Effort versus Sorting: That's the question. One-Shot Contest and Two-Stage Elimination Tournament in Theory and Experiment

Preliminary version

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Abstract

In this paper we analyse two formats of lottery contests between four partially heterogeneous players theoretically and test the predictions in a laboratory experiment. We show that simultaneous one-shot competition dominates the pairwise sequential elimination tournament with respect to sorting out a "strong" agent as winner while the elimination tournament dominates in terms of effort provision at least as loong as all players remain active in the one shot contest which occurs in equilibrium if the weak players are not too ´weak and or. to few in number. We explain the differences between the two formats caused by increasing heterogeneity for a given combination of weak and strong agents by the bigger total effort any player has to respond to in the pooled interaction of the one-shot contest. Experimentally we find substantial overprovision of effort by both types in the one-shot contest and even more so in the tournament. Effort overprovision by the weak players is higher than by the strong which drives the strong players' probability to win below the benchmark. in both formats.

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1 Introduction

Economists use the term contest to refer to a competitive interaction between a set of agents who invest resources of some kind with the aim of acquiring a prize. Contests resp. tournaments[1] are important allocation mechanisms that occur in many aspects of not only human life and span a broad range from naturally evolved and informal like some animals' struggle for a food source to deliberately designed and governed by codified rules like a research and development contest organized by some government. Such situations are a relevant and interesting issue for economists for obvious reasons. Among those is to identify regularities to achieve a certain goal with an appropriately designed contest. For the latter to be relevant the contest resp. some of its defining elements need to be subject to some party's (the designer's) discretion. In which dimensions of the contest and to what extent a designer has the power to intervene respectively if such a designer exists at all depends heavily on context. If the contest is for instance a full scale war, a superior planer with the power to manipulate aspects of the confrontation is rather implausible to exist. In a deliberately designed contest like a promotion competition in a firm, a situation that is frequently referred to in the literature, assuming a designer who can control the number of contestants, the prize structure, the activity to compete in and the contest structure is innocuous. The dimension we investigate in this paper is the interaction format i.e. the contingency plan assigning the agents to certain sub-contests of the grand contest which is certainly among designable aspects in many contexts. Precisely, we study two interaction formats, the one shot multi-agent contest and the sequential elimination tournament with four participants and we think that the set of situations for which either the one or the other is feasible contains a number of interesting and relevant applications: promotion contests, R & D contests, sports contests that allow more than two players resp. teams to directly interact with each other just to name a few. Relating to the literature, Gradstein and Konrad (1999) have studied these two formats for the ratio form contest success function (CSF) with unspecified discriminatory power assuming homogeneous agents. They find that the formats are equivalent in terms of effort provision which is a standard criterion when the CSF has a discriminatory power of one. Based on the assessment that contestants somehow differing in their abilities is probably more the rule than the exception we take the approach of Gradstein & Konrad further and compare the simultaneous and the pairwise sequential knock-out[2] interaction format with heterogeneous players with respect to the following two criteria theoretically as well as in a laboratory experiment:

- total (expected) effort exerted in equilibrium
- sorting efficiency (i.e. the probability of selecting the most productive contestant as winner)

To give a glimpse on the results, our theoretical analysis reveals an interesting trade-off concerning these criteria across the two formats. While the one shot interaction selects a "strong" type of contestant with higher probability than the sequential tournament no matter how different the agents are the latter induces the agents to exert more effort given the "weak" type of players are not discouraged enough as to do drop out. Prohibitive discouragement happens in some circumstances of the one-shot contest depending on the typewise composition of the set of agents and the degree of heterogeneity between the types but never occurs in the pair-wise elimination tournament. For sufficient heterogeneity between the types of players however which prompt the weak to drop out the one-shot contest can produce a higher effort than the tournament.

Introducing heterogeneity among the contestants is in our view a straightforward innovation in analyzing different contest formats but it forces us to modeling choices that might seem arbitrary and therefore only comes at a price to pay in terms of generality with respect to the discriminatory power of the CSF, the size of the set of players and more importantly the heterogeneity of the players. Basically there is no obvious choice concerning dimensions like the CSF or the way to incorporate the heterogeneity of the contestants. For the latter several ways such as differing valuations for the prize, differing ability to transform the resource investment into competitively effective effort and differing cost of effort or combinations of those are possible. The same is true for the choice of the CSF. As our paper relates foremost to that of Gradstein & Konrad we also build our model on the prominent

[1] While this general definition of a contest is standard in the literature there seems to be no general definition of a tournament. In this paper we use the latter specifically to denote interactions in which the allocation of the prize to the winning participant is determined by some dynamic system of component contests between subsets of agents rather than by just one static interaction of all agents involved.

[2] In its standard form the sequential pairwise elimination tournament requires 2^n (with the integer $n \geq 2$) participants although such a tournament can also be run with any number of participants if the contestants in excess of the greatest number $2^n < N$ are paired with another contestant each in a preliminary round for which all other participants have a by.

Tullock or ratio form CSF albeit fixing the discriminatory power to one (lottery contest). The ratio form class of CSFs in general is axiomatically well founded (see Skaperdas 1996) and widely used in the literature. Picking the special case of discriminatory power one rather than leaving it variable might seem as an unecessary restriction at a first glance but as mentioned before this modelling choice is the result of the constraints we face in the theoretical analysis of the general ratio form CSF. The main argument to restrict to the lottery CSF is that it causes the elimination tournament model with heterogeneous agents to be analytically tractable which it isn't for other specifications concerning the discriminatory power (an exception being discriminatory power .5). Furthermore, it produces equilibria in pure strategies which is not the case for sufficiently higher discriminatory power and can nicely be explained to subjects in the experimental implementation by using the lottery analogy. To be able to derive closed form solutions of also have to restrict to the minimal non-trivial setup of four contestants of two different types. Players' types relate to the different constant per unit effort costs which we use to incorporate heterogeneity among the agents. Heterogeneity of the set of contestants can clearly come from two sources: On the one hand from the composition of the set of contestants with respect to the types (a setting with three agents of one type and one of the other can be argued to be less heterogeneous than one with two agents of each type) and on the other hand from how different the costs between the two types are. Speaking of heterogeneity in this paper we will always refer to the latter as we will compare the two interaction formats keeping the four players' types constant.

For simplicity we will work with the basic assumption of the games we analyze being of complete information. For the contestants themselves we deem this to be not particularly implausible as familiarity of the agents with each other due to e.g. past interactions is not per se uncommon. In certain contexts this assumption can however put the contest at odds with the designer's preferences. If sorting out a "strong" agent as winner is important for the planer as without any uncertainty about the contestants' abilities the most able one could simply be picked by hand without bothering to run a contest at all. But we are convinced that also with complete information there are plausible circumstances in which handpicking an agent as winner is not possible. This is basically the case in any situation in which the effort of the agents is the designer's first priority and more important than what type of contestant actually wins. Therefore, awarding the prize to an agent because of some, possibly performance related characteristic, but not the actual performance in the contest itself counteracts the incentives of the agents and will backfire against the planer's own interest. Sports is an important example for such a situation. Even though the paper form of the competitors is often well known in advance the gold medal can not be handed to the athlete with the best past record but only to the one with the best actual performance. Both, concerns for fairness and the effort of the athletes as the reason to be for the contest prohibit doing otherwise. Nevertheless, in sports the track record of the athletes is often used to discriminate between them in certain contest dimensions like the starting position, seeding etc[3]. With such situations in mind and the need for a tractable model we believe it is appropriate to maintain the perfect information setting for the designer as well.

By topic this paper is part of the literature on tournament design and we will briefly summarize some important papers to relate our work to. The paper to start studying dynamic contests in economics is Rosen (1986) who considered a sequential elimination tournament with either homo- or heterogeneous agents when the designer's objective is to allocate the prize such that the efforts in each stage are constant. Heterogeneous agents in a contest have been considered in an important paper by Stein (2002) who solved the one-shot lottery contest[4] with n strictly heterogeneous players. Turning back to sequential contests, the aforementioned work by Gradstein & Konrad and the paper of Amegashie (1999) both introduce effort maximization as the designers objective. Tthe former relates the optimal structure of the contest resp. tournament to discriminatory power of the ratio form CSF. More recently, Fu & Lu (2009) have a similar approach studying multi-stage tournaments with respect to the number of stages, player assignment in the stages and the allocation of the prize using the ratio form CSF with varying imperfect discriminatory power but considering homogeneous agents only. Another contribution to this string of literature set apart by the choice of the contest technology ("all pay auction" i.e. ratio form CSF with perfect discriminatory power) is the paper of Moldovanu & Sela (2006) who analyze the effortwise optimal assignment of agents to sub-contests in a tournament which they refer to as "contest architecture" under different assumptions for the homogeneous agents' effort cost functions. The experimental paper our work is closest related to is the recent study of Sheremeta (2010) who compared one-shot contests and

[3]The whole literature on seeding in tournaments is concerned with the effects of how to assign the heterogeneous agents in the sub-contests on the different stages of a tournament.

