# Strategic Disclosure of Meaningful Information within the Environment with Competing Agents

Young-Ro Yoon*

September 5, 2010


**Abstract**

This article studies the effects of asymmetric payoffs on the informed agent's strategic disclosure of meaningful information to his rival. We show that if the penalty is larger than the reward, only low quality information is disclosed, in order to induce imitation. On the other hand, when the reward is larger than the penalty, which quality type of information is revealed depends on how sufficiently large the reward is. If the reward is weakly large, only low quality information is revealed. If the reward is sufficiently large, there exist both a separating- and a pooling equilibrium in which high quality information is revealed, in order to induce deviation. The derived economic intuitions can also be applied broadly to other case such as the firms' competition for occupying the global standard in technology adoption through R&D.


*Department of Economics, Wayne State University, Detroit, MI 48202, Tel: 313-577-0603, E-mail: yoonyr@wayne.edu.

# 1 Introduction

Agents who compete with one another for a common task are often evaluated based on their relative performance. So when carrying out a task, each agent must consider not only his own performance but also other agents' performances. For example, if, in completing a task, an agent was successful but his peers were also successful, he will be given a less positive evaluation than if he alone had been successful. On the other hand, if an agent was not successful but other competitors also failed, it may result in a less negative evaluation than if he alone was unsuccessful.[1]

Now consider one agent carrying out a given task in a competitive environment. If he has meaningful information related to the task, we can predict that he may not want to reveal it to others, in order to be the only agent who completes the task successfully. If information he has is perfect, our prediction may be correct. However, if information is, although meaningful, not perfect, sometimes concealing it from other agents may not be optimal, because it could lead him in the wrong direction and cause him to be the only agent who fails at the task. Then, given that the same failure of other agents can mitigate the agent's failure by sharing the blame, revealing his information could produce the better situation. This reasoning suggests a possibility that meaningful information could, for this purpose, be revealed in a competitive environment. This type of information disclosure would be more likely to happen if an agent is skeptical about the correctness of his information. Then can we expect the disclosure of information by an agent who has a relatively strong confidence in his information? Although it seems unlikely, the answer can be not certain because how other competitors respond to the revealed information must also be considered.

Then what would be the conditions under which meaningful information is revealed strategically to his rivals? In particular, if it is revealed, which quality type of information will be revealed for which incentive? The aim of this paper is to explore the strategic disclosure of information within the environments with competing agents, by providing the answers to these proposed questions.

The effects of an asymmetry in the reward and the penalty is of particular interest. Sometimes which is greater between the reward and the penalty can be given explicitly. Or agents can interpret the given situation subjectively as the one biased toward either the reward or the penalty. For example, if a given task is relatively easy (hard), in general there is less to gain (lose) and much to lose (gain). Then, the agents may evaluate the situation where easy (hard) task is assigned as one in which the payoff structure is biased toward the penalty (reward). Below it is shown that the asymmetry in reward and penalty plays an important role in deriving the equilibrium in which meaningful information is revealed strategically. The proposed results imply that the information disclosure is related to whether agents hope for the best or prepare for the worst, which is affected by the asymmetry in payoffs.

The model we deal with in this article can be described as follows: Two heterogeneous agents, one partially informed (henceforth M) and one uninformed (henceforth U) about the unknown true state,

[1] Aggarwal and Samwick (1999), Antle and Smith (1986), Gibbons and Murphy (1990), Janakiraman, and Lambert and Larcker (1992) provide the empirical findings which support that relative performance evaluation is being widely used.

compete with each other in a forecasting market. The signal which M observes and its precision are private information. M can decide whether to forecast as the leader in round 1 or forecast simultaneously with U in round 2 after a delay. On the other hand, U is required to forecast only in round 2. After both players forecast, the true state is revealed and each player earns his payoff according to the correctness of both players' forecasts.

The main results of analysis can be summarized as follows. M reveals his private signal truthfully regardless of his timing of forecast. Hence, his decision on the timing of forecast is no more than a decision on the disclosure of his private signal. His equilibrium strategies regarding when to reveal his signal and which quality type of signal he reveals vary according to the asymmetry in the reward and the penalty as follows.

If the penalty is larger than the reward, the unique equilibrium is a separating equilibrium in which M reveals his signal only if it is of low quality. It is in order to induce U's imitation, which enables him to avoid the worst case in which he is penalized alone. On the other hand, if the reward is larger than the penalty, each agent has an incentive to differentiate himself from the other. However, it also bears the risk of being penalized alone. Hence what matters is whether the reward is sufficiently large enough to make agents take this risk. If the reward is weakly large, the incentive to differentiate is not motivated well and there exists a separating equilibrium in which M reveals his signal only if it is of low quality. On the other hand, if the reward is sufficiently large, there exist both a separating- and a pooling equilibrium in which M's high quality signal is always revealed. The disclosure of a signal of this case is in order to induce U's divergence which is a necessary condition for getting reward alone.

In addition, we show that i) the asymmetry in reward and penalty is a necessary condition for the disclosure of M's signal and ii) the imperfect information about the precision of M's signal is a necessary condition for the disclosure of M's high quality signal.

Although we take an example of agents' competition in a forecasting market, the derived intuitions can also be applied to other cases such as R&D and the technology innovation. In particular, the competition between firms for occupying the global standard in technology adoption is a good example which fits the scenario of this model well. We discuss this in Section 6.

The rest of this chapter is organized as follows. Section 2 reviews related literature. Section 3 introduces a model. Section 4 deals with M's truthfulness in revealing his private signal. Section 5 characterizes the equilibrium. Section 6 is a discussion and Section 7 concludes.

## 2 Related literature

Conner and Rumelt (1991) and Conner (1995) explicitly mention the possibility of gaining a strategic advantage by being imitated. Both articles propose that allowing other firms' imitation can be a dominant strategy if a positive network externality is present. In their models, the network externality, defined exogenously, is positive in the sense that other firms' imitation can increase the size of the market; that is, although the pie must be shared, if the pie itself grows, thus increasing each firm's slice, imitation can be advantageous. The positive externality which is present in my model is different in that encouraging other's imitation is for the purpose of securing the minimum amount of pie. Growing the size of pie is not the main objective. Also, whether the externality is positive or negative is derived endogenously from the payoff structure and the information quality. De Fraja (1993) considers the setting in which a firm can have an incentive to reveal knowledge to its rival during the patent race. In his model, although the rival's success may be unfavorable in short-term, if there are the product market benefits as its consequence, a firm may intend to reveal knowledge.

In my model, the strategic incentive to be imitated is for the sake of avoiding the worst case. To my knowledge, Gallini (1984) is the first innovating paper which proposes the possibility that information can be revealed for this purpose. Her model proposes that the licensing can be used to prevent rivals from doing R&D activity which can yield the better technology. However, the model used in Gallini (1984) is different from what is used in my paper. For example, in her model, there is no process of the inference of private information and so the analysis regarding which quality type of information is revealed is not dealt with. Also my model proposes the existence of the equilibrium where information is revealed strategically to induce the other's deviation.

Okuno-Fujiwara, Postlewaite and Suzumura (1990) provide the sufficient condition under which there is a complete revelation of private information. In their model, what the privately informed agents decide is whether to reveal some or all of their information to other agents. That is, the decision is about the optimal amount of information they share with other agents. In our model, what the informed agent decides is the quality of information to disclose, not the amount of information.

Mailath (1993) addresses the topic of market entrance using the model in that only one informed firm has a waiting option and the uninformed firm does not. In his model, because of the unravelling effect, the unique stable equilibrium is the one where the informed firm operates as the leader although its ex-ante profit is higher when it delays and produces simultaneously with the uninformed firm. His model assumes that the informed firm's information is perfect. Hence how the information quality matters is not considered. In my model, on the other hand, as the informed agent's information is imperfect, its quality plays a critical role in a decision on information disclosure. Also what the uninformed agent can infer from the informed agent's delay is not the content of his private signal but its precision.

The topic of information sharing has been analyzed widely in depth in I.O. literature. (Vives (1990), Raith (1996) and Creane (1998) provide a good survey of this issue.) In general, this literature explores the conditions under which competing firms benefit from sharing information and the incentives under which firms commit to the rule of information sharing. It is also assumed that information is revealed truthfully by assuming that an agency is responsible for transferring the information correctly or that information is verifiable. Our model does not address the topic of commitment. Whether to reveal private information is determined by the informed agent's private incentive in the setting of non-cooperative competition.[2]

[2] Unlike other papers in the literature, Ziv (1993) addresses the topic of truthfulness in information revelation. It attempts to endogenize the incentives for truthful information sharing in an oligopoly by providing a mechanism under which the truth-telling of private information is in its own best interest, despite the fact that verification is impossible.

Also the truthfulness of the revealed information, especially when it is not verifiable, is not assumed but derived endogenously.

## 3 Model

Suppose there are two agents $i \in \{M, U\}$ whose jobs are to provide a forecast about the unknown true state of a forthcoming period. These two agents are the agencies which work independently in the forecasting industry.[3] The true state is $w \in \{H, L\}$ and these two states are mutually exclusive. The prior probability of each state is $\Pr(w = H) = \Pr(w = L) = \frac{1}{2}$. Before making a forecast, one agent has the opportunity to observe his own signal $\theta \in \Theta = \{h, l\}$ which is correlated with the true state. The other agent has no chance to observe any informative signal correlated with the true state. Throughout this paper, we denote the agent who can observe $\theta$ as M (informed agent), and the other agent who cannot as U (uninformed agent). It is assumed that $\theta \in \{h, l\}$ is private information, so that U does not know which signal is observed by M.

The signal $\theta$ partially reveals information about the true state in following manner: $\Pr(\theta = h|\, w = H) = \Pr(\theta = l|\, w = L) = p$ and $\Pr(\theta = h|\, w = L) = \Pr(\theta = l|\, w = H) = 1 - p$ where $p \in \left(\frac{1}{2}, 1\right)$. Here, $p$ measures the precision of M's signal $\theta$ which denotes the quality of information. As $p > \frac{1}{2}$, $\theta$ is meaningful information in the sense that it delivers information about the unknown true state. However, as $p < 1$, it is imperfect information. We assume that $p$ is private information and it is drawn from the uniform distribution $Z(p)$ where $p \in \left(\frac{1}{2}, 1\right) = P$.

