Contests on a line and around a circle

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Motivation

- Professional sports leagues as contests
- Unlike most contest models there is not an exogenous prize
- Teams compete to attract fans, who are in turn attracted by success: market share is proportional to winning
- Problem with the demand function for team sports- not clear how contest success relates to the utility of fans
- It doesn’t matter where the prize is, but it does matter where the fans are
- Do fans substitute between teams?

Literature

- Rottenberg (1956) competitive balance and the invariance principle (Coase)
- Quirk and El-Hodiri (1974) invariance principle and revenue sharing
- Fort and Quirk (1995), Vrooman (1995)
- Szymanski and Kesenne (2004) revenue sharing diminishes competitive balance in a Tullock contest
- Dietl et al (2009), (2010)
- Very little written about contests and league structures (contrast the large literature on individual contests)

Location models

- In a location model (e.g. Hotelling) a fan can trade off success against distance (e.g. travel)
- Market share based on team success rather than price
- Entry is typically sequential – expansion franchises the norm
    - 8 teams in National League (baseball) in 1876
    - 30 teams today
    - 12 teams in English Football League in 1888
    - 92 team today, promotion and relegation admits new teams
- Are league members profitable?
- Competitive balance issue- does each team have a reasonable prospect of winning?

Franchise Location

- Significant differences between the US model and the European model
- US- franchise protected by exclusive territories (70 miles), typically one team per city
- Europe- no restriction on entry, leagues organised in hierarchies with promotion and relegation on sporting merit
- These differences stem from different philosophies of the founders

William Hulbert on local competition

“No two clubs should be admitted from the same city.

The evil effects of having more than one club in a city have been shown in Philadelphia this year. First, the Centennials went under, and then the Philadelphias and Athletics divided the interest, so that both of them have end the season at a loss, poorer than poverty and owing their players. One club can live in Philadelphia but two must starve”

Chicago Tribune, October 24, 1875

William Hulbert on numbers

"It may be asked why the advent of more clubs and a more general interest in the game will hurt it. The answer is statistical...

...the nine first named could live respectably, pay good salaries, and perhaps a modest dividend, and put the exhibition on a sound basis. On the other hand, if the whole gang be let in, half of the games will not pay..."