[4]Generalizing the number of heterogeneous participants in the one shot contest requires restricting the ratio form CSF to discriminatory power one in order to perimt explicit solutions.

two-stage tournaments with homogeneous agents. Altmann et al. (2008) also compare strategically equivalent simultaneous and sequential interaction of homogeneous agents but use an additive noise contest technology. Using an additive noise model as well Harbring & Lünser (2008) finally consider heterogeneous agents albeit only in simultaneous interaction. A robust pattern these studies reveal is that subjects are prone to provide excessive effort compared to the standard game theoretical benchmark. The extent of effort overprovision is however sensitive to e.g. the subjects'experience, competitive efficiency, the interaction format, the prize spread etc.

The remainder of the paper is organized as follows: Section 2 features the theory, first we describe the model and then we briefly take a detour to examine the mechanics of the simple two player lottery contest which is helpful to understand why one-shot contest and sequential elimination tournament differ with respect to total effort and sorting out a strong player when agents are heterogeneous. After this we solve for the equilibria in the different settings of the four player game in both interaction formats. From the equilibria our main results follow directly and we go on to explain them using the insights from our detour to the two player contest. Section 3 describes the experiment and presents the experimental data which puts the theoretical results into a behavioral perspective. Section 4 concludes.

2 The model

We consider an interaction of a predefined format in which four players can exert individually costly effort in order to win a monetary prize P which we set to unity in the diagrams appearing in the paper unless otherwise stated. Players are assumed to act as to maximize their expected payoffs. The two interaction formats we analyze are the one-shot contest between all four players (henceforth one-shot contest) and the two-stage pairwise sequential elimination tournament (henceforth tournament). The latter consists of three two-player contests, two take place in the tournament's first stage (semifinals) and a one in the second stage (final) which is played between the semifinal winners. In each contest the players involved simultaneously choose an action resp. effort $x \geq 0$. Let $N = \{1, 2, 3, 4\}$ be the set of players and the cost per unit of effort exerted by some player $i \in N$ be denoted as $c_i \in \{c_s, c_w\}$. W.l.og. we assume $c_w \geq c_s = 1$ and thus suppress c_s in all formulas. From all parameters in the model only c_w will be subject to variation to study the effect of different degrees of heterogeneity between subsets of players in both interaction formats. We assume complete information i.e. that the cost parameters of all agents are common knowledge. The probability for player i to win a contest with a set of opponents $J \subseteq N \setminus i$ (number of opponents denoted n) is given by the Tullock CSF with a discriminatory power of 1 (lottery contest):

$$q_i(x_i, X_j) = \begin{cases} \frac{1}{1+n} & \text{if } x_i = X_j = 0 \\ \frac{x_i}{x_i + X_j} & \text{otherwise} \end{cases} \quad \text{with } X_j = \sum_J x_j \tag{1}$$

The lotto contest game is like e.g. Cournot competition aggregative in the sense that x_i and the total effort of the other players X_j determine q_i irrespective of how X_j is made up by the individual efforts of the players in J. As the objective function for player i we get

$$\pi_i(x_i, X_j) = q_i P_i - c_i x_i \tag{2}$$

with P_i being the possibly personalized prize i receives on winning the contest. Note that the prize at stake is not necessarily P in all interactions of the four player model. In the semifinals of the tournament P_i is the so called option or continuation value i.e. i's expected pay-off of participating in the final for which $P_i \leq P$ holds.

2.1 Two player one-shot contest

Before we proceed to solve the different versions of both interaction formats introduced above we first analyze a simple one shot contest between two players i and j competing for some positive personalized prizes P_i and P_j with arbitrary positive constant marginal costs of c_i and c_j. This yields useful insights for the more complicated four player interactions. The maximization problem of player i reads

$$\max_{x_i} \pi_i(x_i, x_j) = \frac{x_i}{x_i + x_j} P_i - c_i x_i \tag{3}$$

As π_i is concave in x_i the FOC is sufficient for a maximum and we get the reaction function $r_i : \mathbb{R}^+ \to \mathbb{R}^+$

$$r_i(x_j) = \max\{\frac{\sqrt{P_i c_i x_j} - c_i x_j}{c_i}, 0\} \tag{4}$$

By construction of the contest success function $r_i(.)$ is not defined for $x_j = 0$ as an arbitrary small effort of i suffices to gain P_i instead of $\frac{P_i}{2}$. There is however a unique interior pure strategy Nash equilibrium of the game and we get player i's optimal effort as

$$x_i^* = \frac{P_i^2 P_j c_j}{(P_i c_j + P_j c_i)^2} \tag{5}$$

In figure 1 we display the concave and non-monotonic reaction functions of two players and two different parameter settings for each of them

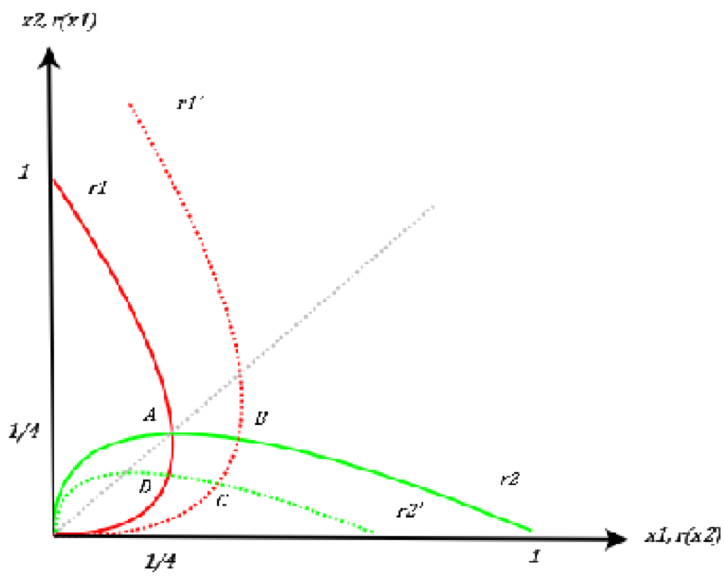

Figure 1 - reaction functions

Concerning the parameters, for any (P, c) and $(P^{'}, c^{'})$ the reaction functions of a player are identical iff $\frac{P}{c} = \frac{P^{'}}{c^{'}}$ holds. Comparing two players' non-zero best responses to some effort exerted by the opponent i is "stronger" (and j "weaker") iff $r_i(x) > r_j(x) \iff \frac{P_i}{c_i} > \frac{P_j}{c_j}$.The solid reaction functions r_1 and r_2 (red player 1, green player 2) are symmetric with $P_1 = P_2 = 1$ and $c_1 = c_2 = 1$ and so is the Nash equilibrium at $(\frac{1}{4}, \frac{1}{4})$. The inflated (dotted) reaction function $r_1^{'}$ of player 1 represents a parameter setting in which the ratio $\frac{P_1}{c_1}$ has increased (player i stronger). This results in a curve which is steeper at each location in the domain with a non-zero best reply (except zero) and the opponent's critical effort for which i ceases to be active is shifted to the right. The deflated (dotted) reaction function $r_2^{'}$ of player 2 represents a parameter setting in which $\frac{P_2}{c_2}$ has decreased and is flatter at each location in the domain with a non-zero best response than the solid (drop out point shifted inward).

A well known insight of contest theory can immediately be recognized in figure 1, namely that the competition is the more intense in terms of individual equilibrium effort relative to the maximum effort the player is prepared to exert the less heterogeneous the contestants are concerning their valuation of the prize resp. their effort costs. In the model of different marginal effort costs as source of heterogeneity this results in the intersection of the players' reaction functions lying the closer to their maxima the more homogeneous the players are. In the case of fully symmetric players the equilibrium is exactly in the maximum of the reaction curves on the 45° line. In any asymmetric equilibrium with $\frac{P_i}{c_i} > \frac{P_j}{c_j}$ the intersection of the players' reaction curves occur on the player i's (stronger player) increasing and player j's (weaker player) decreasing part of the reaction function. Therefore, a further increase in $\frac{P_i}{c_i}$ raises x_i^* but reduces x_j^* while an increase of $\frac{P_j}{c_j}$ (as long as $\frac{P_i}{c_i}$ is not exceeded) raises both agents' equilibrium efforts. How strong a certain change in the prize-cost ratio of a player alters the equilibrium efforts depends on the values determining the original equilibrium. Graphically a parameter change de- resp. inflates the reaction function of a player sweeping it along the reaction function of the other player and so the magnitude of the change of a player's equilibrium effort brought by some change in the prize-cost ratio depends on the slope of the reaction functions out of the point the change occurs from. With some manipulations and defining $\frac{P_i}{c_i} = R_i$ we can rewrite (5) as $\frac{R_i^2 R_j}{(R_i+R_j)^2}$ which increases strictly in R_i and increases in R_j for $R_i \geq R_j$ but decreases in R_j otherwise. Therfore, total effort in euilibrium obviously rises when the weaker player's prize cost ratio increases. Adding up the equilibrium efforts of both players yields $\frac{R_i R_j}{R_i+R_j}$ and we see that total effort is increasing in any player's prize cost ratio converging to R_j for R_i going to infinity. The equilibrium effort of a player is convex in R_i for $R_i < \frac{R_j}{2}$ and concave otherwise.