M's action set is denoted by $\{a_m, t_m\}$. Here, $a_m \in F = \{h, l\}$ denotes M's forecast. If $a_m = h$ ($a_m = l$), it denotes that M's forecast is $w = H$ ($w = L$). M has two rounds during which he can forecast; this forecast is irreversible and $t_m \in T = \{t_1, t_2\}$ denotes M's timing of forecast where $t_1$ ($t_2$) denotes round 1 (round 2). M can decide when to forecast endogenously. If he forecasts in round 1 (round 2), then it is denoted by $t_m = t_1$ ($t_m = t_2$). The waiting option given to M is in order to analyze his strategic decision regarding information disclosure. For example, if an agent is displeased by the fact that others observe his forecast and infer his private information, he may delay his forecast to conceal his information. If, on the other hand, he regards it as beneficial, he may prefer to reveal his information by announcing a forecast without a delay. Also we assume that although M delays his forecast, no cost is imposed. This is in order to rule out the possibility that M, despite his desire to delay, avoids doing so because it is costly. Therefore, M's strategic decision on when to forecast is a voluntary decision which is free from a concern of a costly delay. [4] On the other hand, U is required to forecast only in round 2. Hence, U's action set is $A_u = \{a_u\}$ where $a_u \in F = \{h, l\}$. If $t_m = t_1$, that is, if both agents forecast sequentially,

[3] We want to distinguish our case from those in which the agents are hired by one principal and the discussion regarding the optimal contract provided by the principal can be the main topic. This makes the model free from subordinate concerns, such as why the uninformed agent is hired by the principal.

[4] Although we add the cost for a delay, the additional result would be the following: If M wants to act without a delay, a cost for a delay has no effects. On the other hand, if M wants to delay his action, he considers both the expected gain from a delay and a given cost. If the latter (former) dominates the former (latter), he would act without (after) a delay.

U can observe M's forecast before announcing his own forecast. On the other hand, if $t_m = t_2$, that is if both agents forecast simultaneously, U has no chance to observe M's forecast. In this case, U knows that M did not take the opportunity to forecast in round 1.

Each agent's ex-post payoff, which is given by the market, is determined after the realization of the true state $w$ and is conditional on the correctness of both agents' forecasts, $a_i$ and $a_{-i}$, as follows:

| $w$ | $a_u = w$ | $a_u \neq w$ |
|---|---|---|
| $a_m = w$ | $1, 1$ | $\gamma, -\phi$ |
| $a_m \neq w$ | $-\phi, \gamma$ | $-1, -1$ |

**Table 1: Ex-post payoff structure, $\gamma > 1$ and $\phi > 1$**

We assume that market does not know which agent is informed and not informed. So in evaluating the agents, it considers only the correctness of both agents' forecasts.[5] This payoff structure is designed to incorporate the competitive environment two agents face. Suppose that both agents forecasted identically. Then if their forecasts reveal the true state correctly, both earn $+1$ and, if not, both earn $-1$. On the other hand, if both agents' forecasts are different, the agent who made a correct forecast gets $\gamma > 1$ and the agent who made a false forecast gets $-\phi < -1$. In other words, if an agent's forecast turns out to be correct, the other agent's identical forecast causes a negative externality because the reward must be divided. On the other hand, if an agent's forecast turns out to be wrong, the other agent's identical forecast causes a positive externality because the penalty will be shared. However, as the true state is revealed only after both agents forecast, whether $a_i = w$ or $a_i \neq w$ is not verified in advance and therefore an uncertainty is embedded. In the main part of the analysis, we assume that $\gamma \neq \phi$.[6] Hence, our case is either $\gamma > \phi$ or $\gamma < \phi$ where the first (second) denotes the case in which the payoff structure is biased toward the reward (penalty).

The timing of the game is described as follows:

T1) Nature chooses $\theta \in \{h, l\}$ and $p \in \left(\frac{1}{2}, 1\right)$. M observes both $\theta$ and $p$. The ex-post payoff structure is announced.

T2) Before round 1 starts, M decides when to forecast (either in round 1 or in round 2, but not in both) and whether to be truthful in revealing $\theta$ or not.

T3) Round 1 starts. M forecasts if he decided to do so. If not, he waits until round 2 starts.

T4) Round 2 starts. If M forecasted in round 1, U observes $a_m$ and forecasts as the follower. If not, both M and U forecast simultaneously.

[5] Our model adopts the framework of a forecasting contest model. The bottom line of this scenario is that the market commits ex-ante to a particular payoff structure. Compared to the reputational herding model, in which the market is unable to commit ex ante to a particular evaluation rule and therefore evaluates the agents' type while using all information ex post, a forecasting contest model shows the payoff effects more clearly.

[6] In section 6.1, we go over the case in which the reward and the penalty are symmetric.

T5) After two rounds are over, the true state $w$ is revealed and each player earns his payoff following the ex-post payoff structure.

A pure strategy for M is a pair of functions $\sigma_m = (\sigma_{m1}, \sigma_{m2})$ where i) $\sigma_{m1} : \Theta \times P \longrightarrow T$ is the choice regarding the timing of forecast as a function of $\theta$ and $p$, and ii) $\sigma_{m2} : \Theta \times P \times T \longrightarrow F$ is M's choice of forecast as a function of $\theta, p$ and $t_m$. The pure strategy for U is $\sigma_u = (\sigma_{u1}, \sigma_{u2})$ where $\sigma_{u1} : a_m \longrightarrow a_u$ is U's forecast as a function of M's forecast when $t_m = t_1$ (when $a_m$ is observable) and $\sigma_{u2}$ is a forecast when $t_m = t_2$ (when $a_m$ is not observable). We also allow a mixed strategy for both players.

This model uses the Perfect Bayesian equilibrium concept. Within the model, there exist two types private information, $\theta \in \{h, l\}$ and $p \in \left(\frac{1}{2}, 1\right)$ and U has a chance to infer these according to M's realized timing of forecast. U will have the chance to infer $\theta$ only if $t_m = t_1$. In this case, as $a_m$ is observable, U forms a belief with regard to whether M is truthful in revealing $\theta$ or not. Also, U can always observe M's timing of forecast which conveys information about $p$. That is, after observing $t_m \in \{t_1, t_2\}$, U forms a belief regarding $p$. Let $\lambda(a_m = \theta)$ be U's belief for the truthfulness of M's action and $Z(p|\, t_m)$ be a posterior belief over $p$. Then, each agent's strategy $\sigma_m$, $\sigma_u$ and $\lambda(\theta = a_m)$, $Z(p|\, t_m)$ constitute the Perfect Bayesian equilibrium if each agent's ex-ante expected payoffs are maximized for given beliefs $\lambda(\theta = a_m)$, $Z(p|\, t_m)$, and the other agent's strategy. Also, in equilibrium, $\lambda(\theta = a_m)$ and $Z(p|\, t_m)$ should be consistent with $\sigma_m$ in the Bayesian sense.

## 4 Truthfulness in revealing $\theta$

Note that M's signal $\theta \in \{h, l\}$ is private information. As U can observe only the announced forecast $a_m$, M can make a strategic decision on whether to be truthful in revealing $\theta$ or not. In this section, we show that, in regard to the truthfulness in revealing $\theta$, M's strategy to deviate from $\theta$ is strictly dominated regardless of $t_m \in \{t_1, t_2\}$. That is, M always reveals $\theta$ truthfully. This result simplifies the analysis which will be discussed in the subsequent sections.

**Proposition 1**

*M reveals her signal $\theta$ truthfully for all $p \in \left(\frac{1}{2}, 1\right)$ regardless of the timing of his forecast.*

**Proof of Proposition 1**

In the appendix.

If we recall the given ex-post payoff structure (Table 1), the result that M has no incentive to ignore his signal $\theta$ is quite intuitive. According to Table 1, the ex-post payoffs under $a_i \neq w$ are strictly smaller than those under $a_i = w$. That is, $Min\ \pi_i(a_i = w, a_{-i}) = 1 > -1 = Max\ \pi_i(a_i \neq w, a_{-i})$. Therefore M's primary objective should be to make a correct forecast. Hence, when no other meaningful information is available, M has no incentive to ignore $\theta$. This is why M's strategy which is to deviate from $\theta$ is strictly dominated.

An important implication of Proposition 1 is that, especially when $t_m = t_1$, because of the requirement that U's belief should be consistent in equilibrium, U assigns zero probability to the possibility that M deviates from $\theta$, i.e., $\lambda(\theta = a_m) = 1$ where $\lambda(\cdot)$ is U's belief that M's is truthful in revealing $\theta$. That is, if $a_m$ is observable, U can infer $\theta$ perfectly. From M's standpoint of view, it also implies that the only option he can use to maximize his expected payoff is a decision on the timing of forecast. Moreover, as he should reveal $\theta$ truthfully regardless of his timing of forecast, the decision on the timing of forecast is no more than a decision on the disclosure of his private signal $\theta$: *If $t_m = t_1$, it is in order to reveal $\theta$ to U and if $t_m = t_2$, it is in order to prevent $\theta$ from being revealed to U.*

# 5 Equilibrium

In the following, we check the existence of i) (partially revealing) separating equilibrium and ii) pooling equilibrium. Note that M's information quality, which is his type, is a continuum, i.e., $p \in \left(\frac{1}{2}, 1\right)$, while the action set is discrete, i.e., $t_m \in \{t_1, t_2\}$. Hence M's timing of forecast does not reveal M's type $p$ perfectly. In that sense, the separating equilibrium of this model reveals the type partially. Also regarding pooling equilibrium, the following two pooling strategies, i) "$t_m = t_1$ for all $p \in \left(\frac{1}{2}, 1\right)$" and ii) "$t_m = t_2$ for all $p \in \left(\frac{1}{2}, 1\right)$", can be considered.

**Definition 1**
*We say that it is a revealing equilibrium if $\theta$ is revealed for $p \in (a, b)$ where $a, b \in \left[\frac{1}{2}, 1\right]$.*

As we are primarily interested in M's strategic disclosure of informative signal, in the most part of the paper, we restrict our attention to the revealing equilibrium.[7] Then the separating equilibrium is the revealing equilibrium. The pooling equilibrium where $t_m = t_1$ for all $p \in \left(\frac{1}{2}, 1\right)$ is also the revealing equilibrium.