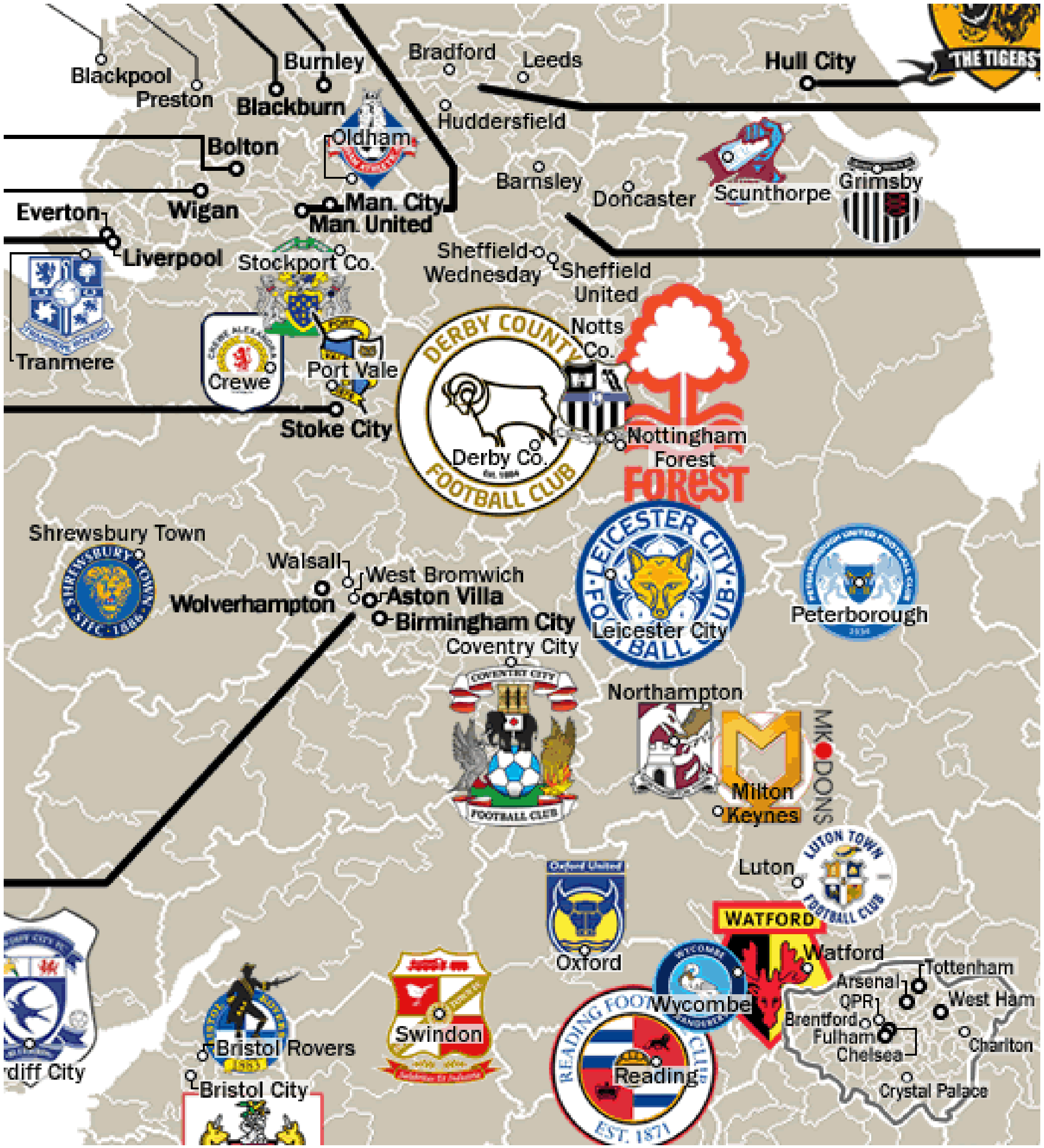
Chicago Tribune, October 24, 1875

Official Rules of the twelve founder members of the English Football League

"4. That there be two classes of League club- First and Second- each to consist of 12 clubs"