2.1.1 Best responses

To respond optimally to the effort of the opponent a player has to trade off the marginal benefit and the marginal cost of own effort. The benefit of own effort comes via the increasing probability to win the prize. The way the probability to win changes with own effort given a certain effort level provided by the opponent drives the shape of the reaction function. With our CSF the probability to win is strictly concave in won effort. The lower the effort to play against the steeper the initial incline of the probability curve but the stronger it flattens down for higher own effort. The higher the effort to play against the flatter is the curve initially but the less strong it flattens down further out. If the effort to play against gets prohibitively high the slope of the probability to win curve is lower than marginal cost already at the origin and the best response is to stay out of the contest. Put together, due to the ratio expression the probability to win is sensitive to own effort if the other's effort is low and insensitive when the other's effort is high and so the best response effort is increasing initially up to a certain level of the opponent's effort and decreases afterwards. This is a result of how the concavity of the probability to win as a function of own effort changes in the effort of the opponent. To show this formally consider the marginal effect of x_j on the marginal benefit of own effort:

$$\frac{\delta\delta q_i}{\delta x_i \delta x_j} P_i = \frac{P_i(x_i - x_j)}{(x_i + x_j)^3} \tag{6}$$

Focusing on the change of the marginal benefit of optimally chosen own effort by substituting $r_i(x_j)$ for x_i in (6) it follows immediately that the maximum best response is located at $x_j = r_i(x_j) = \frac{P_i}{4c_i}$ and that x_j is a strategic complement to x_i to the left and a strategic substitute to the right of this location (given the best response is not zero). Furthermore, (6) with $x_i = r_i(x_j)$ is decreasing and convex in x_j and so is the slope of $r_i(x_j)$. If we consider any pair of $(x_j, x_j^{'})$ with $x_j < x_j^{'}$ and $r_i(x_j) = r_i(x_j^{'}) > 0$ the marginal effect on $\frac{\delta q_i P_i}{\delta x_i}$ is greater for x_j than for $x_j^{'}$. A bigger adjustment in own effort is necessary in the former case to re-establish equality of marginal benefit and marginal cost than in the latter and so the increasing part of the reaction function is steeper than the decreasing part. As one can also see in figure 1 it holds that player i's (non-zero) best response to an opponent's effort x_j are decreasing and convex in c_i and increasing and concave in P_i.

For the comparison of the interaction formats we consider it is important to see how a change in the best response brought by a parameter change varies with the effort of the other player. Differentiating the first derivative of (4) for x_j with respect to c_i yields:

$$\frac{\delta\delta r_i}{\delta x_j \delta c_i} = -\frac{\sqrt{P_i c_i x_j}}{4c_i^2 x_j} \tag{7}$$

By a cost increase the slope of the reaction function becomes smaller for each x_j associated with a non-zero best response and so the decrease in the (non-zero) best response effort for a given increase in c_i rises in x_j - the higher the oppent's effort to respond to the more a cost increase dampens the player's optimal effort. This best response deflating effect is convex in c_i. The marginal impact of an increase of the prize on the slope of the reaction function given by

$$\frac{\delta\delta r_i}{\delta x_j \delta P_i} = \frac{1}{4\sqrt{P_i c_i x_j}} \tag{8}$$

is strictly positive and so the slope of the reaction function gets larger for each x_j with a non-zero best response if P_i increases. This best response effort inflating effect is concave in P_i. Inspecting (7) and (8) it becomes evident that

$$\frac{\delta\delta r_i}{\delta x_j \delta c_i}\frac{c_i}{P_i} = -\frac{\delta\delta r_i}{\delta x_j \delta P_i}$$

To what extent the best response shifts with a change of one of the paramters c_i and P_i depends on their ratio (recall that the best responses of a player are equal for any (P, c) and $(P^{'}, c^{'})$ iff $\frac{P}{c} = \frac{P^{'}}{c^{'}}$). If $P_i > c_i$ (7) is greater in absolute value than (8) and the other way around. For any given x_j a change in either the prize or the marginal cost of effort have the same effect in opposite direction on the player's non-zero best response given they are equal in absolute value.

The mechanics of how changes in P_i and or c_i influence the best response to any x_j drive the widening gap between the r_i and $r_i^{'}$ curves in figure 1 and the higher the other player's effort the further the (non-zero) best responses of a player associated with two different combinations of prizes and or marginal costs lie apart from each other. Thus, a change in prize or marginal cost of a player effortwise looms the larger the higher the total effort of the other players is to best respond against.

2.1.2 Equilibrium winning probabilities

Besides the equilibrium efforts of the players in the contest we are also interested in their immediate consequence, the probability with which a certain player emerges as the winner. If the agents are symmetric the probability to win is clearly trivial, we get $q_i^* = \frac{1}{2}$ and the contest as a device to sort out an agent with specific characteristics is pointless. Allowing for different valuations of the prize and different marginal costs for the two contestants the equilibrium winning probability is given as $q_i^*(P, c) = \frac{P_i c_j}{P_i c_j + P_j c_i}$. As the reaction function of i relates optimally chosen own effort with any effort of the other player each point on the reaction function represents a winning probability given by the expression $\frac{r_i(x_j)}{r_i(x_j)+x_j}$. Assuming a suitable parameter combination (c_j, P_j) each point on i's reaction function can be the intersection with j's reaction function and thus constitutes an equilibrium. Using $\frac{r_i(x_j)}{r_i(x_j)+x_j}$ we can relate the shape of the reaction function directly to the winning probabilities for different equilibria. Parameter combinations that produce equilibria to the left of the maximum of i's reaction function satisfy $P_i > \frac{P_j c_i}{c_j}$ while for equilibria to the right of the maximum $P_i < \frac{P_j c_i}{c_j}$ has to hold. In the origin we have $q_i = .5$ by assumption with an infinitely steep incline as it jumps to 1 for an infinitesimal change in x_i. Moving the reaction curve along to the right q_i goes down to zero convexly. Setting $P_i = P_j$, $c_i = 1$ and varying c_j (what we do in the four player interaction) we can compute the equilibrium winning probability of i directly from c_j and get

$$q_i^*(c_j) = \frac{c_j}{c_j + 1} \tag{9}$$

$$q_j^*(c_i) = \frac{1}{c_j + 1} \tag{10}$$

2.2 One shot contest

Stein (2002) has solved the simultaneous lottery contest for $n > 2$ heterogeneous players with heterogeneity modelled by the agents having different valuations of the prize and or different abilities to translate effort into competitively effective input in the CSF. Our way of modelling heterogeneity is a special case of Stein's which is given by setting the productivity paramter equal to 1 for all agents as only the ratio of the prize and the constant marginal cost are relevant for the game's equilibria. Nevertheless we quickly solve the different constellations of the model.

Each of the four players engaged in the contest resp. tournament can be either weak (w) or strong (s) concerning the per unit effort costs (weak player has high costs, strong player low costs). All players' cost parameters are common knowledge among the participants. All constellations we study are fully symmetric for all players with the same cost type and so are the unique equilibria. We will therefore solve for the Nash equilibrium effort of an arbitrary player of each cost type, suppress the individual player index i and attach the cost type index instead. Multiplication of the representative agent's effort by the number of the other weak resp. strong players and summing up gives the total effort of the other players for the representative agent to react against. We first solve the model for an arbitrary number of strong and weak players denoted by n_s resp. n_w and later present the equilibria for all possible combinations in the four player setting: All strong, three strong - one weak, two strong - two weak, one strong - three weak, all weak; Notation wise we refer to the different scenarios concerning the number of agents by adding them in a superscript. In the four player model we will just index the number of weak players to all variables which have to be distinguished across the settings for brevity. Recall, that the reaction functions of the representative strong and weak player with X_{-s} and X_{-w} denoting the total effort of all other players from the perspective of an individual strong resp. weak player are

$$r_s(X_{-s}) = \max\{\sqrt{PX_{-s}} - X_{-s}, 0\} \quad (11)$$

$$r_w(X_{-w}) = \max\{\frac{\sqrt{Pc_w X_{-w}} - c_w X_{-w}}{c_w}, 0\} \quad (12)$$

Total effort of the other contestants takes on $X_{-s} = (n_s - 1)x_s + n_w x_w$ and $X_{-w} = n_s x_s + (n_w - 1)x_w$, substituting these expressions into (11) and (12) and solving the system of equations yields

$$x_s^{n_w n_s *} = \frac{P(c_w n_w - n_w + 1)(n_w + n_s - 1)}{(n_s + n_w c_w)^2} \quad (13)$$

$$x_w^{n_w n_s *} = \frac{P(c_w + n_s + c_w n_s)(n_w + n_s - 1)}{(n_s + n_w c_w)^2} \quad (14)$$

It is important to note, that depending on the parameters the weak players' efforts can get negative for which case their equilibrium action is zero effort and they cease to be active in the contest. In the four player model this occurs in the one weak and the two weak setting.

