{c_W^2 + 2c_W + 5} > c_W$$

does hold. Rearranging gives

$$\begin{aligned} 5c_W^3 + 2c_W^2 + c_W &> c_W^3 + 2c_W^2 + 5c_W \\ \Leftrightarrow c_W(c_W^2 - 1) &> 0 \\ \Leftrightarrow c_W > 1 &\vee \quad -1 < c_W < 0 \end{aligned}$$

This proves the claim $\mathcal{S}_{\texttt{SWSW}} > \mathcal{S}_{\texttt{SSWW}}$ for all $c_W > 1$.

Proof of Proposition 3: The proposition consists of two parts (a) and (b). We will prove those parts separately.
(a) The first part of this proof is trivial. To establish the result $\mathcal{E}_{\texttt{2S2W}} = \mathcal{E}_{\texttt{SSSS}}$ for $c_W = c_S$, we can simply insert this condition in equations (13) and (14). Noting that $F(c_W) = 1$ if $c_W = c_S$, we see that both $\mathcal{E}_{\texttt{SSWW}}$ and $\mathcal{E}_{\texttt{SWSW}}$ equal $\mathcal{E}_{\texttt{SSSS}}$; consequently, it must hold that $\mathcal{E}_{\texttt{2S2W}} = \mathcal{E}_{\texttt{SSSS}}$, since $\mathcal{E}_{\texttt{2S2W}}$ is a weighted sum of the previous expressions. Next, we will prove the claim that $\lim_{c_W \to \infty} \mathcal{E}_{\texttt{2S2W}} = \frac{P}{2c_S}$. As above, we separately consider $\mathcal{E}_{\texttt{SSWW}}$ and $\mathcal{E}_{\texttt{SWSW}}$, since $\mathcal{E}_{\texttt{2S2W}}$ is a weighted sum of the two. Without loss of generality,

[27]Note that squaring is without loss of generality here, since we are only interested in solutions for $c_W > 1$.

we assume that $c_S = 1$, which implies that we can use the expressions in (13) and (14). We get:

$$\begin{aligned}
\lim_{c_W\to\infty} \mathcal{E}_{\texttt{SSWW}} &= \lim_{c_W\to\infty}\left[\frac{1+c_W^3}{2c_W(1+c_W)^2}P+\frac{1}{1+c_W}P\right] = \lim_{c_W\to\infty}\left[\frac{\frac{1}{c_W^3}+1}{2+\frac{4}{c_W}+\frac{2}{c_W}}P+\frac{\frac{1}{c_W}}{\frac{1}{c_W}+1}P\right] = \frac{P}{2} \\
\lim_{c_W\to\infty} \mathcal{E}_{\texttt{SWSW}} &= \lim_{c_W\to\infty}\left[\frac{(1+c_W)^2F^*(c_W)+4c_W^2}{2(1+c_W)^2[1+F^*(c_W)]^2}P+\frac{(1+c_w)[c_wF^*(c_W)^2+1]+4c_WF^*(c_W)}{2c_W(1+c_W)[1+F^*(c_W)]^2}P\right] \\
&= \lim_{c_W\to\infty}\left[\frac{\frac{1}{F^*(\cdot)}+\frac{2}{c_WF^*(\cdot)}+\frac{2}{c_W^2F^*(\cdot)}+\frac{4}{c_W[F^*(\cdot)]^2}}{2+\frac{4}{F^*(c_W)}+\frac{2}{[F^*(\cdot)]^2}+\frac{4}{c_W}+\frac{8}{c_WF^*(\cdot)}+\frac{4}{c_W[F^*(\cdot)]^2}+\frac{2}{c_W^2}+\frac{4}{c_W^2F^*(\cdot)}+\frac{2}{c_W^2[F^*(\cdot)]^2}}\right. \\
&\qquad\left.+\quad\frac{1+\frac{1}{c_W}+\frac{1}{c_W[F^*(\cdot)]^2}+\frac{1}{c_W^2F^*(\cdot)}+\frac{4}{c_WF^*(\cdot)}}{2+\frac{2}{c_W}+\frac{4}{c_WF^*(\cdot)}+\frac{2}{c_W[F^*(\cdot)]^2}+\frac{4}{F^*(\cdot)}+\frac{2}{[F^*(\cdot)]^2}}\right]P \\
&= \frac{P}{2}
\end{aligned}$$

Recall that $\lim_{c_W\to\infty} F(c_W) = \infty$ (follows from Lemma 1). Therefore, it holds that $\lim_{c_W\to\infty} \mathcal{E}_{\texttt{2S2W}} = \frac{P}{2}$.

(b) The proof for this part is completely analogous to the one in (a), except that we use equations (15) and (16) instead of (13) and (14).

B Tables & Figures

Table 1: Aggregate Incentive Provision and Selection in A: `SSSS` and D: `2S2W`

		A: `SSSS`	D: `2S2W` (low het.)	D: `2S2W` (high het.)
Total Effort ($\mathcal{E}$)	*Data*	**282.283**	**272.349**	**256.800**
		(5.862)	(6.518)	(7.249)
	Theory	180	149.16	128.36
Selection ($\mathcal{S}$)	*Data*	**1.000**	**0.576**	**0.775**
		(0.000)	(0.029)	(0.025)
	Theory	1.000	0.674	0.816
Observations		300	300	300

Note: Aggregate incentives in terms of the sum of total effort exerted by all players on all stages (in experimental currency, Taler). Selection refers to the expected probability of a strong type winning the tournament, conditional on the Data effort exertion of all players and the Tullock CSF. See text for details. Standard errors in parentheses.