## 5.1 Separating equilibrium

In round 2, U faces one of the following two situations: $t_m = t_1$ and $t_m = t_2$. If M did not forecast in round 1, U's best response in round 2 is to use a mixed strategy, i.e., $z = [0, 1]$ where $z = \Pr(a_u = h)$ because no information correlated with the true state is available and therefore both $a_u = h$ and $a_u = l$ attain the same expected payoffs.

Next, consider the case in which M forecasted in round 1, i.e., $t_m = t_1$. From Proposition 1, U can infer $\theta$ perfectly from observing $a_m$. Hence, what we have to consider now is solely U's posterior belief regarding $p$ inferred from observing that $t_m = t_1$. Here, we assume that M's strategy regarding the timing of forecast is a monotone function of the information quality.

**Definition 2**
*1) Monotone increasing strategy: $\exists x \in \left(\frac{1}{2}, 1\right)$ s.t. for $p \in \left(\frac{1}{2}, x\right)$, $t_m = t_1$ and for $p \in (x, 1)$, $t_m = t_2$.*

[7] The existence of the non-revealing pooling equilibrium where $t_m = t_2$ for all $p \in \left(\frac{1}{2}, 1\right)$ is discussed in section 6.4.

*2) Monotone decreasing strategy:* $\exists y \in \left(\frac{1}{2}, 1\right)$ *s.t. for* $p \in (y, 1)$, $t_m = t_1$ *and for* $p \in \left(\frac{1}{2}, y\right)$, $t_m = t_2$.

Even M knows that if he forecasts in round 1, U can infer $\theta$ perfectly. As no cost is imposed for a delay of forecast, if M forecasts in round 1 this is his voluntarily decision which is in order to give U a chance to infer $\theta$ perfectly. Also, the monotone increasing (decreasing) strategy means that he wants to reveal his informative signal if it is of low (high) quality. The analysis yields the following separating equilibrium.

**Proposition 2: Separating equilibrium**

*1) Suppose that* $\gamma < \phi$. *Then there exists the unique equilibrium in which if* $p \in \left(\frac{1}{2}, \frac{\phi-1}{\gamma+\phi-2}\right)$, $t_m = t_1$ *and if* $p \in \left(\frac{\phi-1}{\gamma+\phi-2}, 1\right)$, $t_m = t_2$. *If* $t_m = t_1$, *U imitates* $a_m$.

*2) Suppose that* $\phi < \gamma < 3\phi+2$. *Then there exists the unique equilibrium in which if* $p \in \left(\frac{1}{2}, \frac{3\gamma-\phi+2}{2\gamma+2\phi+4}\right)$, $t_m = t_1$ *and if* $p \in \left(\frac{3\gamma-\phi+2}{2\gamma+2\phi+4}, 1\right)$, $t_m = t_2$. *If* $t_m = t_1$, *U imitates or deviates from* $a_m$ *with probability* $\frac{1}{2}$.

*3) Suppose that* $\gamma > 3\phi + 2$. *Then there exists the unique equilibrium in which if* $p \in \left(\frac{1}{2}, \frac{\gamma-\phi}{\gamma+\phi+2}\right)$, $t_m = t_2$ *and if* $p \in \left(\frac{\gamma-\phi}{\gamma+\phi+2}, 1\right)$, $t_m = t_1$. *If* $t_m = t_1$, *U imitates or deviates from* $a_m$ *with probability* $\frac{1}{2}$.

*For all three cases, if* $t_m = t_2$, *U uses a mixed strategy, i.e.,* $\Pr(a_u = h) = [0, 1]$.

**Proof of Proposition 2**

In the appendix.

Consider the situation in which the payoff structure is biased toward the penalty, i.e., $\gamma < \phi$. If M forecasts without a delay, U infers that M's signal $\theta$ is of low quality. Hence, the credibility he assigns to the correctness of M's signal may not be high. However, as the payoff structure is biased toward the penalty, U's main concern is to avoid being penalized alone. Hence, in spite of his weak belief for the correctness of $\theta$, U imitates M's forecast. By forecasting identically, at least U can avoid the worst case in which he earns the lowest payoff $\pi_u = -\phi$. Of course, as $\theta$ is correlated with the true state, it seems reasonable for U to rely on it. However, it should be noted that the payoff structure biased toward the penalty also affects U's strategic behavior of imitating $a_m$. In particular, U's different response when the payoff structure is biased toward the reward, as stated below, clearly shows how the given payoff structure affects U's strategic response against the inferred $\theta$.

Although M observes the informative signal, he is not free from the concern that he could be penalized because his information is not perfect. However, at least as M observes the signal correlated with true state, his belief for the correctness of his signal matters. If it is high, i.e., $p \in \left(\frac{\phi-1}{\gamma+\phi-2}, 1\right)$, M gives more weight to the possibility that his signal is correct. If forecast turns out to be correct, U's identical forecast causes a negative payoff externality because the reward must be shared. Thus, he regards U's identical forecast as a strategic substitute and therefore wants to prevent his signal from being revealed. This is why M delays his forecast when his signal is of relatively high quality. On the other hand, if the precision of his signal is low, i.e., $p \in \left(\frac{1}{2}, \frac{\phi-1}{\gamma+\phi-2}\right)$, he is greatly concerned about the possibility that he could be penalized alone. If his forecast turned out to be wrong, the other's identical forecast would cause a positive payoff externality because the penalty can be shared. Hence, in this case, M regards U's identical forecast as a strategic complement and therefore wants to induce U's imitation by revealing his signal to U. This explains why M forecasts without a delay when his signal is of low quality, even when a waiting option is not costly.

Next, consider the case in which the payoff structure is biased toward the reward, i.e., $\gamma > \phi$. In this case, there exist two different separating equilibria according to how sufficiently large the reward is. In both separating equilibria, U uses a mixed strategy. That is, not like the previous case in which the penalty is greater than the reward, U deviates from $a_m$ with a positive probability even though he knows that $a_m$ delivers meaningful information about true state. Note that the reward $\gamma$ can be earned only if an agent is the unique one who forecasted correctly. That is, alongside being correct, the different forecast by both players is the other necessary condition for earning $\gamma$. So the incentive to differentiate is initiated, which yields the possibility that U deviates with a positive probability.

Such an incentive also affects M. However, how sufficiently large the reward is matters because a different forecast also bears a risk of announcing a wrong forecast alone. Hence, whether the reward is sufficiently large enough to make him take such a risk affects M's decision regarding which quality type of signal he intends to disclose. If the reward is weakly greater than the penalty, i.e., $\phi < \gamma < 3\phi + 2$, the incentive to differentiate is not motivated much and he may expect that it is same to U. Then he is more likely to be biased toward avoiding the risk of being penalized alone, so he reveals his signal if it is of low quality. If his signal is relatively precise, he may conceal it because he still has an incentive to differentiate. On the other hand, if the reward is sufficiently large, i.e., $\gamma > 3\phi + 2$, interestingly he reveals his signal only if it is of high quality. If payoffs are given in such a way, M may have a relatively strong incentive to differentiate and expect that the same applies to U. In this case, if he intends to reveal his signal, it is better to do so when the signal is of high quality for the following reasoning. If U deviates from what the low quality signal says, U's forecast is more likely to be correct compared to the case where U deviates from what the high quality signal says. That is, if signal is relatively precise, it is better to reveal it and if not, it is better to conceal it.

### 5.2 Pooling equilibrium

Consider the following strategy for M: $t_m = t_1$ for all $p \in \left(\frac{1}{2}, 1\right)$. If U observes that $t_m = t_1$ which is on the equilibrium path, U's posterior belief regarding $p$ is $p \in \left(\frac{1}{2}, 1\right)$. On the other hand, for $t_m = t_2$ which never occurs along the equilibrium path, we can assign arbitrary U's off-the equilibrium posterior belief regarding $p$. However, it is important to note that, regardless of U's off-the equilibrium belief, U's best response in round 2 is still to use a mixed strategy, i.e., $z = [0, 1]$ where $z = \Pr(a_u = h)$ because $a_m$ is not observable and therefore $\theta$ cannot be inferred. That is, U's off-the equilibrium belief does not affect U's best response and therefore it does not affect M's expected payoff when $t_m = t_2$. Then what M should consider is only whether to i) show $\theta$ in order to induce U's imitation or deviation or ii) conceal $\theta$ and make U use a mixed strategy. The analysis yields the following result:

**Proposition 3: Pooling equilibrium**

*1) Suppose that* $\gamma > 3\phi + 2$*. Then there exists the unique pooling equilibrium in which* $t_m = t_1$ *for all* $p \in \left(\frac{1}{2}, 1\right)$*. If* $t_m = t_1$*, U deviates from* $a_m$*.*

*2) Suppose that* $\gamma = 3\phi + 2$*. Then there exists the unique pooling equilibrium in which* $t_m = t_1$ *for all* $p \in \left(\frac{1}{2}, 1\right)$*. If* $t_m = t_1$*,* $z < \frac{1}{2}$ *where* $z = \Pr(a_u = a_m)$*.*

*For both cases, if* $t_m = t_2$*, U uses a* mixed strategy, i.e., $z = [0, 1]$ where $z = \Pr(a_u = h)$.

**Proof of Proposition 3**

In the appendix.

Proposition 3 indicates that for the revealing pooling equilibrium to be supported, i) the reward should be sufficiently large and ii) U deviates or is more likely to deviate from $a_m$ when observable. If the reward is strongly stressed, even U, who is uninformed about the true state, intends to deviate from $a_m$ if observable because he is willing to take the risk of being penalized alone for the sake of earning $\gamma$. This also applies to M. If M expects that U intends to differentiate himself by deviating from $a_m$, then it would be better for M to always reveal his signal to U. If U has no chance to observe $a_m$, sometimes it can be that $a_m = a_u$ because U uses a mixed strategy. On the other hand, if he forecasts without a delay, the necessary condition for earning $\gamma$, which is that $a_m \neq a_u$, is always guaranteed. This explains why M reveals his signal to U for all $p \in \left(\frac{1}{2}, 1\right)$.