All strong: $n_s = 4$, $n_w = 0$;

$$x_s^{0*} = \frac{3P}{16} \quad (15)$$

All weak: $n_s = 0$, $n_w = 4$:

$$x_w^{4*} = \frac{3P}{16c_w} \quad (16)$$

One weak: $n_s = 3$, $n_w = 1$;

$$x_s^{1*} = \begin{cases} \frac{3Pc_w}{(c_w+3)^2} & \text{if } c_w \leq \frac{3}{2} \\ \frac{2P}{9} & \text{otherwise} \end{cases} \quad (17)$$

$$x_w^{1*} = \begin{cases} \frac{3P(3-2c_w)}{(c_w+3)^2} & \text{if } c_w \leq \frac{3}{2} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

Two weak: $n_s = 2$, $n_w = 2$;

$$x_s^{2*} = \begin{cases} \frac{3P(2c_w-1)}{4(c_w+1)^2} & \text{if } c_w \leq 2 \\ \frac{P}{4} & \text{otherwise} \end{cases} \quad (19)$$

$$x_w^{2*} = \begin{cases} \frac{3P(2-c_w)}{4(c_w+1)^2} & \text{if } c_w \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

Three weak: $n_s = 1$, $n_w = 3$;

$$x_s^{3*} = \frac{3P(3c_w - 2)}{(3c_w + 1)^2} \quad (21)$$

$$x_w^{3*} = \frac{3P}{(3c_w + 1)^2} \quad (22)$$

From the equilibrium efforts we get the probability to have a strong contestant as winner of the one-shot contest (subscript O refers to the format) given by the ratio of the strong players' total effort divided by the total effort of both types of players:

$$q_{sO}^{n_w*} = \frac{(4-n_w)x_s^{n_w*}}{(4-n_w)x_s^{n_w*} + n_w x_w^{n_w*}}$$

2.2.1 Effects of increasing heterogeneity in the one-shot contest

Because of the deflating effect of increasing marginal cost on the reaction function it is straightforward that the (non-zero) equilibrium effort of a weak player decreases with c_w. For the strong players however the reaction functions are not altered by varying c_w and so all changes in their equilibrium efforts occur as movements along one and the same reaction function. In any equilibrium of a homogeneous one shot contest with more than two players resp. a mixed contest involving $n_s \geq 2$ and $n_w \geq 1$ agents and $c_w : x_w^{n_w n_s*} > 0$ it is obvious that the total effort of the other players is strictly greater than the own effort of any individual contestant. As the maximum of each reaction function lies on the 45° line the equilibrium in any such contest is on the decreasing part of the reaction function of each player. Thus, in equilibrium the efforts are strategic substitutes and increases in c_w which reduce $x_w^{n_w n_s*}$ lead to an increase in $x_s^{n_w n_s*}$. If the weak players' marginal costs get sufficiently high so that they cease to exert effort the we get a symmetric "strong player only" contest with equilibrium efforts $x_s^{n_w n_s*} = \frac{P(n_s-1)}{n_s^2}$. In a mixed setting with $n_s = 1$ and $n_w > 1$ the equilibrium can lie on the rising part of the strong player's reaction function (efforts strategic complements) whereas for the weak players it is always on the declining part. Thus, an increase in c_w reduces $x_w^{n_w n_s*}$ but can either raise or lower $x_s^{n_w n_s*}$ depending on the value c_w takes on. Of the type combinations we study the "three weak" setting falls in the latter category with the equilibrium on the maximum of the strong players' reaction functions iff we have $c_w : x_w^{3*}(c_w) = \frac{x_s^{3*}(c_w)}{3}$ which is true for $c_w = \frac{5}{3}$. From (7) we can see that a cost increase has a stronger negative effect on the best response of a player the higher the effort of the other player(s) to respond to is. This causes cost increases to loom large in the pooled simultaneous competition and is ultimately a consequence of the concavity of the CSF associated with higher oponnent effort. Furthermore, as the efforts of weak and strong players in equilibrium with the exception of the situation mentioned above are strategic substitutes cost increases of the weak cause the strong to raise their efforts which additionally reduces the equilibrium effort of the weak. Thus, a cost increase causes the equilibrium effort of the weak to move downward on a deflated reaction function.

2.3 Tournament

In the tournament the set of players is assigned to two pairwise first stage contests (semifinals) the winners of which interact in the second stage contest (final). The winner of the final receives the prize P. In all contests of the tournament the players choose their efforts simultaneously with the cost type of the opponent and also that of the other players in case of the semifinals being common knowledge. Concerning the notation we will use the usual subscripts s, w and i, j etc. to denote cost type resp. identity of an agent and the subscript F, S to denote the stage of the tournament. The superscripts s, w and i, j etc. denote the cost type resp. identity of the opponent in the given stage while the number of weak agents as above refers to the cost type combination (only relevant in semifinal stage).We solve for the subgame-perfect Nash equilibrium specifying the optimal efforts of all players in the semifinals and all possible finals. The finals are simple two player one shot contests analyzed in section 2.1. Three constellations are possible: all strong, all weak and mixed. Substituting the cost parameters of the players involved into (5) we get the equilibrium efforts in the final:

$$x_{sF}^{s*} = \frac{P}{4} \tag{23}$$

$$x_{sF}^{w*} = \frac{Pc_w}{(c_w+1)^2} \tag{24}$$

$$x_{wF}^{s*} = \frac{P}{(c_w+1)^2} \tag{25}$$

$$x_{wF}^{w*} = \frac{P}{4c_w} \tag{26}$$

Using (9) and the equilibrium effort we get the expected payoffs in equilibrium for each type of player

$$\pi_{sF}^{s*} = \frac{P}{4} \tag{27}$$

$$\pi_{sF}^{w*} = \frac{Pc_w^2}{(c_w+1)^2} \tag{28}$$

$$\pi_{wF}^{s*} = \frac{P}{(c_w+1)^2} \tag{29}$$

$$\pi_{wF}^{w*} = \frac{P}{4} \tag{30}$$

The win probability for a homogeneous final is $\frac{1}{2}$ while for a heterogeneous final it is given by (9). With the final efforts being known for all contingencies it remains to solve for the efforts in the semifinals. The prize the players compete for in the semifinal is the expected profit gained by taking part the final i.e. the right to contest the prize P against either possible opponent weighted by their winning probabilities. In case of the semifinal between the two other players being of the same type this lottery is of course degenerate. The effort choices of heterogeneous players in one semifinal influence the continuation or option value of qualifying for the final for the contestants in the other semifinal by determining the probability to meet the one or the other opponent. If this other semifinal is heterogeneous as well the two players involved in it necessarily evaluate any lottery over the prospective final opponents differently which feeds back into their effort choices. This dependency works in both directions and in equilibrium the semifinal efforts have to be best responses to the effort choice of the semifinal opponent and also to the win probability (and thus implicitly to the efforts) in the other semifinal. Suppose that agents in $A = \{1,2\}$ play each other in one semifinal and agents in $B = \{3,4\}$ in the other such that q_{iS}^j is a function of x_{iS}^j and x_{jS}^i etc. A profile of semifinal efforts is part of the unique subgame perfect equilibrium of the tournament iff

$$x_{iS}^{j*} = \arg\max q_{iS}^j[q_{kS}^l \pi_{iF}^k + (1-q_{kS}^l)\pi_{iF}^l] - c_i x_{iS}^j \ \forall i \in A \wedge j \in A \backslash i \wedge k = 3 \wedge l = 4 \text{ and } \forall i \in B \wedge j \in B \backslash i \wedge k = 1 \wedge l = 2 \tag{31}$$

In the tournament there are the same five combinations of cost types as in the one-shot contest which we will denote by the number of weak players as before. Additionally, in the "two weak" setting we have to distinguish how the contestants are seeded in the semifinals, which either creates two mixed or two homogeneous stage 1 contests. The case of two mixed semifinals produces either a mixed or a homogeneous final that of the homogeneous semifinals produces a mixed final with certainty. Put together, we have the following settings in which we will use symmetry between players of the same cost type in one semifinal to solve for the equilibrium stage 1 efforts for a representative agent of each type. The subgame perfect Nash equilibria are given by:

- All strong (x_{sS}^{0*}, x_{sF}^*)
- All weak (x_{wS}^{4*}, x_{wF}^*)
- One weak $(x_{wS}^{s1*}, x_{sS}^{w1*}, x_{sS}^{s1*}; x_{wF}^{s*}, x_{sF}^{w*}, x_{sF}^{s*})$
- Two weak (mixed semifinals) $(x_{wS}^{s2*}, x_{sS}^{w2*}; x_{wF}^{w*}, x_{wF}^{s*}, x_{sF}^{w*}, x_{sF}^{s*})$
- Two weak (homogeneous semifinals) $(x_{wS}^{w2*}, x_{sS}^{s2*}; x_{wF}^{s*}, x_{sF}^{w*})$
- Three weak $(x_{wS}^{s3*}, x_{sS}^{w3*}, x_{wS}^{w3*}; x_{wF}^{s*}, x_{wF}^{w*}, x_{sF}^{w*})$

