# Competing for the Attention of Decision Makers

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**Abstract**

A decision maker wants to divide a budget between multiple agents, awarding a larger share of the budget to the "more-qualified" agents. Agents are privately informed about their own qualifications, which may be independent of their valuations. Although the decision maker can learn about qualifications by reviewing agents (e.g., meeting with agents, conducting interviews, calling references), he is time constrained and cannot review everyone before choosing a budget division. The way in which the decision maker decides which agents to review affects his ability to learn agent qualifications. Instead of assuming that the decision maker reviews agents randomly, or based on some observable characteristic, we introduce "Contests for Attention," where agents compete for the decision maker's limited attention. When agents compete for attention, the decision maker learns about the qualifications by observing their efforts to gain attention (signaling), as well as by reviewing agents (direct revelation). In equilibrium, the decision maker has correct beliefs about the qualifications of *all* agents, even though he only reviews *some* agents. He chooses the same budget division that he would have chosen if he were able to review all agents. (*JEL* D44, D78, D82)

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# 1 Introduction

A decision maker must allocate a limited resource between multiple agents. The large literature on the topic focuses on the case where the optimal allocation involves the resource going to the agents who value it most. This is a fine assumption most of the time, such as when an auction house wants to raise the most money from the sale of a painting, or when a procurement officer wants to hire a firm that can perform a given service at the lowest cost. In contrast to the literature, we assume that the decision maker wants to allocate the resource based on agent qualifications when the most-qualified agents are not necessarily the agents with the highest valuations.[1]

The analysis starts with the case in which the decision maker allocates a perfectly divisible resource. All else equal, the decision maker prefers to award a larger share of the resource to more-qualified agents. Each agent is privately informed about its own qualifications, and the decision maker can "review" an agent to learn its qualifications–e.g., he can meet with or interview agents, call references, or otherwise conduct investigations. Due to time constraints, the decision maker is unable to review all agents' qualifications before choosing an allocation. The decision maker must decide which agents to review, before deciding an allocation.

The way in which the decision maker decides which agents to review affects his ability to learn agent qualifications. For example, the decision maker could randomly review agents, or select agents to review based on some observable applicant characteristic (e.g., expected qualifications). Doing so leaves the decision maker completely uniformed about the qualifications of those not reviewed. The decision maker could alternatively require that an agent make a payment in exchange for review, e.g., charge an application fee. Only agents with high-enough qualifications (i.e., a high-enough expected allocation if the decision maker learns their qualifications) are willing to pay the fee (see for example, Austen-Smith 1998, Leslie 2005, Cotton 2010). Compared to randomly reviewing applications, the use of fees makes the decision maker more informed about the qualifications of the agents who do not receive access. Although he remains uncertain about their qualifications and the ideal allocation.

This paper considers an alternative mechanism through which the decision maker may determine which agents to review: "Contests for Attention." Agents compete for the decision maker's attention. The decision maker then reviews the qualifications of the agents that gained his attention before choosing an allocation based on his beliefs about agent qualifi-

[1]Because qualifications are not perfectly correlated with valuations, known mechanisms for allocating the prizes to the highest-value agents (e.g., a Vickrey-Clarke-Groves mechanism) do not guarantee that the prizes go to the most-qualified agents.

cations. That is, the decision maker decides which agents to review by holding a contest for attention in which agents make costly payments (e.g., money, time, effort), with an agent's probability of being reviewed increasing in its own payment and decreasing in the payment of the other agents. In equilibrium, an agent with higher qualifications is willing to pay more to reveal its qualifications to the decision maker than an otherwise similar agent with lower qualifications. This monotonic relationship between qualifications and effort in the contest for attention means it is always possible to design a contest for attention that results in the decision maker becoming fully informed about the qualifications of *all* agents, even though she only reviews *some* agents. Under such a contest structure, the decision maker is certain to implement her fully-informed allocation.

When agents differ only in their qualifications, the optimal contest for attention takes the form of a standard all-pay auction, where the decision maker reviews the agents who provide the highest payment. The all-pay auction framework is typically used to allocate the prizes themselves (e.g., Siegel 2009, Baye et al. 1993, 1996, Hillman and Riley 1989), but in our framework it is used to allocate the decision maker's attention. When agents differ in terms of prize valuation, or the distribution from which their qualifications are drawn, a standard all-pay auction is not sufficient for qualification maximization. Rather, full revelation requires that the all-pay contest handicap payments to account for agent asymmetries. (Think of a head-start in a footrace or a golf handicap, which allow contestants of different types to compete on more-equal terms.) A qualification-maximizing contest for attention fully handicaps for known asymmetries, and in equilibrium the most-qualified agents are reviewed.[2]

The main result is striking: When agents compete for a decision maker's attention, the decision maker learns about the qualifications of the agents through the signaling power of their effort to gain attention. Although the decision maker is unable to review all agents, he still learns about all agents' qualifications. In equilibrium, the decision maker chooses the same allocation that he would have choose, if he were able to review all agents; that is, he implements his ideal allocation.