## 5.3 Summary of the revealing equilibrium

The derived revealing equilibrium can be summarized as follows:

| | $\gamma < \phi$ | $\phi < \gamma < 3\phi + 2$ | $\gamma = 3\phi + 2$ | $\gamma > 3\phi + 2$ |
|---|---|---|---|---|
| **Separating equilibrium** | | | | |
| Disclosure | √ (low $p$) | √ (low $p$) | | √ (high $p$) |
| No disclosure | √ (high $p$) | √ (high $p$) | | √ (low $p$) |
| **Pooling equilibrium** | | | | |
| Disclosure | | | √ (for all $p$) | √ (for all $p$) |

**<Table 2: Revealing equilibrium>**

If $\gamma < 3\phi + 2$ ($\gamma = 3\phi + 2$), the unique revealing equilibrium is the separating (pooling) equilibrium. On the other hand, if $\gamma > 3\phi + 2$, there exist both separating- and pooling equilibrium as the revealing equilibrium. Moreover interestingly in both equilibria, high quality signal is revealed always. In particular, note that the pooling equilibrium, where $t_m = t_1$ for all $p \in \left(\frac{1}{2}, 1\right)$, is free from the concern that it can be based on the irrational belief for the off-the-equilibrium path because U's belief for $t_m = t_2$ does not affect M's expected payoff when $t_m = t_2$. Figure 1 depicts the regions in which different type of equilibrium configurations exist.

< *Figure 1 here* >

# 6 Discussion

## 6.1 When the reward and the penalty are symmetric

When we consider the revealing equilibrium in above section, it was already checked that the there exists no revealing equilibrium if the reward and the penalty are symmetric. Now we consider the following M's pooling strategy: "$t_m = t_2$ for all $p \in \left(\frac{1}{2}, 1\right)$" when $\gamma = \phi$. If we can show that this pooling strategy is sustained as an equilibrium strategy, it is the unique equilibrium when $\gamma = \phi$. Under this given pooling strategy, if U observes that $t_m = t_2$, $z = [0,1]$ where $z = \Pr(a_u = h)$. On the other hand, for $t_m = t_1$ which is off-the-equilibrium path, any arbitrary U's belief over $p$ can be assigned. In the following, we show that regardless of U's off-the-equilibrium belief, non-revealing pooling equilibrium is the unique equilibrium of the case in which $\gamma = \phi$.

**Proposition 4**

*Suppose that the reward and the penalty are symmetric, i.e., $\gamma = \phi$. Then the unique equilibrium is the non-revealing pooling equilibrium in which $t_m = t_2$ for all $p \in \left(\frac{1}{2}, 1\right)$.*

**Proof of Proposition 4**

In the appendix.

This implies that the asymmetry in reward and penalty is the necessary condition for the strategic disclosure of informative signal.

## 6.2 When information quality is public information

In this section, in the interest of comparison, we consider the case in which the precision of M's signal is public information. Proposition 5 verifies that if $\gamma \neq \phi$, the revealing equilibrium is derived always. However, there exists no equilibrium where M's high quality signal is disclosed, which is the most distinctive result compared to the case where $p$ is private information.

**Proposition 5**

*Suppose that $p$ is public information.*

*1) Suppose $\gamma < \phi$. If $p \in \left(\frac{1}{2}, \frac{\phi-1}{\gamma+\phi-2}\right)$, $t_m = t_1$ and if $p \in \left(\frac{\phi-1}{\gamma+\phi-2}, 1\right)$, $t_m = t_2$. If $t_m = t_1$, U imitates $a_m$ for all $p \in \left(\frac{1}{2}, 1\right)$.*

*2) Suppose $\gamma > \phi$. If $p \in \left(\frac{1}{2}, \frac{\gamma+1}{\gamma+\phi+2}\right)$, $t_m = t_1$ and if $p \in \left(\frac{\gamma+1}{\gamma+\phi+2}, 1\right)$, $t_m = t_2$. If $t_m = t_1$, for $p \in \left(\frac{1}{2}, \frac{\gamma+1}{\gamma+\phi+2}\right)$, U deviates from $a_m$ and for $p \in \left(\frac{\gamma+1}{\gamma+\phi+2}, 1\right)$, U imitates $a_m$. If $p = \frac{\gamma+1}{\gamma+\phi+2}$, in the mixed strategy equilibrium, $\Pr\left(t_m = t_1\right) \in [0,1]$ and $\Pr(a_u = a_m) = \frac{1}{2}$.*

*3) Suppose $\gamma = \phi$. Then, for all $p \in \left(\frac{1}{2}, 1\right)$, $t_m = t_2$. If $t_m = t_1$, U imitates $a_m$ for all $p \in \left(\frac{1}{2}, 1\right)$.*

**Proof of Proposition 5**

In the appendix.

Consider the case where the payoff structure is biased toward the reward. In this case, U has an incentive to be differentiated. However, if he is perfectly informed that $\theta$ is of high quality, i.e., $p \in \left(\frac{\gamma+1}{\gamma+\phi+2}, 1\right)$, the incentive to be differentiated is dominated because ignoring quite precise signal is not attractive. Hence, U imitates $a_m$ if it is observable. In the case of M, he intends to induce both agents' different forecasts in order to be differentiated. He knows that if high quality $\theta$ is revealed, U imitates it and the opportunity for him to earn $\gamma$ is foregone. Hence, he prevents his signal from being revealed to U. On the other hand, if the precision is relatively low, i.e., $p \in \left(\frac{1}{2}, \frac{\gamma+1}{\gamma+\phi+2}\right)$, U does not give much credit to the correctness of M's signal. Therefore the incentive to be differentiated dominates and U diverges from $a_m$ if observable. Then it would be better for M to reveal his signal in order to guarantee that $a_m \neq a_u$, which is a necessary condition for earning $\gamma$. If $\gamma < \phi$, whether $p$ is private or public information, U's equilibrium strategy when $t_m = t_1$ is same. Hence the same equilibrium is derived for both cases.

## 6.3 Application: Competition between firms for acquiring the global standard

In this section, we discuss how the economic intuitions derived in the model can be applied to the firms' strategic behaviors regarding information disclosure in a competitive market. As the motivating examples, suppose that two competing firms, Firm 1 and Firm 2, consider adopting the new technology when there are two potential alternatives, X and Y. One firm can select only one, either X or Y, because of the budget constraint for R&D. If both technologies are introduced to a market, only one technology can be successful, because both are substitutes. Which is the better technology or will be preferred by the market is uncertain.

The war between firms for acquiring the global standard in technology would be a good example which fits this scenario because only one technology can survive as the unique standard. The competition for video standard between Sony's BETA and JVC's VHS is a famous example. Also the competition between LCD TV and PDP TV, the one between Sony's Blu-ray and Toshiba's HD-DVD, and the one between NAND flash and NOR flash in flash memory market are recent examples.

Actually, the payoffs structure, provided in the model, can be applied to this scenario. Assume that, out of two technologies X and Y, the situation turns out to be favorable for X. If both firms adopted X, then because of market sharing, a firm may earn a smaller profit than if it had been the only firm that adopted X. On the other hand, if both firms adopted Y, the products will be purchased anyway because there is no alternative choice in the market. Hence, both firms' same mistake of adopting Y is better than one firm making the mistake by itself. Here, it would not be ad hoc to assume that when both firms adopt Y, each firm earns a smaller market-sharing profit than the case in which both adopted X. The reasoning that there can be a potential increase in demand when the favorable technology was adopted and introduced in the market would support this.

Now suppose that Firm 1, which is a leading firm, has favorable information toward X. However, if it has a weak confidence in the correctness of its information, concealing its information can induce the following worst case: If its information is concealed and Firm 2 selects Y, then both X and Y will be introduced and the market will have a chance to compare both technologies. In this case, if the market prefers Y over X, then Firm 1 will lose a market, especially after spending tremendous money, time and effort for R&D without retrieving those. Now if Firm 1 is so concerned about this possibility, then disclosing its information to Firm 2 can be attractive. If it reveals information that X is favorable and Firm 2 follows it, only X will be introduced in the market. Then although Y was actually the better technology, because the option of purchasing Y is not given, X would be purchased. Of course, in this case, disclosing its information makes Firm 1 give up the best case in which it dominates a market as a monopolist. However the worst case where it loses a market can be prevented and instead the positive amount of profit from market sharing can be guaranteed. Firm 2 would be more likely to follow Firm 1's information as it is also concerned more about the worst case. Then this would be no more than the case where information can be disclosed strategically for the sake of being imitated in order to avoid the worst case, which is one of the main results of this model. In this example, the informed firm's information disclosure is for the sake of market-sharing through depriving the market of its opportunity to compare all options and consume the better and favorable technology.

Especially, nowadays the loss incurred from losing in the competition becomes bigger because the astronomical amount of money is needed in a stage of initial investment for R&D. For example, on February 2008, Toshiba announced plans to discontinue development and manufacturing of HD-DVD players after losing in the competition against Blu-ray by Sony. Its loss is expected as $986 million in its HD-DVD business.[8] Also in flash memory market, NOR flash memory is expected to lose in a competition against NAND flash memory. On March 2009, the firm Spansion, the major company which has been providing NOR flash memory, filed for Chapter 11 bankruptcy protection while having laid off 3000 workers (approximately 35% reduction).[9] In this way, as the loss incurred from losing in a competition becomes bigger, firms would be more likely to avoid the worst case in which it loses in a competition. In this case, the information disclosure for this purpose can be the attractive option.[10]

On the other hand, if each firm expects that the monopoly profit is sufficiently large enough to offset the loss incurred when it loses in the competition, this would correspond to the case where the reward is sufficiently larger than the penalty. Then what we can expect is the possibility that information can be disclosed strategically in order to induce the competing firm to adopt the alternative technology. Implicitly, the competition between firms in the war for global standard, provided in the above examples, can be understood as the results of such a case in which each firm was biased toward hoping for the best rather than preparing for the worst.

[8] Source: Reuters March 13th 2009

[9] Source: New York Times, March 2, 2009

[10] Actually, there were some attempts between competing firms to avoid the costly cutthroat competition. The Blu-ray Disc Association and DVD Forum attempted to negotiate a compromise in early 2005. Also before the start of web browser war, on May 1995, Microsoft suggested Netscape a 'market sharing' rather than competition.