All strong: In the homogeneous semifinals the players compete for the prize $\pi_{sF}^{s*} = \frac{P}{4}$ which yields equilibrium semifinal efforts

$$x_{sS}^{0*} = \frac{P}{16} \tag{32}$$

All weak: The option value of participation in the final is $\pi_{wF}^{w*} = \frac{P}{4}$ which yields equilibrium semifinal efforts

$$x_{wS}^{4*} = \frac{P}{16c_w} \tag{33}$$

One weak: There is a homogeneous semifinal between two strong players who face the prospect of playing either a strong or a weak opponent in the final. In the heterogeneous semifinal the players' prize to contest is the expected pay-off from playing a strong opponent in the final which is $\pi_{sF}^{s*} = \frac{P}{4}$ for the strong and $\pi_{wF}^{s*} = \frac{P}{(c_w+1)^2}$ for the weak player. Substituting these expressions for P_i and P_j into (5) for either the weak or strong player as players i and j gives the equilibrium semifinal efforts

$$x_{sS}^{w1*} = \frac{Pc_w(c_w+1)^2}{(c_w^3+2c_w^2+c_w+4)^2} \tag{34}$$

$$x_{wS}^{s1*} = \frac{4P}{(c_w^3+2c_w^2+c_w+4)^2} \tag{35}$$

These efforts determine the winning probability of the strong player in the mixed semifinal independent from P as

$$q_{sS}^{w1*} = \frac{c_w(c_w+1)^2}{c_w^3+2c_w^2+c_w+4} \tag{36}$$

and thus the expected value of qualifying for the final for the two strong agents in the homogeneous semifinal which takes on $q_{sS}^{w1*}\frac{P}{4} + (1-q_{sS}^{w1*})\frac{Pc_w}{(c_w+1)^2}$. Substituting this common option value of playing the final into (5) gives the equilibrium semifinal effort

$$x_{sS}^{s1} = \frac{Pc_w(c_w^4+4c_w^3+6c_w^2+20c_w+1)}{16(c_w+1)^2(c_w^3+2c_w^2+c_w+4)} \tag{37}$$

The probability to sort out a strong contestant as winner of the tournament (subscript T refers to this) becomes

$$q_{sT}^{1*} = (1-q_{sS}^{w1*})\frac{1}{c_w+1} \tag{38}$$

Three weak: This setting is analogous to the previous bare from the homogenous semifinal being between to weak agents and so the players in the mixed semifinal compete for the option values of $\pi_{sF}^{w*} = \frac{Pc_w}{(c_w+1)^2}$ and $\pi_{wF}^{w*} = \frac{P}{4}$. Substituting these expressions into (5) for either the weak or strong player gives the equilibrium semifinal efforts

$$x_{sS}^{w3*} = \frac{4Pc_w^5}{(4c_w^3+c_w^2+2c_w+1)^2} \tag{39}$$

$$x_{wS}^{s3*} = \frac{Pc_w(c_w+1)^2}{(4c_w^3+c_w^2+2c_w+1)^2} \tag{40}$$

which determine the strong player's win probability as

$$q_{sS}^{w3*} = \frac{4c_w^3}{4c_w^3+c_w^2+2c_w+1} \tag{41}$$

For the weak players in the other semifinal the option value is therefore $q_{sS}^{w3*}\frac{P}{(c_w+1)^2} + (1-q_{sS}^{w3w*})\frac{P}{4}$. Substituting into (5) gives their equilibrium semifinal efforts

$$x_{wS}^{w1} = \frac{Pc_w(c_w^4+20c_w^3+6c_w^2+4c_w+1)}{16(c_w+1)^2(4c_w^3+c_w^2+2c_w+1)} \tag{42}$$

Using the equilibrium efforts we get the probability for the strong player to win the tournament as

$$q_{sT}^{3*} = q_{sS}^{w3*}\frac{c_w}{c_w+1} \tag{43}$$

Two weak (homogeneous semifinals): This is the easier to solve case of those with two contestants of each type. In each semifinal the players know to face an agent of the other type when reaching the final and so we have the option values $\frac{Pc_w}{(c_w+1)^2}$ (strong) and $\frac{P}{(c_w+1)^2}$ (weak). Substituting these expressions for P_i and P_j into (5) for either the weak or strong player as players i and j gives the equilibrium semifinal efforts

$$x_{sS}^{s2*} = \frac{P{c_w}^2}{4(c_w+1)^2} \tag{44}$$

$$x_{wS}^{w2*} = \frac{P}{4c_w(c_w+1)^2} \tag{45}$$

The seeding of the contestants produces a mixed final for sure and so the probability of one of the strong players to win the tournament is simply

$$q_{sThSF}^{2*} = \frac{c_w}{c_w+1} \tag{46}$$

Two weak (mixed semifinals): This is the most delicate of all versions of the tournament we consider. As explained above a player in a semifinal not only has to best respond to the direct opponent's effort but also has to take into account the efforts in the other semifinal which determine how valuable it is for him to reach the final at all. This dependence goes in either direction and we can show that as it gets more likely that the strong player qualifies for the final out of one semifinal the mutual best responses of the agents in the other semifinal change such that the probability of the strong player winning increases. Therefore, a strong player's semifinal effort causes the positive direct effect of increasing the probability to advance to the final but also the negative indirect effect of making it more likely to meet the other strong contestant there. The indirect effect has a quite subtle source however: Suppose player 1 and 2 are engaged in one semifinal and 3 and 4 in the other with the lower identity indicating the strong agent in both cases. If 1 increases his effort the option value of competing in the final for both, 3 and 4 decreases implying that their reaction functions in the semifinal contest get deflated which leads to lower efforts from both as 1's effort gives rise to a negative externality on 3 and 4. This effort dampening effect is more pronounced for the strong agent 3 than for 4 but as 3's effort is overall higher the ratio of the optimally chosen own to total effort nevertheless increases. To solve for x_{sS}^{w2*} and x_{wS}^{s2*} we proceed as follows: For any exogenously given winning probability of the strong player in the other semifinal (shortly denoted by q_{sS}) there is a unique option value for each player in the given semifinal which we denote by $V_s(q_{sS})$ and $V_w(q_{sS})$ for brevity:

$$\begin{aligned} V_s(q_{sS}) &= q_{sS}\frac{P}{4} + (1-q_{sS})\frac{Pc_w}{(c_w+1)^2} \\ V_w(q_{sS}) &= q_{sS}\frac{P}{(c_w+1)^2} + (1-q_{sS})\frac{P}{4} \end{aligned} \tag{47}$$

Both option values are simply weighted averages of the expected payoffs of playing either the strong or the weak opponent in the final and thus for all $q_{sS}^{w2} \in [0,1]$ there is a unique optimal semifinal effort for both players given by appropriately substituting V_s and V_w into (5). With these mutually optimal semifinal efforts $x_{sS}^{w2w}(q_{sS})$ and $x_{wS}^{w2w}(q_{sS})$ the winning probability of the strong player in the semifinal under consideration is determined. As the two semifinals are symmetric in equilibrium it must hold that

$$\frac{x_{sS}^{w2*}(q_{sS})}{x_{sS}^{w2*}(q_{sS}) + x_{wS}^{w2*}(q_{sS})} = q_{sS} \tag{48}$$

Solving the equilibrium condition (48) for q_{sS} yields the equilibrium winning probability of the strong contestant q_{sS}^{w2*} in any of the two semifinals as a function of c_w:

$$q_{sS}^{w2*} = \frac{c_w - f(c_w) - c_w^2 + 7c_w^3 + 1}{6c_w^3 + 2c_w^2 - 2c_w - 6} \text{ with } f(c_w) = \sqrt{c_w^6 + 2c_w^5 - c_w^4 + 60c_w^3 - c_w^2 + 2c_w + 1} \tag{49}$$

Note that q_{sS}^{w2w*} is independent from P. With q_{sS}^{w2w*} determined the equilibrium option values V_s and V_w are known and substituting these as the prizes in (5) yields the equilibrium semifinal efforts

$$x_{sS}^{w2*} = \frac{Pc_w(12c_w + c_w f(c_w) + 3f(c_w) + 22c_w^2 - 4c_w^3 - c_w^4 + 3)(3c_w f(c_w) - 4c_w + f(c_w) + 22c_w^2 + 12c_w^3 + 3c_w^4 - 1)^2}{8(c_w+1)^2(3c_w^2 + 2c_w + 3)^3(3c_w + f(c_w) + 3c_w^2 + c_w^3 + 1)^2} \quad (50)$$

$$x_{wS}^{s2*} = \frac{P(12c_w + c_w f(c_w) + 3f(c_w) + 22c_w^2 - 4c_w^3 - c_w^4 + 3)^2(3c_w f(c_w) - 4c_w + f(c_w) + 22c_w^2 + 12c_w^3 + 3c_w^4 - 1)}{4(c_w+1)^2(3c_w^2 + 2c_w + 3)^2(6c_w^2 + 4c_w + 6)(3c_w + f(c_w) + 3c_w^2 + c_w^3 + 1)^2} \quad (51)$$

The expressions for the optimal semifinal efforts are much more complex compared to those of the other (even mixed) settings which is a consequence of their mutual dependency across the semifinals via the option values. Using the equilibrium semifinal efforts to derive the probability of a strong player winning the tournament yields

$$q_{sTmSF}^{2*} = (q_{sS}^{w2*})^2 + 2(1 - q_{sS}^{w2*})q_{sS}^{w2*}\frac{c_w}{c_w+1} \quad (52)$$