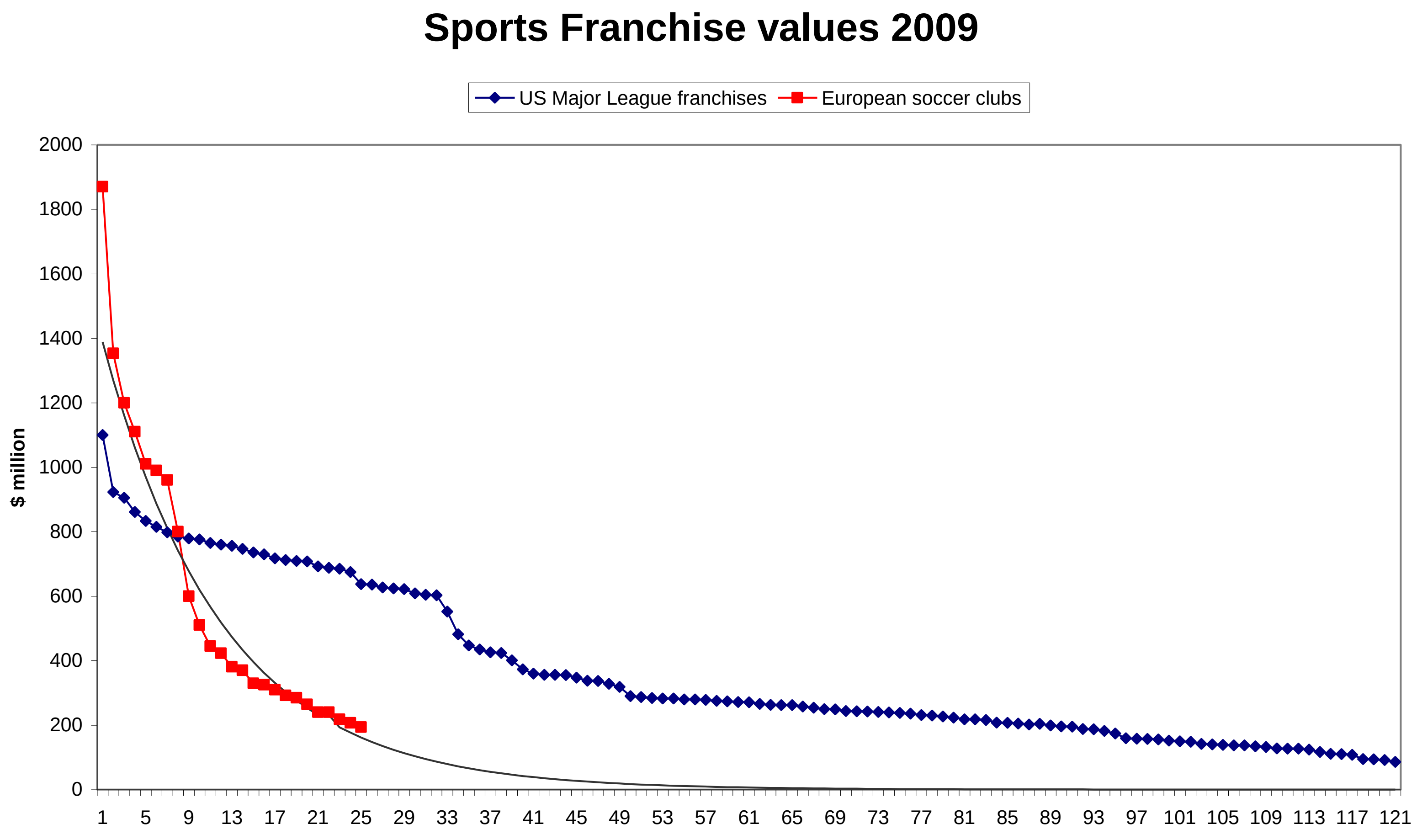
Rules dated 11 January 1889

The "second class" did not start until 1892, but by 1923 the League had expanded to 88 teams in four divisions



Blackpool
Preston
Burnley
Blackburn
Bradford
Leeds
Hull City
THE TIGERS
Huddersfield
Oldham
Bolton
Wigan
Everton
Liverpool
Man. City
Man. United
Barnsley
Doncaster
Scunthorpe
Grimsby
Stockport Co.
Sheffield Wednesday
Sheffield United
Tranmere
Crewe
Port Vale
Stoke City
DERBY COUNTY FOOTBALL CLUB
Derby Co.
Notts Co.
Nottingham Forest
FOREST
LEICESTER CITY FOOTBALL CLUB
Leicester City
Peterborough
Shrewsbury Town
Walsall
West Bromwich
Aston Villa
Wolverhampton
Birmingham City
Coventry City
Northampton
MK DONS
Milton Keynes
Luton
LUTON TOWN FOOTBALL CLUB
Oxford
WATFORD
Watford
Tottenham
Arsenal
QPR
West Ham
Brentford
Fulham
Chelsea
Charlton
Crystal Palace
Wycombe
Swindon
Reading
READING FOOTBALL CLUB
EST. 1871
Bristol Rovers
Bristol City
rdiff City