### 6.4 Non-revealing pooling equilibrium

In the main text, in order to focus on the matter of strategic information disclosure, we only dealt with the equilibrium where M's informative signal is disclosed. However, we can also consider the possibility that M intends to conceal his signal for all $p$ (Non-revealing pooling equilibrium). The corresponding question is whether M's pooling strategy, "$t_m = t_2$ for all $p \in \left(\frac{1}{2}, 1\right)$", can be sustained as the equilibrium strategy or not. Under given this pooling strategy, for the case in which $t_m = t_1$, we can assign an arbitrary U's posterior off-the-equilibrium belief over $p$. In particular, unlike the case of the revealing pooling equilibrium, the off-the-equilibrium belief matters in deriving the equilibrium because M's expected payoff when $t_m = t_1$ is affected by it.

If we maintain the assumption that M's strategy is either monotone increasing or monotone decreasing, we can derive the existence of the non-revealing pooling equilibrium. The common features of the sufficient conditions under which M's signal is not revealed for all $p$ are as follows[11]:

*1) the reward is greater than the penalty*

*2) if $t_m = t_1$, U forms a off-the-equilibrium belief over p under which he imitates $a_m$.*

The intuition for these two conditions is straightforward; if the reward is greater than the penalty, M's incentive to differentiate is initiated. Hence, if M expects that U will imitate his forecast and therefore a necessary condition for earning $\gamma$ will not be satisfied, M intends to conceal his private signal through a delay.[12]

## 7 Concluding remarks

In this article, we explore the informed agent's strategic incentive to reveal his meaningful information to rival within a competitive environment. We show that, when the precision of the informed agent's information is private information, the asymmetry in the reward and penalty of the payoff structure is a necessary condition for the disclosure of meaningful information. It also plays an important role in the informed agent's decision regarding which quality type of information he reveals for which incentive. If the penalty is larger than the reward, his information is revealed only if it is of low quality; this disclosure reflects the desire to be imitated for the sake of avoiding the worst case. On the other hand, if the reward is larger than the penalty, which quality type of information is revealed is contingent on how large the reward is. In particular, if the reward is sufficiently large, there exist both separating- and pooling equilibrium in which high quality information is always revealed. It is also verified that

[11] The detailed analysis for this non-revealing pooling equilibrium is skipped.

[12] It can be worthwhile to ask whether U's off-the-equilibrium belief over $p$, which yields the non-revealing pooling equilibrium, is a rational belief or not. However, in the present model, the type (information quality) is continuum and the action set is discrete. Hence there is a limit in applying the well-known Intuitive criterion, D1 or D2 criterion directly, in order to do the equilibrium refinement, into this model. As our main focus is on the existence of the revealing equilibrium where informative signal is disclosed, we leave the the analysis on this matter as a future work.

incomplete information about the precision of the informed player's signal is a necessary condition for the disclosure of high quality information.

The current model has some limits in following sense. First, it is assumed that the prior belief for the true state is 1/2. This may preclude analyzing the question of how initial bias toward true state affect the disclosure of information. For example, if the informed player goes against the biased prior, it may signal that he has a really accurate information. That is, the uninformed player may have an advantage of being able to infer the precision of signal the informed player has from the revealed private signal itself, in addition to the timing of revelation. Second, we extend our analysis while assuming that the informed player uses a monotone strategy. So the current model does not provide an answer as to whether there exists an equilibrium where non-monotone strategy can be sustained. Third, in our model, only one player is informed and the other is not. The more general case would be the one where both players are informed but observe the signals with different precision. The analysis for this case, while maintaining the assumption that information quality is private information, would be worthwhile but demanding because each player's belief for the other player's information quality should be considered interactively. The current model does not allow the tractable analysis regarding these points. The extension of the model for considering these cases awaits future work.

# 8 Appendix

## 8.1 Proof of Proposition 1

**Case 1: When M forecasts in round 1**

M's pooling strategy is either "$a_m = h$ whether $\theta = h$ or $\theta = l$." or "$a_m = l$ whether $\theta = h$ or $\theta = l$.". Note that whether $\theta = h$ or $\theta = l$, both cases are ex-ante symmetric to M. Hence, the ex-ante expected payoff from being truthful (telling a lie) when $\theta_A = h$ should be the same as that for the case in which $\theta_A = l$. Hence, if being truthful (or telling a lie) is A's best response when $\theta_A = h$, it should be the same when $\theta_A = l$. Then, obviously M's pooling strategy cannot constitute an equilibrium because it means that sometimes being truthful is better and sometimes telling a lie is better, according to $\theta \in \{h, l\}$ although ex-ante symmetric.

Next M's separating strategies are "$a_m = h$ ($a_m = l$) if $\theta = h$ ($\theta = l$)" and "$a_m = h$ ($a_m = l$) if $\theta = l$ ($\theta = h$)". In the following, we show that the only separating strategy which constitutes an equilibrium is the first one, which implies that M is truthful in revealing $\theta$.

Without loss of generality, assume that $\theta = h$. In the following, $E\pi_m (a_m = \theta, \cdot)$ is M's expected payoff when he is truthful in revealing $\theta$ and $E\pi_m (a_m \neq \theta, \cdot)$ is the expected payoff when he deviates from $\theta$. If $t_m = t_1$ and therefore $a_m$ is observable, U decides whether to imitate or deviate from $a_m$.

If we consider the case in which U imitates $a_m$,

$$E\pi_m (a_m = \theta, \cdot) = \sum_{w \in \{H,L\}} \Pr(w | \theta = h) \pi_m (a_m = \theta, a_u = a_m, w) = 2p - 1$$

$$E\pi_m (a_m \neq \theta, \cdot) = \sum_{w \in \{H,L\}} \Pr(w | \theta = h) \pi_m (a_m \neq \theta, a_u = a_m, w) = 1 - 2p$$

Also if we consider the case in which U deviates from $a_m$,

$$E\pi_m (a_m = \theta, \cdot) = \sum_{w \in \{H,L\}} \Pr(w | \theta = h) \pi_m (a_m = \theta, a_u \neq a_m, w) = p\gamma - (1-p)\phi$$

$$E\pi_m (a_m \neq \theta, \cdot) = \sum_{w \in \{H,L\}} \Pr(w | \theta = h) \pi_m (a_m \neq \theta, a_u \neq a_m, w) = -p\phi + (1-p)\gamma$$

Now, let $\mu \in [0, 1]$ be the probability that U imitates M's action. Then,

$$E\pi_m (a_m = \theta, a_u, w) - E\pi_m (a_m \neq \theta, a_u, w) = -(\gamma\mu - 2\mu - \phi - \gamma + \mu\phi)(2p - 1) > 0$$

because $\gamma\mu - 2\mu - \phi - \gamma + \mu\phi < 0$ for all $\mu \in [0, 1]$.[13] Thus, regardless of U's best response in round 2, M's best response is to reveal $\theta$ truthfully. The case in which $\theta = l$ should yield the same result. This then implies that if $t_m = t_1$, the unique equilibrium is the separating equilibrium in which M reveals $\theta$ truthfully.

[13] $\gamma\mu - 2\mu - \phi - \gamma + \mu\phi = \mu(\gamma + \phi - 2) - \gamma - \phi$. So $\mu \gtreqless \frac{\gamma+\phi}{\gamma+\phi-2} \Longrightarrow \gamma\mu - 2\mu - \phi - \gamma + \mu\phi \gtreqless 0$. However as $\frac{\gamma+\phi}{\gamma+\phi-2} - 1 = \frac{2}{\gamma+\phi-2} > 0$, for all $\mu \in [0, 1]$, $\gamma\mu - 2\mu - \phi - \gamma + \mu\phi < 0$.

**CASE 2: When M forecasts in round 2**

In this case, M knows that U uses a mixed strategy, i.e., $z = [0,1]$ where $z = \Pr(a_u = h)$ because U has only the prior belief for the true state that $\Pr(w = H) = \Pr(w = L) = 0.5$. Although U forms a posterior belief for $p$ from observing $t_m = t_2$, as $a_m$ is not observable, no information about the true state is conveyed.

**Assumption A.1**[14]
*Suppose U has no chance to observe $a_m$. In this case, M believes that* $\Pr(a_u = h) = \Pr(a_u = l) = 0.5$.

Without loss of generality, assume that $\theta = h$. Then,

$$\begin{aligned} E\pi_m\left(a_m = h, a_u, w\right) &= \frac{1}{2}\left(\sum_{a_u\in\{h,l\}}\left(\sum_{w\in\{H,L\}}\Pr\left(\left.w\right|\theta\right)\pi_m\left(a_m = h, a_u, w\right)\right)\right) \\ &= \frac{1}{2}\left(2p - \phi + p\gamma + p\phi - 1\right) \end{aligned}$$

$$\begin{aligned} E\pi_m\left(a_m = l, a_u, w\right) &= \frac{1}{2}\left(\sum_{a_u\in\{h,l\}}\left(\sum_{w\in\{H,L\}}\Pr(\left.w\right|\theta)\pi_m\left(a_m = l, a_u, w\right)\right)\right) \\ &= \left(-\frac{1}{2}\right)\left(2p - \gamma + p\gamma + p\phi - 1\right) \end{aligned}$$

and

$$E\pi_m\left(a_m = h, a_u, w\right) - E\pi_m\left(a_m = l, a_u, w\right) = \frac{1}{2}\left(2p - 1\right)\left(\gamma + \phi + 2\right) > 0$$

Therefore, for all $p \in \left(\frac{1}{2}, 1\right)$, $\gamma > 1$ and $\phi > 1$, if $t_m = t_2$, M's best response is to reveal $\theta$ truthfully.

Finally, M reveals $\theta$ truthfully whether $t_m = t_1$ or $t_m = t_2$. ■

## 8.2 Proof of Proposition 2

The following result will be used frequently in the analysis.

**Result 1**: If $\gamma \geq \phi$, $\frac{\phi-1}{\gamma+\phi-2} \leq \frac{1}{2}$ and if $\gamma < \phi$, $\frac{1}{2} < \frac{\phi-1}{\gamma+\phi-2} < 1$.