Table 2: Aggregate Incentive Provision and Selection in A: `SSSS`, B: `SSWW`, and C: `SWSW`

			low het.		high het.	
		A: `SSSS`	B: `SSWW`	C: `SWSW`	B: `SSWW`	C: `SWSW`
		(1)	(2)	(3)	(4)	(5)
Total Effort ($\mathcal{E}$)	*Data*	**282.283**	**317.993**	**249.527**	**275.440**	**247.480**
		(5.862)	(6.780)	(7.545)	(5.923)	(8.332)
	Theory	180	152	146.32	133	123.72
Selection ($\mathcal{S}$)	*Data*	**1.000**	**0.473**	**0.627**	**0.620**	**0.853**
		(0.000)	(0.041)	(0.040)	(0.040)	(0.026)
	Theory	1.000	0.604	0.744	0.710	0.921
Observations		300	150	150	150	150

Note: Aggregate incentives in terms of the sum of total effort exerted by all players on all stages (in experimental currency, Taler). Selection refers to the expected probability of a strong type winning the tournament, conditional on the Data effort exertion of all players and the Tullock CSF. See text for details. Standard errors in parentheses.

Table 3: Total Effort ($\mathcal{E}$): Different Rounds

		Rounds	A: SSSS (1)	low het. B: SSWW (2)	low het. C: SWSW (3)	high het. B: SSWW (4)	high het. C: SWSW (5)
Total Effort ($\mathcal{E}$)	*Data*	1-10	**340.77**	**363.82**	**312.82**	**290.26**	**274.42**
			(9.920)	(11.075)	(14.093)	(10.408)	(13.407)
		11-20	**281.73**	**323.64**	**220.62**	**280.40**	**240.14**
			(9.117)	(10.387)	(11.209)	(9.595)	(14.133)
		21-30	**225.97**	**266.52**	**215.56**	**255.66**	**227.88**
			(7.953)	(9.551)	(8.590)	(10.326)	(15.167)
	Theory		180	152	146.32	133	123.72
Observations			100	50	50	50	50

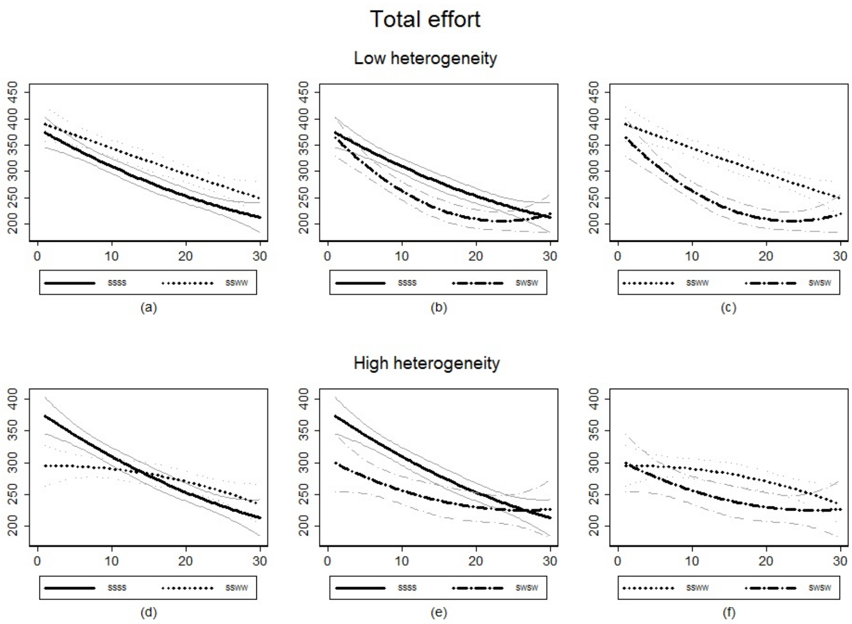
Note: Aggregate incentives in terms of the sum of total effort exerted by all players on all stages (in experimental currency, Taler). Selection refers to the expected probability of a strong type winning the tournament, conditional on the Data effort exertion of all players and the Tullock CSF. See text for details. Standard errors in parentheses.

Table 4: Selection ($\mathcal{S}$): Different Rounds

		Rounds	A: SSSS (1)	low het. B: SSWW (2)	low het. C: SWSW (3)	high het. B: SSWW (4)	high het. C: SWSW (5)
Selection ($\mathcal{S}$)	*Data*	1-10	**1.00**	**0.44**	**0.70**	**0.58**	**0.88**
			(0.000)	(0.071)	(0.065)	(0.071)	(0.046)
		11-20	**1.00**	**0.54**	**0.58**	**0.58**	**0.84**
			(0.000)	(0.071)	(0.071)	(0.071)	(0.052)
		21-30	**1.00**	**0.44**	**0.60**	**0.70**	**0.84**
			(0.000)	(0.071)	(0.071)	(0.065)	(0.052)
	Theory		1.000	0.604	0.744	0.710	0.921
Observations			100	50	50	50	50

Note: Aggregate incentives in terms of the sum of total effort exerted by all players on all stages (in experimental currency, Taler). Selection refers to the expected probability of a strong type winning the tournament, conditional on the Data effort exertion of all players and the Tullock CSF. See text for details. Standard errors in parentheses.

Figure 1: Aggregate Incentive Provision over the Course of the Experiment



Note: Polynomial fitted regressions. Black lines depict means of respective experimental round, grey lines correspond to 95% confidence intervals.