2.3.1 Increasing heterogeneity in the tournament - cost and option value effects

Increasing the marginal cost of the weak players creates two potential effects on the agents' reaction functions which either de- or inflate them and thereby also change the equilibrium in the contests. Weak players' reaction functions get deflated by increasing c_w in any stage of the tournament as the parameter change alters the trade-off between marginal benefit and marginal cost of own effort. The cost increases leave the weak players less competitive which benefits strong players facing a weak opponent in the tournament actually or prospectively. Thus, the option value as the prize at stake in the semifinal for any contestant varies with c_w whenever a player of the other type is a possible opponent in the final and it is composed of the probability to meet the one or the other type as opponent times the expected pay-off against either of them. The option value effect on the effort choices is the cost effect in the final anticipated at the semifinal stage. For strong players the option value is either non-monotonic inverse U shaped (when the other semifinal is mixed[5]) or monotonically increasing (when the other semifinal is all weak). For the weak players the option value is monotonically decreasing but less steep declining when the other semi-final is all weak than when it is mixed. From (5) we see that in homogeneous semifinals the equilibrium efforts of the players are given as $x_{sS}^* = \frac{V_s}{4}$ and $x_{wS}^* = \frac{V_w}{4c_w}$ while in mixed semifinals $x_{sS}^* = \frac{V_s^2 V_w c_w}{(V_s c_w + V_w)^2}$ and $x_{wS}^* = \frac{V_s V_w^2 c_w}{(V_s c_w + V_w)^2}$ hold. In all settings a strong players' option value is negatively related with the expected effort of the player provided in the final as a high option value implies meeting a weak opponent with some probability. Thus, the option value effect does not have a very big impact on the expected total effort provided by the strong player. The more a strong player's reaction function gets inflated in the semifinal due to the option value effect the more likely a low effort equilibrium with high expected pay-off for this player arises in the final. This becomes especially clear if one considers the case of the two weak tournament with homogenenous semifinals: rising costs of the weak players' let the all strong semifinal converge to a effort intense contest for P directly as the necessarily mixed final degenerates and becomes a sure win for the strong contestant. The weak player's decreasing option value reflects the increasing probability to meet a strong player in the final (mixed other semifinal) and or the decreasing expected profit of competing against a strong type. Therefore, the option value is positively related with the expected effort provided by the weak player in the final given.

2.4 Total effort and sorting in one-shot contest and tournament

Having solved all possible settings of both interaction formats we can investigate the two magnitudes of interest, namely expected total effort in equilibrium and for the mixed settings also the ex-ante probability to sort out a strong agent as winner for varying heterogeneity between strong and weak players. The meaningful comparison thereby is between the one-shot contest and the tournament with the costwise same set of agents. It has to be noted that unlike in the one-shot contest in the tournament the weak agents have a strictly positive equilibrium

[5] In this case the option value effect is initially positive inflating the reaction function (expected pay-off from participating in the final increases with c_w) for sufficiently high c_w it becomes negative deflating the reaction function again as it gets ever more sure to meet the strong opponent in the final (option value inverse U-shaped taking on $\frac{P}{4}$ for $c_w = 1$ and for c_w going to infinity). If the other semifinal is all weak the option value increases monotonically with c_w going to P.

effort for all c_w and the tournament is less discouraging for weak contestants than the one-shot interaction format. In figures 2, 3 and 4 we present figures for the total expected effort[6] (denoted X, left panel) and the probability to win for a strong agent in one shot contest and tournament for varying c_w.

All strong: Total effort is obviously independent from c_w and constant equaling $\frac{3P}{4}$ in both formats.

All weak: Total effort for both formats is $\frac{3P}{4c_w}$.

One weak: Weak agent drops out of the one-shot contest for $c_w = 1.5$

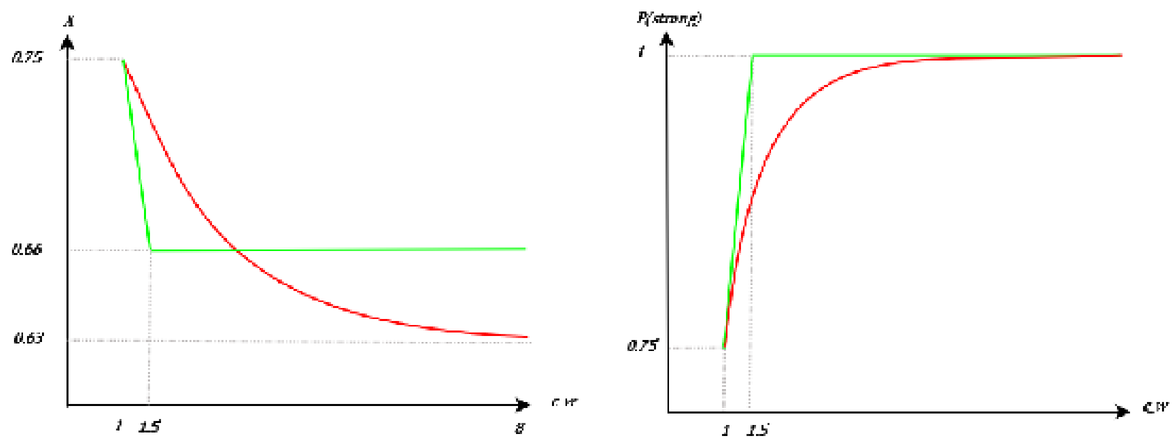

Figure 2 - total effort and sorting in one weak setting

Two weak: Weak agents drop out of the one-shot contest for $c_w = 2$. From all constellations in both formats, total effort in the mixed semifinals tournament is non-monotonic.

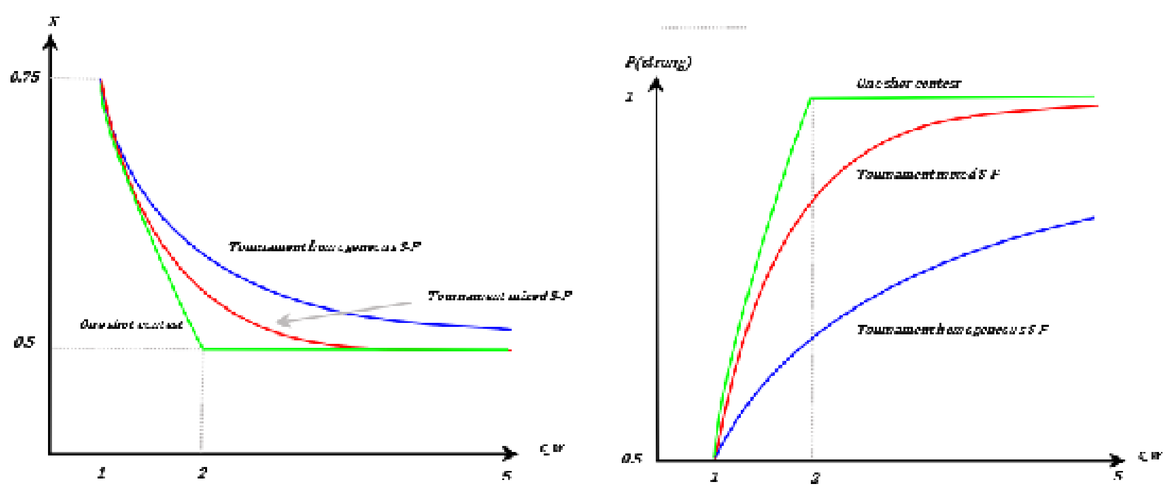

Figure 3 - total effort and sorting in two weak setting

Three weak: The effort of the strong agent does not suffice to make the three weak drop out for any c_w.

[6] In the one-shot contest the expeceted total effort is degenerate as there is no uncertainty about the agents individual equilibrium efforts. In the tournament with both types of player involved however the realized total effort is stochastic in all but the two weak homogeneous semi-final setting and depends on which agents actually move on to the final.

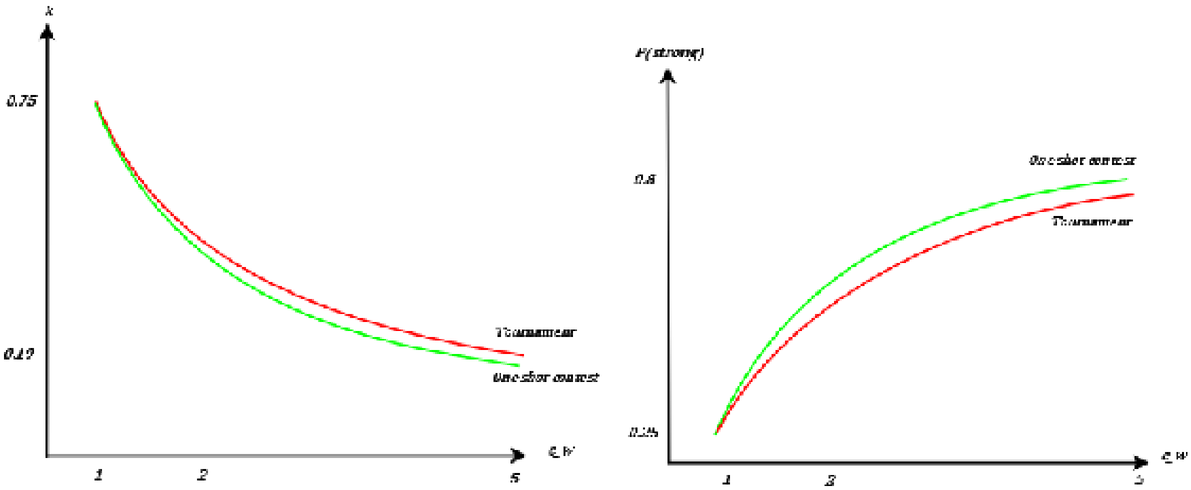

Figure 4 - total effort and sorting in three weak setting

The three results listed below become evident looking at the figures and can also be checked analytically by comparing the added up Nash equilibrium efforts resp. comparing the strong players winning probabilities in the different settings of the two formats. For the sake of brevity we relegate these exercises to the appendix and rather give intuitive explanations for the three main results.