[14] We can actually show that, when U uses a mixed strategy equilibrium, i.e., $z \in [0,1]$ where $z = \Pr(a_u = h)$, any M's belief $q = \Pr(a_u = h) \in [0,1]$ is rational belief for following reasons: i) For any M's belief $q \in [0,1]$, M's best response is to reveal $\theta$ truthfully. ii) As U cannot observe M's $\theta$, U's posterior belief should be about both the true state and M's private signal, i.e., $\Pr\left(w, \theta\right)$. This yields that $E_u\left(a_u = h\right) = E_u\left(a_u = l\right)$ and therefore any M's belief $q \in [0,1]$ is a consistent belief. We pick one specific value $q^* = \frac{1}{2}$, which seems the reasonable M's belief, compared to others. We use this assumption throughout the paper.

First, we assume that M's strategy is monotone increasing. Without loss of generality, assume that $a_m = h$. Then,

$$\begin{aligned} E\pi_u(a_u = a_m, w) &= \int \left( \sum_{w \in \{H,L\}} \Pr(w|\, \theta = h) \pi_u(a_u = a_m, w) \right) dZ(p|\, t_m = t_1) \\ &= \int_{p \in \left(\frac{1}{2}, x\right)} (p - (1-p))\, dZ(p) \\ &= \frac{1}{2}(2x - 1) \end{aligned} \quad (1)$$

$$\begin{aligned} E\pi_u(a_u \neq a_m, w) &= \int \left( \sum_{w \in \{H,L\}} \Pr(w|\, \theta = h) \pi_u(a_u \neq a_m, w) \right) dZ(p|\, t_m = t_1) \\ &= \int_{p \in \left(\frac{1}{2}, x\right)} (p(-\phi) + (1-p)\gamma)\, dZ(p) \\ &= \left(-\frac{1}{4}\right)(\phi - 3\gamma + 2x\gamma + 2x\phi) \end{aligned} \quad (2)$$

where (1) is the expected payoff when U imitates M's forecast and (2) is the expected payoff when U deviates from M's forecast. Then,

$$E\pi_u(a_u = a_m, w) - E\pi_u(a_u \neq a_m, w) = \frac{1}{4}(4x - 3\gamma + \phi + 2x\gamma + 2x\phi - 2)$$

We denote

$$f(x) \equiv 4x - 3\gamma + \phi + 2x\gamma + 2x\phi - 2$$

Then, U's best response in round 2 when $a_m$ is observable can be described as follows: *i)* $f(x) > 0 \Longrightarrow$ *U imitates* $a_m$*, ii)* $f(x) < 0 \Longrightarrow$ *U deviates from* $a_m$*, and iii)* $f(x) = 0 \Longrightarrow$ *U is indifferent between imitating and deviating from* $a_m$*.*

In the following, we denote $E\pi_m(t_m)$ as M's ex-ante expected payoff when he forecasts at $t_m \in \{t_1, t_2\}$. Note that if $t_m = t_2$, U use a mixed strategy, i.e., $z = [0, 1]$ where $z = \Pr(a_u = h)$. Then, from Assumption A.1.,

$$\begin{aligned} E\pi_m(t_m = t_2) &= \frac{1}{2}\left( \sum_{a_u \in \{h,l\}} \left( \sum_{w \in \{H,L\}} \Pr(w|\, \theta = h) \pi_m(a_m, a_u, w) \right) \right) \\ &= \frac{1}{2}(2p - \phi + p\gamma + p\phi - 1) \end{aligned} \quad (3)$$

CASE 1: When $f(x) > 0$.

In this case, if M forecasts in round 1, U imitates M's forecast in round 2. Then,

$$\begin{aligned} E\pi_m(t_m = t_1) &= \sum_{w \in \{H,L\}} \Pr(w|\, \theta = h) \pi_m(a_m, a_u = a_m, w) \\ &= 2p - 1 \end{aligned} \quad (4)$$

From (3) and (4),

$$E\pi_m(t_m = t_1) - E\pi_m(t_m = t_2) = \left(-\frac{1}{2}\right)(p\gamma - \phi - 2p + p\phi + 1) \tag{5}$$

The computation then yields the following result.

**Lemma A.2**

*Suppose that $\gamma < \phi$. If $p \in \left(\frac{1}{2}, \frac{\phi-1}{\gamma+\phi-2}\right)$, $t_m = t_1$ and if $p \in \left(\frac{\phi-1}{\gamma+\phi-2}, 1\right)$, $t_m = t_2$. If $t_m = t_1$, U imitates $a_m$.*

**Proof of Lemma A.2**

From (5), if $p \gtreqless \frac{\phi-1}{\gamma+\phi-2}$, $E\pi_m(t_m = t_1) \lesseqgtr E\pi_m(t_m = t_2)$. This satisfies that M's strategy is monotone increasing. From result 1, i) if $\gamma \geq \phi$, $E\pi_m(t_m = t_1) < E\pi_m(t_m = t_2)$ for all $p \in \left(\frac{1}{2}, 1\right)$, ii) if $\gamma < \phi$, for $p \in \left(\frac{1}{2}, \frac{\phi-1}{\gamma+\phi-2}\right)$, $E\pi_m(t_m = t_1) > E\pi_m(t_m = t_2)$ and for $p \in \left(\frac{\phi-1}{\gamma+\phi-2}, 1\right)$, $E\pi_m(t_m = t_1) < E\pi_m(t_m = t_2)$. Hence the separating equilibrium is supported only if $\gamma < \phi$. In this case, $x = \frac{\phi-1}{\gamma+\phi-2}$ and moreover $f\left(x = \frac{\phi-1}{\gamma+\phi-2}\right) = \frac{(3\gamma+3\phi-2)(\phi-\gamma)}{(\gamma+\phi-2)} > 0$. So the condition that $f(x) > 0$ is also satisfied. ■

CASE 2: When $f(x) < 0$.

In this case, if M forecasts in round 1, U deviates from M's forecast in round 2. Then,

$$\begin{aligned} E\pi_m(t_m = t_1) &= \sum_{w \in \{H,L\}} \Pr(w|\,\theta)\pi_m(a_m \neq a_u) \\ &= p\gamma - \phi + p\phi \end{aligned} \tag{6}$$

From (3) and (6),

$$E\pi_m(t_m = t_1) - E\pi_m(t_m = t_2) = p\left(\frac{1}{2}\gamma + \frac{1}{2}\phi - 1\right) + \frac{1}{2} - \frac{1}{2}\phi \tag{7}$$

Then, if $p \gtreqless \frac{\phi-1}{\gamma+\phi-2}$, $E\pi_m(t_m = t_1) \gtreqless E\pi_m(t_m = t_2)$. However, this is contradictory to the conjecture that M's strategy is monotone increasing. Hence, this case is excluded.

CASE 3: When $f(x) = 0$.

In this case, if M forecasts in round 1 U is indifferent to imitating or deviating from M's forecast. Now, suppose that $z = \Pr(a_u = a_m)$ when $a_m$ is observable. Then

$$\begin{aligned} E\pi_m(t_m = t_1) &= z(p - (1-p)) + (1-z)(p\gamma - (1-p)\phi) \\ E\pi_m(t_m = t_2) &= \frac{1}{2}(p - (1-p)) + \frac{1}{2}(p\gamma - (1-p)\phi) \end{aligned} \tag{8}$$

If $z \neq \frac{1}{2}$, the separating equilibrium is not derived. If $z = \frac{1}{2}$, M is indifferent between forecasting in round 1 or round 2 for all $p \in \left(\frac{1}{2}, 1\right)$. Then as $f(x) = 0$ at $x = \frac{3\gamma-\phi+2}{2\gamma+2\phi+4}$, from the condition of consistency, we only have to check the condition under which $x = \frac{3\gamma-\phi+2}{2\gamma+2\phi+4} \in \left(\frac{1}{2}, 1\right)$ is satisfied.

**Lemma A.3.**

*Suppose that* $\phi < \gamma < 3\phi + 2$*. Then, for* $p \in \left(\frac{1}{2}, \frac{3\gamma-\phi+2}{2\gamma+2\phi+4}\right)$*,* $t_m = t_1$ *and for* $p \in \left(\frac{3\gamma-\phi+2}{2\gamma+2\phi+4}, 1\right)$*,* $t_m = t_2$*. If* $t_m = t_1$*, U imitates or deviates from* $a_m$ *with probability* $\frac{1}{2}$*.*

**Proof of Lemma A.3.**

We check the condition under which $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} \in \left(\frac{1}{2}, 1\right)$ is satisfied. Note that $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} - \frac{1}{2} = -\frac{(\phi-\gamma)}{(\gamma+\phi+2)}$ and $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} - 1 = -\frac{(3\phi-\gamma+2)}{2(\gamma+\phi+2)}$. So i) if $\gamma > \phi$, $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} > \frac{1}{2}$ and if $\gamma \leq \phi$, $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} \leq \frac{1}{2}$, ii) if $\gamma < 3\phi + 2$, $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} < 1$ and if $\gamma \geq 3\phi + 2$, $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} \geq 1$. Hence, a) if $\phi \geq \gamma$, $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} \notin \left(\frac{1}{2}, 1\right)$, b) if $\phi < \gamma < 3\phi + 2$, $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} \in \left(\frac{1}{2}, 1\right)$ and c) if $\gamma \geq 3\phi + 2$, $\frac{3\gamma-\phi+2}{2\gamma+2\phi+4} \notin \left(\frac{1}{2}, 1\right)$. Therefore, $x = \frac{3\gamma-\phi+2}{2\gamma+2\phi+4} \in \left(\frac{1}{2}, 1\right)$ is satisfied only if $\phi < \gamma < 3\phi + 2$. ■