Financial implications



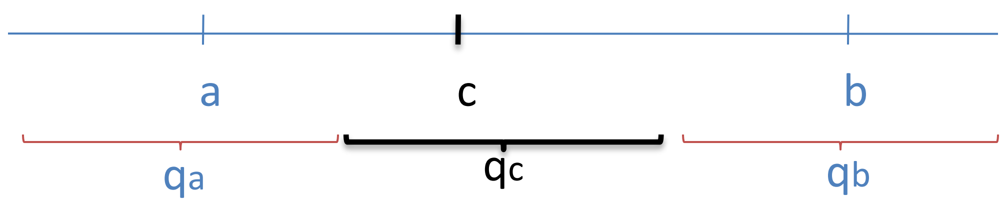
Source: Forbes

Location on a line without talent investment

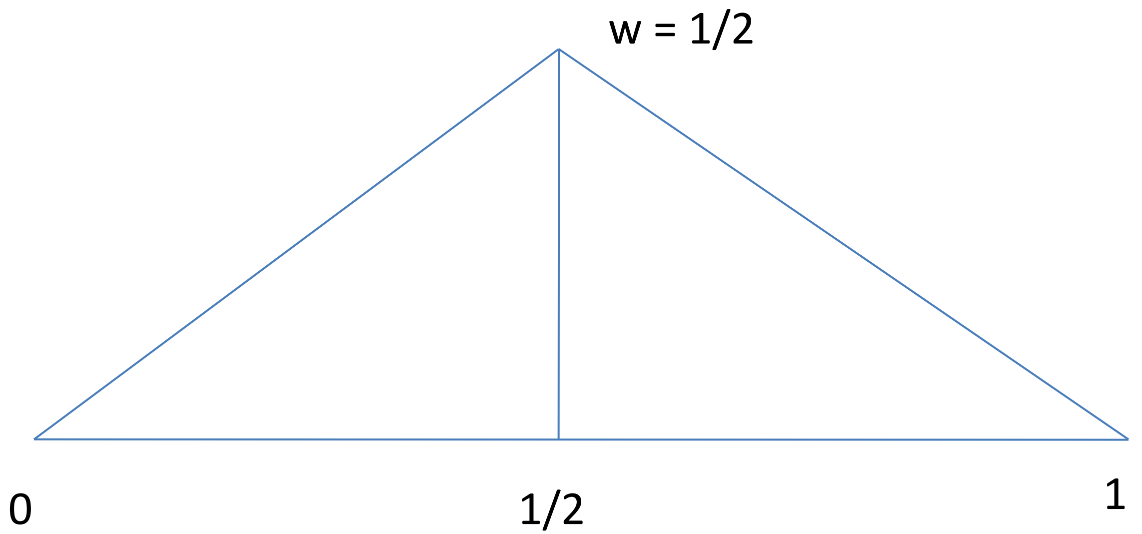
- Prescott and Visscher (1977), sequential entry decision, locating along a line of length 1
- Two types of investment- talent and stadium – the latter entails commitment
- Customers like winning teams, but dislike travelling to games, assume disutility is linear in distance
- Market allocation- players on the outside (no team to left or to right) take all customers on these segments, otherwise players get half the segment between themselves and the nearest player
- Perfect competitive balance, each team has equal probability of winning ($\Sigma w = 1$)

Location on a line without talent investment

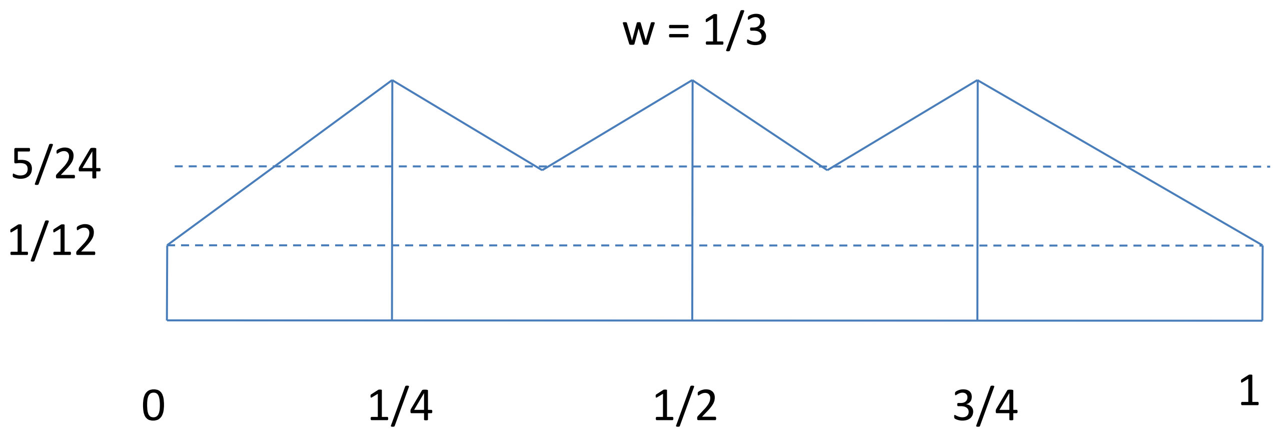
- **2 teams**: Hotelling result, each team locates at the centre and obtains half the market
- **3 teams**: working backwards, the third team has three location choices (far left, far right or middle), and location choices of 1 and 2 should leave team 3 indifferent
- Solution- first two teams (a and b) locate at ¼ and ¾ , so that their market shares are 3/8 each, while the entrant (team c) locates anywhere between a and b and obtains ¼ of the market