1. Both interaction formats are equivalent in total effort exerted in equilibrium if $c_w = 1$ (Gradstein & Konrad).

2. For all c_w given the weak players remain active, total expected effort is lower in the one shot contest than in the tournament

3. For all mixed settings and all $c_w > 1$ the probability of a strong contestant to emerge as winner is higher in the one shot contest than in the tournament

Results 2 and 3 imply that there is a trade-off between effort provision and sorting out a strong contestant. The one shot contest dominates the tournament in the latter while in the former it is the other way around. Turning back to our analysis of the simple two player one-shot contest we can offer an intuitive explanation for results 2 and 3.

Ad 2: If the heterogeneity between the strong and weak players increases total effort is reduced more in the one-shot contest than in the tournament. The negative impact on a weak player's non-zero best responses induced by increasing costs is growing with the total effort of the other player(s) to respond to as can be seen from (7) (see also the widening gap between the two reaction functions of both players in figure 1). As the efforts of weak and strong players are strategic substitutes in the one shot contest in all but one constellation cost increases of the weak raise the strong players' equilibrium efforts and reduce those of the weak. In the two player interactions of the tournament the strong players' efforts can rise only due to the option value effect in the semifinals but as a high option values are the consequence of an expectedly low intensity final this effect is not very strong. The weak players' efforts decrease because of cost and possibly also option value effect but these occur with lower total effort to play against and consequently do not loom as large as in the one-shot contest. These effects and their relation to each other can be nicely seen in the following three diagrams depicting the average effort of an agent of each type from an ex ante perspective (left panel) and total effort per player type (right panel). Given that all players remain active in all but the 3 weak setting with $c_w > x$ the average strong player's effort is higher in the one shot contest than in the tournament and so in order for the aggregate effort in the one-shot contests drop below that of the tournament the average weak player's effort must be sufficiently low in the one-shot contest as to compensate the higher effort on behalf of the strong. This is in fact the case and can be explained as a consequence of the higher costs having a stronger effort reducing effect on the weak players in the pooled interaction of the one shot contest than in the pairwise interactions of the tournament.

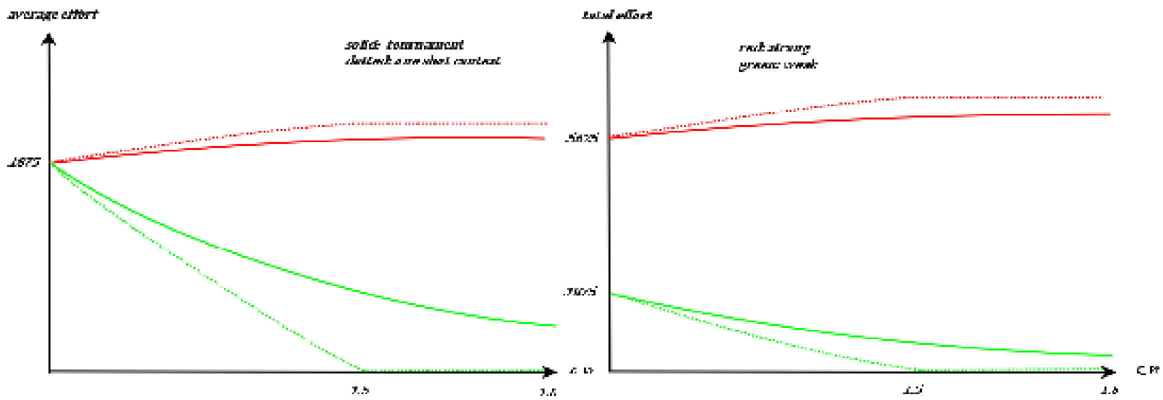

Figure 5 - average effort for types (one weak setting)

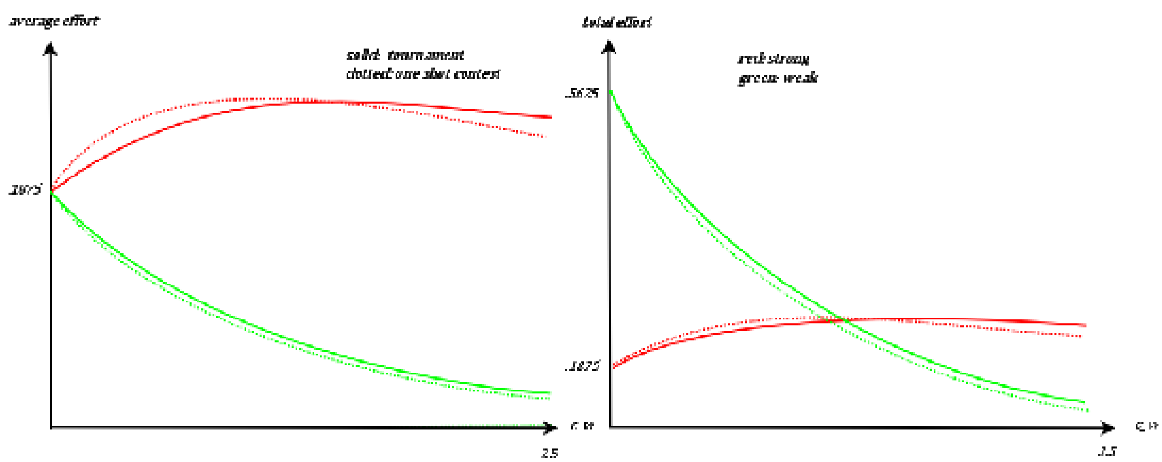

Figure 6 - average effort for types (three weak setting)

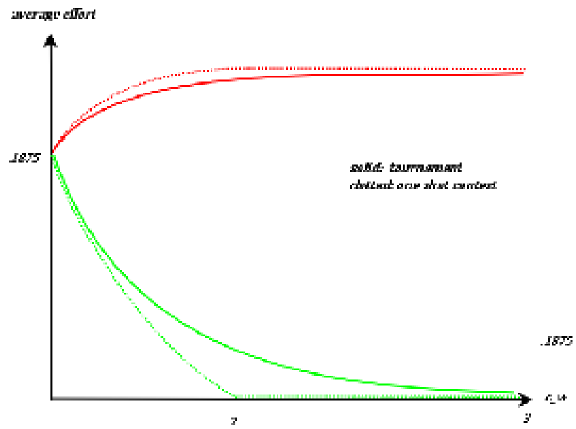

Figure 7 - average effort for types (two weak setting)

Ad 3: The higher probability to win of a strong player is the straightforward consequence of how the efforts of the different types of players change in the two formats. Increasing heterogeneity in the one shot contest lifts the average effort of the strong players beyond that in the tournament (efforts strategic substitutes on the downward sloping parts of the reaction functions) while at the same time the weak player's efforts in the one shot contest are reduced more than in the tournament.

2.4.1 The planer's trade-off

Given these results there is a clear trade-off for the contest designer: If emphasis is on incentivizing the agents to exert effort the tournament is the dominant format for at least sufficiently low levels of heterogeneity for which the weak do not drop out. If there should be two weak and two strong agents it furthermore holds that the seeding variant with two homogeneous semifinals dominates the one with mixed semifinals.

In the one weak constellation for

For $c_w \leq 1.5$ the tournament clearly dominates also in an incomplete information environment in which the planer only knows a player's type with a certain probability as no weak player will drop out of the one-shot contest even under the worst circumstances. For $c_w > 2.97$ and even more so for $c_w > 3.5$ it is not unambiguous which interaction format produces more aggregate effort from the ex-ante perspective of the incompletely informed designer as in the one weak setting and for the latter also in the two weak constellation (provided heterogeneous semifinals in the tournament) the one-shot contest produces more effort than the tournament. It can however be shown that for any probability of a player being strong resp. weak and the set of agents determined by random draws (which implies that the seeding in the two weak tournament is random as well) nevertheless the tournament produces higher aggregate effort.

If the designer's priority however is to sort out a strong agent and institutional reasons resp. the lack of information rule out hand picking such a contestant the one-shot contest dominates irrespectively of c_w and the planer's beliefs.

3 Experimental implementation

The experimental test of the relation between the two interactions formats given a certain typewise composition of the group of contestants is at this stage more a supplement to the theoretical analysis and we will thus only briefly summarize is without going much into the details. To implement the contest games each of the 20 participants of a session was randomly assigned the role of a "strong" resp. "weak" player at the session's beginning remaining in this role for its entirety[7]. The subjects played 30 periods in both interaction formats (one-shot contest and tournament) with the sequence varied across sessions to control for order effects. In each

[7] Experiments conducted at the Center of Experimental Economics at the University of Innsbruck/Austria.

session the typewise composition of the groups of four but not the persons themselves to interact with each other was kept constant. The interaction groups were reshuffled after every period with subjects of each type being randomly put together in the appropriate numbers. Additionally to this random matching procedure the participants' identities remained undisclosed throughout the experiment to restrain repeated game effects and we assume the four person groups as independent observations.