Next we consider the case where M's strategy is monotone decreasing. Suppose that M's strategy is monotone decreasing. Without loss of generality, assume that $a_m = h$. After observing $a_m = h$, U infers that $\theta = h$. Also, U's posterior belief from observing $t_m = t_1$ is that $p \in (y, 1)$. Then

$$\begin{aligned} E\pi_u \left(a_u = a_m, w\right) &= \int \left( \sum_{w \in \{H,L\}} \Pr(w|\, \theta = h) \pi_u \left(a_u = a_m, w\right) \right) dZ \left(p|\, t_m = t_1\right) \\ &= \int_{p \in (y,1)} \left(p - (1-p)\right) dZ\left(p\right) \\ &= y \end{aligned} \tag{9}$$

$$\begin{aligned} E\pi_u \left(a_u \neq a_m, w\right) &= \int \left( \sum_{w \in \{H,L\}} \Pr(w|\, \theta = h) \pi_u \left(a_u \neq a_m, w\right) \right) dZ \left(p|\, t_m = t_1\right) \\ &= \int_{p \in (y,1)} \left(p\left(-\phi\right) + (1-p)\gamma\right) dZ\left(p\right) \\ &= \left(-\frac{1}{2}\right) \left(\phi - \gamma + y\gamma + y\phi\right) \end{aligned} \tag{10}$$

where (9) is the expected payoff when U imitates M's action and (10) is the expected payoff when U deviates from M's action. Then,

$$E\pi_u \left(a_u = a_m, w\right) - E\pi_u \left(a_u \neq a_m, w\right) = \frac{1}{2} \left(2y - \gamma + \phi + y\gamma + y\phi\right)$$

We denote

$$g\left(y\right) \equiv 2y - \gamma + \phi + y\gamma + y\phi$$

U's best response in round 2 can thus be described as follows: *i)* $g(y) > 0 \Longrightarrow$ *U imitates* $a_m$*, ii)* $g(y) < 0 \Longrightarrow$ *U deviates from* $a_m$*. iii)* $g(y) = 0 \Longrightarrow$ *U is indifferent between imitating and deviating from* $a_m$*.*

CASE 1) When $g(y) > 0$

In this case, if $t_m = t_1$, U imitates $a_m$. Then from (5), $p \gtreqless \frac{\phi-1}{\gamma+\phi-2} \Longrightarrow E\pi_m \left(t_m = t_1\right) \lesseqgtr E\pi_m \left(t_m = t_2\right)$. However, this contradicts the conjecture that M's strategy is monotone decreasing. Hence, this case is excluded.

CASE 2) When $g(y) < 0$

In this case, if $t_m = t_1$, U deviates from $a_m$. Then from (7), $p \gtrless \frac{\phi-1}{\gamma+\phi-2} \Longrightarrow E\pi_m(t_m = t_1) \gtrless E\pi_m(t_m = t_2)$. This satisfies the condition that M's strategy is monotone decreasing. From result 1, i) if $\gamma \geq \phi$, $E\pi_m(t_m = t_1) > E\pi_m(t_m = t_2)$ for all $p \in \left(\frac{1}{2}, 1\right)$, ii) if $\gamma < \phi$, for $p \in \left(\frac{\phi-1}{\gamma+\phi-2}, 1\right)$, $E\pi_m(t_m = t_1) > E\pi_m(t_m = t_2)$ and for $p \in \left(\frac{1}{2}, \frac{\phi-1}{\gamma+\phi-2}\right)$, $E\pi_m(t_m = t_1) < E\pi_m(t_m = t_2)$. Thus, the separating equilibrium is supported only if $\gamma < \phi$. In this case $y = \frac{\phi-1}{\gamma+\phi-2}$ and $g\left(y = \frac{\phi-1}{\gamma+\phi-2}\right) = \frac{(\gamma-\phi+\gamma\phi-\gamma^2+2\phi^2-2)}{(\gamma+\phi-2)}$. If we denote the numerator as $h(\phi) \equiv 2\phi^2 + \phi(\gamma-1) + \gamma - \gamma^2 - 2$, then the following points can be checked: a) $h(\phi)$ is a convex function, b) $h(\phi)$ attains the minimized value at $\phi = -\frac{\gamma-1}{4} < 0$ and c) $h(\phi = \gamma) = 2(\gamma-1)(\gamma+1) > 0$. Hence, for $\phi$ such that $\gamma < \phi$, $h(\phi) > 0$. As the denominator is always positive, $g\left(y = \frac{\phi-1}{\gamma+\phi-2}\right) > 0$ for $\gamma < \phi$. But this contradicts the condition that $g(y) < 0$. Therefore this case is excluded.

CASE 3) When $g(y) = 0$

In this case, if $t_m = t_1$, U is indifferent between imitating and deviating from M's forecast. From (8), if U imitates (or deviates from) $a_m$ with probability $\frac{1}{2}$, M is indifferent between forecasting in round 1 and round 2 for all $p \in \left(\frac{1}{2}, 1\right)$. Then, for the requirement that U's posterior belief over $p$ should be consistent, we only have to check whether $\frac{\gamma-\phi}{\gamma+\phi+2} \in \left(\frac{1}{2}, 1\right)$ for which $g\left(y = \frac{\gamma-\phi}{\gamma+\phi+2}\right) = 0$. Note that $\frac{\gamma-\phi}{\gamma+\phi+2} - \frac{1}{2} = -\frac{(3\phi-\gamma+2)}{2(\gamma+\phi+2)}$ and $\frac{\gamma-\phi}{\gamma+\phi+2} - 1 = -\frac{2(\phi+1)}{(\gamma+\phi+2)} < 0$. So, if $\gamma \leq 3\phi + 2$, $\frac{\gamma-\phi}{\gamma+\phi+2} \notin \left(\frac{1}{2}, 1\right)$, and if $\gamma > 3\phi + 2$, $\frac{\gamma-\phi}{\gamma+\phi+2} \in \left(\frac{1}{2}, 1\right)$. Hence, a separating equilibrium can be supported only if $\gamma > 3\phi + 2$.

Hence the result which supports that M's strategy is monotone decreasing is as follows.

**Lemma A.4.**

*Suppose that* $\gamma > 3\phi + 2$. *Then,* for $p \in \left(\frac{\gamma-\phi}{\gamma+\phi+2}, 1\right)$, $t_m = t_1$ and for $p \in \left(\frac{1}{2}, \frac{\gamma-\phi}{\gamma+\phi+2}\right)$, $t_m = t_2$. *If* $t_m = t_1$, *U imitates or deviates from* $a_m$ *with probability* $\frac{1}{2}$.

Then Lemma A.2-A.4 prove Proposition 2. ■

## 8.3 Proof of Proposition 3

Suppose that U observes that $t_m = t_1$. Without loss of generality, assume that $a_m = h$. Then

$$\begin{aligned} E\pi_u(a_u = a_m, w) &= \int \left( \sum_{w \in \{H,L\}} \Pr(w|\,\theta = h)\pi_u(a_u = a_m, w) \right) dZ(p|\,t_m = t_1) \\ &= \int_{p \in \left(\frac{1}{2}, 1\right)} (p - (1-p))\, dZ(p) \\ &= \frac{1}{2} \end{aligned} \tag{11}$$

$$
\begin{aligned}
E\pi_u\left(a_u \neq a_m, w\right) &= \int \left( \sum_{w \in \{H,L\}} \Pr(w|\, \theta = h)\pi_u\left(a_u \neq a_m, w\right) \right) dZ\left(p|\, t_m = t_1\right) \qquad (12) \\
&= \int_{p \in \left(\frac{1}{2},1\right)} \left(p\left(-\phi\right) + (1-p)\gamma\right) dZ\left(p\right) \\
&= \left(-\frac{1}{4}\right)\left(3\phi - \gamma\right)
\end{aligned}
$$

Then,

$$
E\pi_u\left(a_u = a_m, w\right) - E\pi_u\left(a_u \neq a_m, w\right) = \frac{1}{4}\left(3\phi - \gamma + 2\right)
$$

Hence, U's best response in round 2 for $t_m = t_1$ is as follows: *i) if* $3\phi + 2 > \gamma$, *U imitates, ii) if* $3\phi + 2 < \gamma$, *U deviates, iii) if* $3\phi + 2 = \gamma$, *U is indifferent between imitating and deviating from* $a_m$.

Case 1) When $\gamma < 3\phi + 2$

If $t_m = t_1$, U imitates $a_m$. Recall (5) and Result 1. Then i) if $\gamma \geq \phi$, for all $p \in \left(\frac{1}{2}, 1\right)$, $E\pi_m\left(t_m = t_1\right) < E\pi_m\left(t_m = t_2\right)$, ii) if $\gamma < \phi$, for $p \in \left(\frac{\phi-1}{\gamma+\phi-2}, 1\right)$, $E\pi_m\left(t_m = t_1\right) < E\pi_m\left(t_m = t_2\right)$ and for $p \in \left(\frac{1}{2}, \frac{\phi-1}{\gamma+\phi-2}\right)$, $E\pi_m\left(t_m = t_1\right) > E\pi_m\left(t_m = t_2\right)$. Therefore, there is no case in which the revealing pooling equilibrium is supported.

Case 2) When $\gamma > 3\phi + 2$

If $t_m = t_1$, U deviates from $a_m$. Recall (7) and Result 1. Then i) if $\gamma \geq \phi$, for all $p \in \left(\frac{1}{2}, 1\right)$, $E\pi_m\left(t_m = t_1\right) > E\pi_m\left(t_m = t_2\right)$, ii) if $\gamma < \phi$, for $p \in \left(\frac{\phi-1}{\gamma+\phi-2}, 1\right)$, $E\pi_m\left(t_m = t_1\right) > E\pi_m\left(t_m = t_2\right)$ and for $p \in \left(\frac{1}{2}, \frac{\phi-1}{\gamma+\phi-2}\right)$, $E\pi_m\left(t_m = t_1\right) < E\pi_m\left(t_m = t_2\right)$. Therefore, the revealing pooling equilibrium is supported only if $\gamma \geq \phi$ and the given condition that $\gamma > 3\phi + 2$ satisfies this. Hence, if $\gamma > 3\phi + 2$, the pooling strategy can be supported as the equilibrium strategy.