Welfare

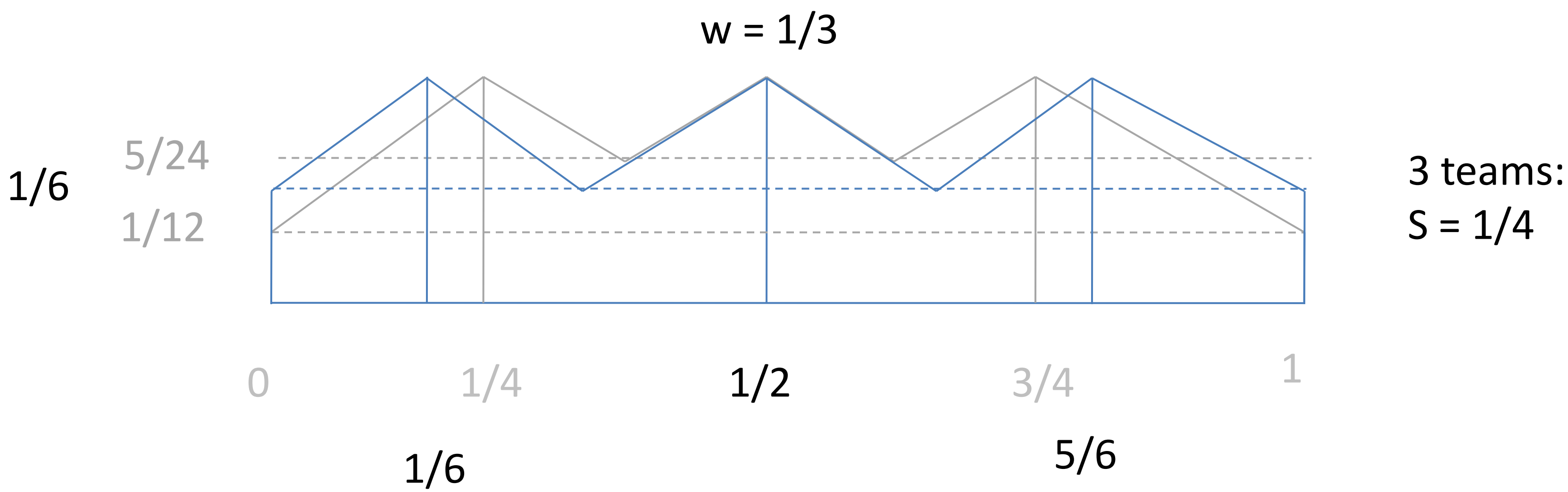


2 teams:
S = 1/4

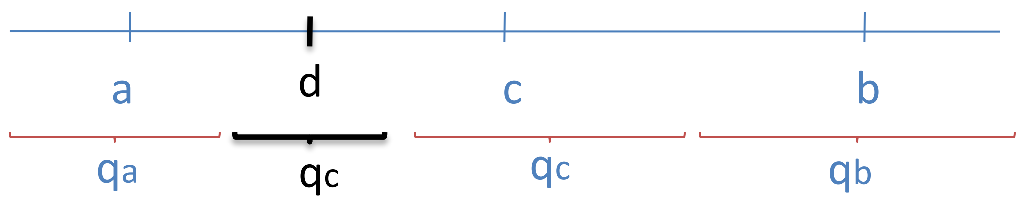


3 teams:
S = 23/96

Optimal location



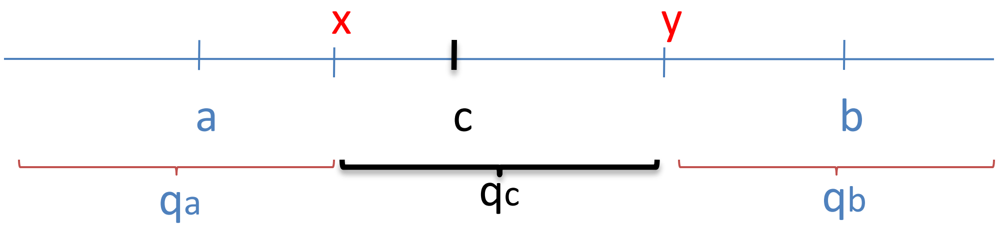
Four team case



- Four segments available for the fourth entrant, who is indifferent in equilibrium
- a and b must be symmetrical, c must locate at the centre
- Hence d should be indifferent between locating just to the left of a and between a and c (=1/2)
- This yields a = 1/6 = b. A and b are equally likely to be located next to d, hence the expected market shares are $q_a = q_b = 7/24$, $q_c = 1/4$ and $q_d = 1/6$
- Hence market shares (and profits) decrease with entry