We have explained the two games separately using the lottery analogy with the instructions of the format to be run as second not referencing those of the other run before. The wording was neutral avoiding loaded terms. The lottery explanation was only hypothetical as all subjects' decisions were entered at and processed by the computer (experiment programmed with ZTREE, Fischbacher 2006) and also the random draws to decide about the winners were computerized. For each period of the one-shot contest resp. for both of the two stages of a period of the tournament (involvement in the second of course being conditional on winning the first round) the participants were given an endowment of 240 Taler (200 Taler = 1 Euro) which they could, according to the explanation, use to buy identifiable balls which would be placed in an urn and one ball would be drawn randomly to choose the winner of the interaction. Subjects' in the role of a weak player thereby had to pay a price of 1.5 Taler per ball while subjects' in the strong type role had to pay 1 Taler. In both interaction formats the winner (person whose ball was drawn in the one shot contest resp. person whose ball was drawn in the semifinal and the final of the tournament) received a prize of 240 Taler plus the endowment not being spent on purchasing balls (the latter being the pay-off for all other persons in the period).

After each period of the one-shot contest the subjects were informed about the total number of balls purchased by the other players and their own pay-off in this period. In the tournament the subjects were informed about the number of balls the person they were interacting with had bought after each stage they were actively involved in and after the last stage they were active they also got informed about their own pay-off in the period. In both formats four of the 30 periods played were randomly chosen and paid out. After the two contest games the subjects were additionally taking part in two short experiments eliciting risk and distributional preferences. All four experiments together gave a range for the payoffs from 13 to 21 Euro (average about 16 Euro, duration of a session about 100').

3.1 Treatments and theoretical benchmarks

Treatment variation concerned the typewise composition of the groups, we have implemented all possible settings in both interaction formats in a total of 14 sessions. The following table summarizes the treatments and presents the theoretical benchmarks (total equilibrium effort, sorting i.e. probability of a strong player to win) as derived in the previous section[8]:

Setting	One-shot				Tournament			
	Sessions	Obs.	Total effort	Sorting	Sessions	Obs.	Total effort	Sorting
All strong	2	600	180	1	2	600	180	1
All weak	1	300	120	0	1	300	120	0
One weak	1	300	160	0.994	1	300	164	0.884
Two weak homogeneous[9]	2	600	144	0.806	1	300	152	0.604
Two weak mixed	*X*	*X*	*X*	*X*	1	300	146.3	0.744
Three weak	1	300	131	0.45	1	300	135	0.41

3.2 Results

We will present the data of the experiments in this version of the paper on the aggregate level only. Nevertheless or maybe even because of this the most important aspects of the subjects' behavior becomes quite clear anyway. The following tables compare total effort and sorting for all settings across the one-shot contest and the tournament. It should be noted that the total effort data in the tournament consists only of actual decisions and unlike the theoretical benchmark does not feature the weighted average of the efforts in all typewise possible constellations

[8]Transforming the theoretical model with its continuous action spaces into the discrete version of the experiment may cause problems with multiple equilibria resp. non.existence of a pure strategy equilibrium. We have checked this and made sure none of these problems occur.

[9]The homogenous-mixed distinction is only relevant for the tournament and refers to the seeding of the contestants in the semi-finals.

of the final but due to the large number of observations this should not have an effect. Getting around this minor problem would have required to run the tournament interaction format applying the strategy method and ask the subjects for their effort choices for all contingencies before resolving the uncertainty who would move on to the final. As we did not want to complicate the demanding experiment any further we refrained from doing so.

Setting	Total effort data (theory)		H_0: one shot = tournament	
	One-shot	Tournament	t-Test	Rank-sum
All strong	241.39 (180)	276.46 (180)	0.00	0.00
All weak	160.53 (120)	179.3 (120)	0.0067	0.0005
One weak	231.23 (160)	256.85 (164)	0.0048	0.0011
Two weak homogeneous	203.73 (144)	288.41 (152)	0.000[10]	0.000
Two weak mixed		219.58 (146.3)		
Three weak	183.17 (131)	233.96 (135)	0.0067	0.0005

As we can see in the table, there is significant overprovision of effort in all settings: Effort provision exceeds equilibrium predictions by up to 90\%. There are clear patterns w.r.t.~overprovision: First of all, it is higher in the tournament than in the one-shot contest. Second, it is quite interesting to see that overprovision in the two homogeneous settings is equal in both the one-shot contest (where equilibrium predictions are exceeded by 30\%) and the tournament (where overprovision reaches a level of approximately 50\%). Nevertheless, the quantitative predictions of theory are not matched by the experimental data. The same holds for qualitative predictions for homogeneous settings for which the theoretical prediction is that total effort is the same for each format. Experimental evidence, however, suggests that total effort is significantly higher in sequential tournaments. Even though contrary to theoretical predictions, this finding confirms previous experimental evidence by Altmann2008 and Sheremeta2010b.

The qualitative features are in line with theoretical predictions, for the one weak, two weak and three weak settings efforts are higher in two-stage than in one-stage tournaments, and we can always reject the hypothesis of equal means. Furthermore, the data shows that the tournament efforts in the two weak setting are higher in the homogeneous than in the mixed semifinal seeding and again, we can reject the null hypothesis of mean equality.

Summing up, the experimental data allow us to reject quantitative predictions of the theoretical model in all treatments. Qualitative predictions are in line with theory in all but the homogeneous settings. Let's now turn to the data for the sorting property of the interaction formats. Analogous to total effort the data to calculate the average winning probability of a strong player for the tournament involves only the actual type pairings in the final and not a weighted average of all possible constellations. As above, the high number of observations should however avoid any substantial deviation in the average win probability we consider.

Setting	Sorting (theory)		H_0: one shot = tournament	
	One-shot	Tournament	t-Test	Rank-sum
All strong	1	1	X	X
All weak	0	0	X	X
One weak	0.83 (0.991)	0.8 (0.881)	0.3449	0.3444
Two weak homogeneous	0.5517 (0.806)	0.5367 (0.604)	0.1801[11]	0.18
Two weak mixed	X	0.6433 (0.744)		
Three weak	0.39 (0.45)	0.3167 (0.41)	0.0605	0.0604

As for the the total effort criterion we will first discuss the quantitative fit of experimental results and predictions of the theoretical model: We see that the sorting property of both formats is between 6% and 25% lower in the experiment than suggested by theory. This indicates that effort (over)provision of the weak agents is relatively bigger than by the strong agents. Generally speaking, the probability that a strong type wins is usually not much higher than the probability to draw a strong type randomly from the pool of agents. In the three weak setting even a random draw is better in terms of sorting than the tournament.

Qualitative the theoretical predictions are somewhat better: Sorting seems to be better in the one-shot contest compared to the tournament in settings one weak and three weak and; especially in case of one weak, we cannot reject the null hypotheses that the two means are equal. Further, we see that sorting is much better in the

[10] Both tests compare two weak one-shot with the aggregate data of both seeding versions of the two weak tournament.
[11] Both tests compare two weak one-shot with the aggregate data of both seeding versions of the two weak tournament.

homogeneous than in the mixed semifinal seeding of the two weak setting which is also in line with qualitative theoretical predictions. Summing up, experimental evidence for the sorting properties of the two tournament formats is rather weak; even qualitative predictions are not always met.

4 Conclusion

This paper examines how one-shot contest and sequential pairwise elimination tournament with four players of two different types relate to each other in terms of total (expected) effort in equilibrium and sorting out a "strong" agent as winner using the ratio form CSF with discriminatory power 1. It can be seen that the one-shot contest dominates the elimination tournament in the sorting property irrespective of the typewise composition of the set of contestants and irrespective of the degree of heterogeneity between the types which we introduce by increasing the constant marginal effort costs of the weak type of players (keeping those of the strong types constant). On the other hand for at least low levels of heterogeneity and or not too few weak players in the group the elimination tournament dominates the one-shot contest in terms of expected total effort in equilibrium. If there are only one or two weak agents among the contestants this result can however be reversed under certain circumstances concerning their effort costs which cause weak players to drop out[12] of the one-shot contest (but never out of the tournament). The driving force of the different properties of the formats with respect to these criteria is that increasing marginal costs of effort result in increasingly lower equilibrium efforts for the weak players the higher the total effort of the other players to react to is. As the one-shot contest pools all players, the total effort of the others is higher than in the two player interactions of the elimination tournament and so the discouraging effect of increasing heterogeneity on the weak players' efforts looms larger in this format driving aggregate effort below and the probability for a strong contestant emerging as winner above that in the tournament. We can only show these properties for this minimalistic model concerning the number of players and types and the special CSF we have chosen. Even modest generalizations of any of these dimensions of the model lead to at least some of the interactions not being analytically solveable anymore although certain results as the uniquness of the subgame perfect equilibrium in the elimination tournament with four different players can still be shown. Further research with respect to these limitations is necessary and especially the effect of the CSF with higher discriminatory power seems interesting.

The pattern of total effort and strong player winning probability across the two formats does in the big picture also materialize in the supplementary experiment we have run but quantitatively the data is quite far off from the theoretical predictions. Whatever format we consider, subjects substantially overprovide effort although this is even stronger in the tournament than in the one-shot contest. As overprovision on behalf of the weak agents is even more pronounced than from the strong agents the sorting property of the one-shot contest which performs at least as good as the tournament in this aspect but rarely that much better than theory predicts gets overall pretty dismal and is more or less only marginally more accurate than a random draw.

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