Case 3) When $\gamma = 3\phi + 2$

If $t_m = t_1$, U is indifferent between imitating and deviating from $a_m$. Then from (8),

$$
E\pi_m\left(t_m = t_1\right) - E\pi_m\left(t_m = t_2\right) = z\left(2p - p\gamma + \phi\left(1-p\right) - 1\right) + \frac{1}{2}\left(p\gamma - \phi - 2p + p\phi + 1\right)
$$

Note that if $p \lesseqgtr \frac{\phi-1}{\gamma+\phi-2}$, $2p - p\gamma + \phi\left(1-p\right) - 1 \gtreqless 0$. From result 1, i) if $\gamma \geq \phi$, for all $p \in \left(\frac{1}{2}, 1\right)$, $2p - p\gamma + \phi\left(1-p\right) - 1 < 0$ and ii) if $\gamma < \phi$, for $p \in \left(\frac{1}{2}, -\frac{\phi-1}{2-\phi-\gamma}\right)$, $2p - p\gamma + \phi\left(1-p\right) - 1 > 0$ and for $p \in \left(-\frac{\phi-1}{2-\phi-\gamma}, 1\right)$, $2p - p\gamma + \phi\left(1-p\right) - 1 < 0$. Hence, when $\gamma \geq \phi$, if $z \lesseqgtr \frac{1}{2}$, $E\pi_m\left(t_m = t_1\right) \gtreqless E\pi_m\left(t_m = t_2\right)$ for all $p \in \left(\frac{1}{2}, 1\right)$. On the other hand, if $\gamma < \phi$, M's pooling strategy cannot be sustained. Therefore, the revealing pooling equilibrium can be supported only if $\gamma \geq \phi$ and $z < \frac{1}{2}$. Moreover, the given condition that $\gamma = 3\phi + 2$ satisfies the condition that $\gamma \geq \phi$. ■

## 8.4 Proof of Proposition 4

First, consider the case in which U believes that M's strategy is monotone increasing; That is, $\exists x \in \left(\frac{1}{2}, 1\right]$ such that if $p \in \left(\frac{1}{2}, x\right)$, $t_m = t_1$ and if $p \in (x, 1)$, $t_m = t_2$. Here, as we consider the case in which U observes that $t_m = t_1$, the case in which $x = \frac{1}{2}$ can be excluded. Then for $t_m = t_1$ which is off-the-equilibrium path, U's posterior belief is $p \in \left(\frac{1}{2}, x\right)$ where $x \in \left(\frac{1}{2}, 1\right]$. Then, from (1) and (2),

$$\begin{aligned} E\pi_u (a_u = a_m, w) &= \frac{1}{2}(2x - 1) > 0 \\ E\pi_u (a_u \neq a_m, w)|_{\phi=\gamma} &= \left(-\frac{1}{2}\right)\gamma (2x - 1) < 0 \end{aligned}$$

So U's best response for $t_m = t_1$, which is off-the-equilibrium path, is always to imitate $a_m$ if it is observable.

Second, consider the situation in which U believes that M's strategy is monotone decreasing; that is, $\exists y \in \left[\frac{1}{2}, 1\right)$ such that If $p \in (y, 1)$, $t_m = t_1$ and if $p \in \left(\frac{1}{2}, y\right)$, $t_m = t_2$. As we consider the case in which U observes that $t_m = t_1$, the case in which $y = 1$ is excluded. Then for $t_m = t_1$ which never occurs along the equilibrium path, U's posterior belief is $p \in (y, 1)$ where $y \in \left[\frac{1}{2}, 1\right)$. If U's belief is formed in this way, from (9) and (10),

$$\begin{aligned} E\pi_u (a_u = a_m, w) &= y > 0 \\ E\pi_u (a_u \neq a_m, w)|_{\phi=\gamma} &= -\gamma y < 0 \end{aligned}$$

So U's best response for $t_m = t_1$, which is off-the-equilibrium path, is always to imitate $a_m$ if it is observable.

Therefore, regardless of U's posterior belief, if $\gamma = \phi$, U's best response is to imitate $a_m$ if it is observable. Then from (4), M's expected payoff from deviating from the given pooling strategy is $E\pi_m (t_m = t_1) = 2p - 1$. On the other hand, if he follows the given pooling strategy, from (3), $E\pi_m (t_m = t_2)|_{\phi=\gamma} = \frac{1}{2}(2p - 1)(\gamma + 1)$. Then

$$E\pi_m (t_m = t_1) - E\pi_m (t_m = t_2)|_{\phi=\gamma} = \left(-\frac{1}{2}\right)(2p - 1)(\gamma - 1) < 0$$

So if $\phi = \gamma$, $t_m = t_2$ for all $p \in \left(\frac{1}{2}, 1\right)$. ■

## 8.5 Proof of Proposition 5

In the following, recall (3).

(Step 1) U's best response when $t_m = t_1$.

Suppose that $t_m = t_1$. Without loss of generality, assume that $a_m = h$. Then, U infers that $\theta = h$. Then

$$E\pi_u (a_m, a_u = a_m, w) = \sum_{w \in \{H,L\}} \Pr(w | \theta = h)\pi_u (a_m, a_u, w) = 2p - 1 \quad (13)$$

$$E\pi_u(a_m, a_u \neq a_m, w) = \sum_{w \in \{H,L\}} \Pr(w|\,\theta = h)\pi_u(a_m, a_u, w) = -p\phi + (1-p)\gamma \tag{14}$$

where (13) is U's expected payoff when he imitates M's forecast and (14) is the one when U deviates from M's forecast. Then,

$$E\pi_u(a_m, a_u = a_m, w) - E\pi_u(a_m, a_u \neq a_m, w) = p(\gamma + \phi + 2) - \gamma - 1$$

Hence, if $p \gtreqqless \frac{\gamma+1}{\gamma+\phi+2}$, $E\pi_u(a_u = a_m) \gtreqqless E\pi_u(a_u \neq a_m)$. Note that $\frac{\gamma+1}{\gamma+\phi+2} - \frac{1}{2} = -\frac{(\phi-\gamma)}{2(\gamma+\phi+2)}$ and $\frac{\gamma+1}{\gamma+\phi+2} - 1 = -\frac{(\phi+1)}{(\gamma+\phi+2)} < 0$. Therefore, 1) if $\gamma > \phi$, for $p \in \left(\frac{1}{2}, \frac{\gamma+1}{\gamma+\phi+2}\right)$, U deviates from $a_m$ and for $p \in \left(\frac{\gamma+1}{\gamma+\phi+2}, 1\right)$, U imitates $a_m$. 2) if $\gamma \leq \phi$, U imitates $a_m$ for all $p \in \left(\frac{1}{2}, 1\right)$.

(Step 2) M's best response

Now using backward induction, we derive M's decision on the timing of forecast.

First, suppose that $\gamma \leq \phi$. In this case, if $t_m = t_1$, U imitates $a_m$ for all $p \in \left(\frac{1}{2}, 1\right)$. Then from (5) and result 1, if $\gamma < \phi$, for $p \in \left(\frac{1}{2}, \frac{\phi-1}{\gamma+\phi-2}\right)$, $E\pi_m(t_m = t_1) > E\pi_m(t_m = t_2)$ and for $p \in \left(\frac{\phi-1}{\gamma+\phi-2}, 1\right)$, $E\pi_m(t_m = t_1) < E\pi_m(t_m = t_2)$. On the other hand, if $\gamma = \phi$, for all $p \in \left(\frac{1}{2}, 1\right)$, $E\pi_m(t_m = t_1) < E\pi_m(t_m = t_2)$.

Second, suppose that $\gamma > \phi$. When $p \in \left(\frac{1}{2}, \frac{\gamma+1}{\gamma+\phi+2}\right)$, if $t_m = t_1$, U deviates from $a_m$. Then from (7) and result 1, if $\gamma > \phi$, $E\pi_m(t_m = t_1) > E\pi_m(t_m = t_2)$ for all $p$. On the other hand if $p \in \left(\frac{\gamma+1}{\gamma+\phi+2}, 1\right)$, if $t_m = t_1$, U imitates $a_m$. Then from (5) and result 1, if $\gamma > \phi$, for $p \in \left(\frac{\gamma+1}{\gamma+\phi+2}, 1\right)$, $E\pi_m(t_m = t_1) < E\pi_m(t_m = t_2)$. Finally, if $p = \frac{\gamma+1}{\gamma+\phi+2}$, U is indifferent between imitating and deviating from $a_m$. In a mixed strategy equilibrium, $\Pr(a_u = a_m) = \frac{1}{2}$ and $\Pr(t_m = t_1) \in [0, 1]$. ■

# 9 References


Aggarwal, R.K. and Samwick, A.A., "Executive compensation, strategic compensation, and relative performance evaluation: Theory and evidence.", Journal of Finance, Vol 54 (1999), pp. 1999-2043.

Antle, R., and Smith, A., "An empirical investigation of the relative evaluation of corporate executives.", Journal of Accounting Research, Vol 24 (1986), pp. 1-39.

Conner, K.R., "Obtaining Strategic Advantage from Being Imitated: When Can Encouraging "Clones" pay?.", Management Science, Vol. 41. (1995), pp. 209-225.

Conner, K.R. and Rumelt, R.P., "Software piracy, an analysis of protection strategies." Management Science, Vol. 37 (1991), pp. 125-139.

Creane, A., "Risk and revelation: Changing the value of information." Economica, Vol. 65 (1998), pp. 247-261.

De Fraja, G., "Strategic spillovers in patent races.", International Journal of Industrial Organization, Vol. 11 (1993), pp. 139-146.

Gallini, N., "Deterrence by Market Sharing: A Strategic Incentive for Licensing.", American Economic Review, Vol. 74 (1984), pp. 931-941.

Gibbons, R. and Murphy, K. J., "Relative performance evaluation for chief executive officers.", Industrial and Labor Relations Review, Vol. 43 (1990), pp. 30-51.

Janakiraman, S., Lambert, R., and Larcker, D., "An empirical investigation of the relative performance evaluation hypothesis.", Journal of Accounting Research, Vol. 30 (1992), pp. 53-69.

Mailath, G.J., "Endogenous sequencing of firm decisions." Journal of Economic Theory, Vol. 59 (1993), pp. 169-182.

Okuno-Fujiwara, M., Postlewaite, A. and Suzumura, K., "Strategic information revelation.", Review of Economic Studies, Vol. 57 (1990), pp. 25-47.

Raith, M. "A general model of information sharing in oligopoly." Journal of Economic Theory, Vol. 71 (1996), pp. 260-288.

Vives, X. "Trade association disclosure rules, incentives to share information, and welfare." RAND Journal of Economics, Vol. 21 (1990), pp. 409-430.

Ziv, A. "Information sharing in oligopoly: the truth-telling problem." RAND Journal of Economics, Vol. 24(3) (1993), pp.455-465.