Location on a line with endogenous investment

- 3 team model
- Location at the centre for the first two teams cannot be an equilibrium (entrant will locate at the centre too and the middle team will have no market)
- Need to identify the indifferent consumers (x and y)
- x: $w_a - (x - a) = w_c - (c - x)$, y: $w_c - (y - c) = w_b - (b - y)$
- Where w_a is the share of league games won by team a, determined by a simple Tullock contest success function



Profits

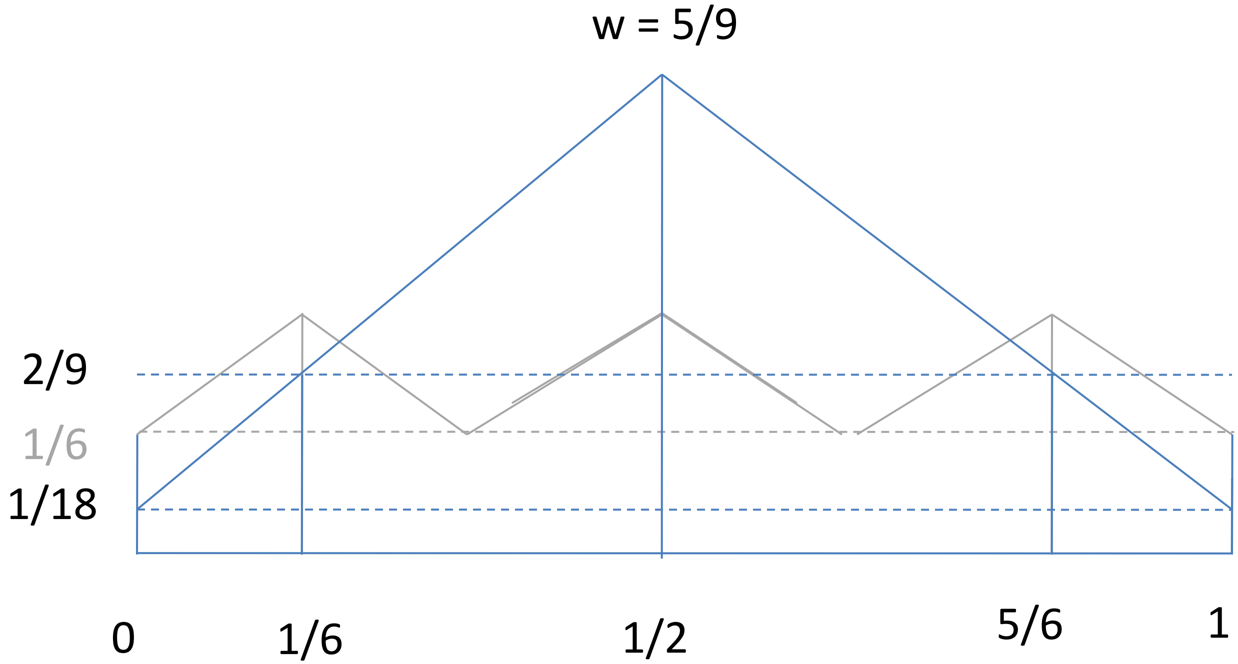
- $x = (w_a - w_c)/2 + (a + c)/2$, $y = (w_c - w_b)/2 + (b + c)/2$
- Market share of team a = x, team b = 1 – y, team c = y – x
- Profits equal market share minus talent investment
- $w_a = t_a / (t_a + t_b + t_c)$
- $\Pi_a = (w_a - w_c)/2 + (a + c)/2 - t_a$
- $\Pi_c = w_c - (w_a + w_b)/2 + (b - a)/2 - t_c$

Equilibrium

- Assume a and b locate symmetrically, while c is located in the middle (= ½)
- $t_a = t_b = 4/27$, $t_c = 10/27$ and so $w_a = w_b = 2/9$, $w_c = 5/9$
- Now $\Pi_a = - 1/6 + a/2 + 1/4 – 4/27$ which is increasing in a
- But it is also necessary that fans to the left of team a do not prefer team b, i.e. For a fan to the left of a at location z we require :

  $5/9 – (c – z) \leq 2/9 – (a – z)$ => $a \leq 1/6$
- Thus we conclude a = 1/6 and so
- $\Pi_a = \Pi_a = 1/54$, $\Pi_c = 8/27$
- Hence entrant's profits are far higher than incumbents'
- (NB a = z = 1/6, thus all fans to the right of a go to team c, a only has fans to the left)

Welfare



3 teams:
S = 11/36

Welfare

- With endogenous investment locations are optimal
- Welfare higher than in the 2 team case
- Fixed costs
- Competitive imbalance is welfare enhancing
- Is aggregate success constant ($\Sigma w = 1$) or increasing ($\Sigma w = n/2$)?

Sensitivity

- If fans prefer incumbent teams (x: $\theta w_a - (x - a) = w_c - (c - x)$, $\theta > 1$), then team c will still dominate – investment levels will be much higher, and the teams may not be profitable
- If Tullock parameter $\gamma < 1$, then model becomes more symmetric, but even if $\gamma \to 0$ $w_c > w_a$
- Because incumbents compete on only one side their incentive to invest is much weaker
- Circles are different...

Contest around a circle

Fans prefer team 1 if

$w_1 – d_{i1} > w_2 – d_{i2}$

For the indifferent consumer

$w_1 – d_1 = w_2 – d_2$

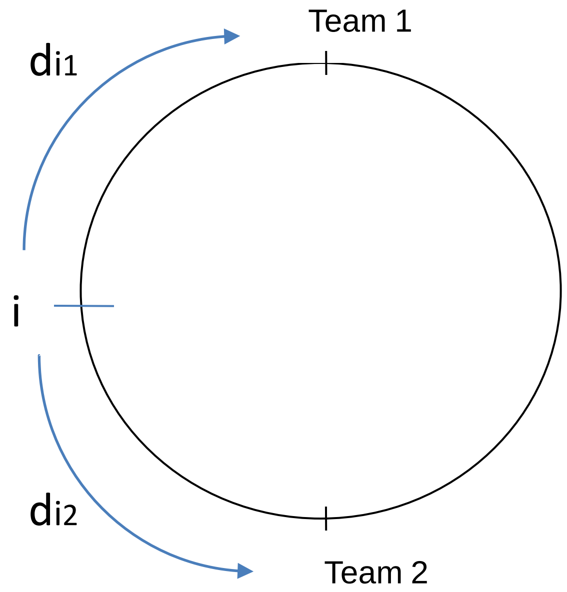
Assume $d_1 + d_2 = ½$

Which implies

$w_1 – w_2 = 2d_1 – ½$

Given symmetry, the market share for team 1 is

$2d_1 = 2w_1 - ½$



Two team equilibrium

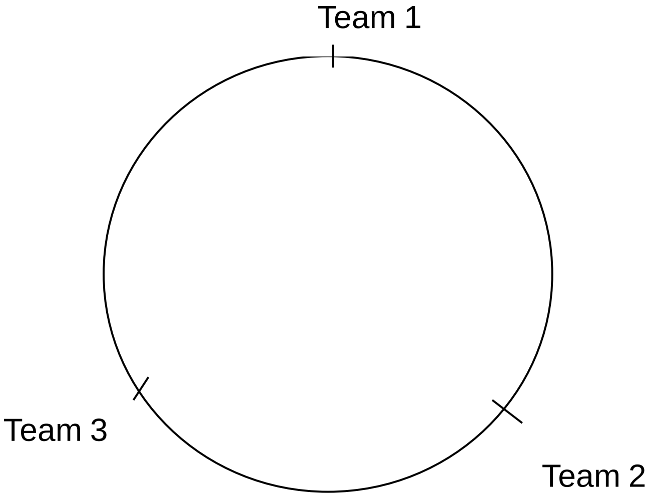
- $\pi_1 = 2w_1 - ½ – t_1$, assume a Tullock contest $w_1 = \frac{t_1^\gamma}{(t_1^\gamma + t_2^\gamma)^2}$
- Assuming the same contest success function as in the standard case we obtain

$$\frac{\partial \pi_1}{\partial t_1} = \frac{2\gamma t_1^{\gamma-1} t_2^\gamma}{(t_1^\gamma + t_2^\gamma)^2} - 1 = 0$$

- Which in the symmetric case reduces implies
- $t_1 = \gamma/2$
- and so profits $\pi_1 = 2 \times ½ - ½ - \gamma/2 = (1-\gamma)/2$

Symmetric 3 team model

Indifference : $w_1 – d_{12} = w_2 – d_{21}$, $w_1 – d_{13} = w_3 – d_{31}$,
$w_2 – d_{23} = w_3 – d_{32}$



$d_{12} + d_{21} = 1/3$, $d_{13} + d_{31} = 1/3$, $d_{23} + d_{32} = 1/3$

Rearranging $d_{13} = (w_1 – w_3)/2 + 1/6$

$\pi_1 = d_{12} + d_{13} - t_1 = w_1 – (w_2 +w_3)/2 + 1/3 - t_1$
$= 3/2\ w_1 + 1/3 - t_1$

$$\frac{\partial \pi_1}{\partial t_1} = \frac{3\gamma t_1^{\gamma-1}(t_2^\gamma + t_3^\gamma)}{2(t_1^\gamma + t_2^\gamma + t_3^\gamma)^2} - 1 = 0$$

Which in the symmetric case implies $t_1 = \gamma/3$

and so profits $\pi_1 = 1/3 - \gamma/3 = (1-\gamma)/3$

General symmetric case

This generalises so that with equally spaced teams the league the profits of any team i are

$$\pi_i = w_i - \left(\frac{w_{i-1} + w_{i+1}}{2} \right) + \frac{1}{n} - t_i$$

So the symmetric solution is $t_i = \gamma/n$ and $\pi_i = (1-\gamma)/n$

NB total talent investment and total profits are independent of n

Fixed costs; relocation is not costless

Asymmetric entry

$d_{12} + d_{21} = ½$, $d_{13} + d_{31} = ¼$, $d_{23} + d_{32} = ¼$

$$\pi_1 = w_1 - \left(\frac{w_2 + w_3}{2}\right) + \frac{3}{8} - t_i$$

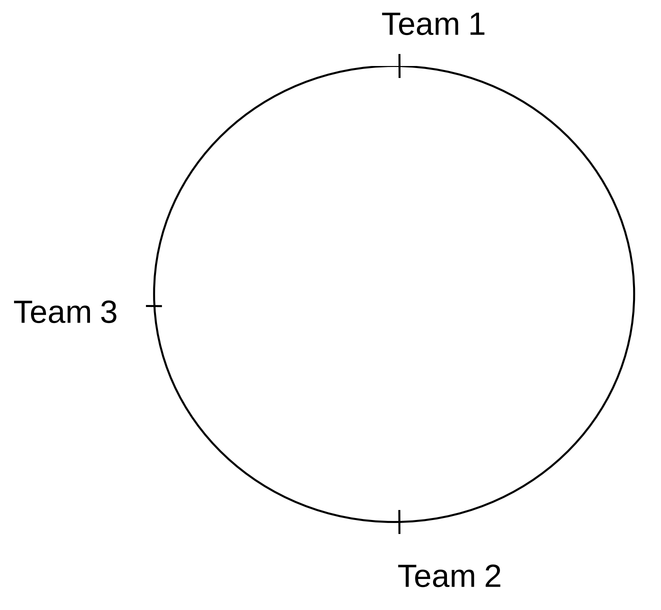
$$\pi_3 = w_3 - \left(\frac{w_1 + w_2}{2}\right) + \frac{1}{4} - t_i$$

Thus $\pi_1 = \pi_2 = (9-8\gamma)/24$ and $\pi_3 = (3-4\gamma)/12$

Note that for teams 1 and 2 post-entry profits are higher so long as

$(9-8\gamma)/24 > (1-\gamma)/2 \quad => \quad \gamma > ¾$

I.e. if the outcome of competition is sufficiently sensitive to expenditures (i.e. high γ), then entry can actually raise the profits of incumbents



Concluding observations

- Location matters in sports contests
- Location models with contests offer a better way to approach welfare issue
- Implications for competitive balance