Our model is applicable to a variety of settings. For example, a politician may meet with special interest before deciding how to divide earmark funding, or how to divide her time between different projects. The more time the politician spends working on or promoting a

[2]There exists a growing literature on handicapping contests. Siegel (2010) develops a general all-pay contest framework with handicaps in an environment with complete information. See also Feess et al. (2008). Kirkegaard (2010) considers the use of handicaps in a model with private information about valuations. Eso and Szentes (2007) show that a version of a handicap auction can maximize revenue in a game in which the auctioneer chooses how much information to reveal to the bidders about the value of the good. In none of these articles is a handicapped contest used to elicit information about bidder types.

piece of legislation, the more likely it is to pass. The time constrained politician may not have time to meet with all interest groups, and therefore interest groups may hire lobbyists and provide political contributions in competition for this limited amount of access. Our results suggest that competition between interest groups for access to the politician improves the politician's ability to optimally allocate a budget or time between projects.

Similarly, a government bureaucrat must allocate an agency's funding across multiple projects. The US Department of Defense, for example, decides how much of their budget to allocate to different defense and research projects. The private firms and organizations that benefit from this defense spending may spend money on lobbyists and consultants in competition for the department's attention. Also, a firm's budget director or upper level management must decide how to divide an operating budget between departments; and a research funding organization must decide how to divide research funding across proposals. Although monetary payments and lobbying is less prevalent in these later examples, the beneficiaries of funding (e.g., department heads or researchers) may still undertake costly actions in competition for the decision maker's attention. Depending on the setting, these costly activities may include working long hours, having personal connections make phone calls or send letters, spending time developing detailed funding proposals, or even making an effort to kissing up to the decision maker. In all of these instances, the decision maker becomes better informed about the merits of the different projects by observing the efforts made (or lack thereof) in competition for his attention.

After deriving the results for the case in which the decision maker allocates a divisible resource, we then consider an alternative setting where the decision maker must allocate a limited number of non-divisible "prizes." This case is a better description of settings in which an employer hires new employees, a procurement officer hires a supplier to fulfill a contract, a university awards limited number of merit scholarships, or a country club lets in new members. In each of these situations, the decision maker often interviews a subset of applicants before allocating prizes based on qualifications. In this case as well, a contest for attention allows the decision maker to become fully informed about the qualifications of all agents and be certain of awarding the prizes to the most-qualified agents.

At the heart of our analysis is the contest for attention, a handicapped all-pay contest used by the decision maker to determine which agents she reviews. This framework differs from the majority of the contest literature, where contests are used to directly allocate prizes. The two articles most-closely related to our analysis, Fullerton and McAfee (1999) and Cotton (2009), deviate from the majority of the literature and use contests at early stages of a game to elicit information about player types.

The contest for attention in our paper plays a similar role as the participant-selection contest in Fullerton and McAfee (1999). In their paper, a contest designer must select a subset of agents to participate in a research tournament. Fullerton and McAfee show that using a contest to allocate entry into the research tournament ensures that only the highest-ability agents enter the tournament. In our setting and in Fullerton-McAfee, higher-qualification agents are willing to pay more to "participate" in the next stage of the game, whether the next stage is our paper's review process or Fullerton and McAfee's research tournament.

Cotton (2009) develops a stylized model in which interest groups may compete for access to a politician. The interest group that provides the highest political contribution earns the right to disclose evidence to the politician about the merits of their policy before the politician chooses which policy to implement.[3] The analysis shows how allocating access through an all-pay auction allows the politician to learn about the evidence of both interest groups, even when he only gives access to one of the groups. The political access model in Cotton (2009) is a special case our model. Our article generalizes the framework to allow for multiple prizes, numerous applicants, and applicant asymmetries.

When the decision maker reviews an application (e.g., holds an interview, calls references, or otherwise conducts an investigation), the applicant's qualifications are fully revealed. This process is related to the literature on costly information verification and auditing. Various articles show how carefully selected auditing policy can increase payoffs in a variety of situations (e.g., Townsend 1979, Baron and Besanko 1984, Border and Sobel 1987, Mookherjee and Png 1989, Kofman and Lawarree 1993, Khalil and Lawarree 2001). We identify a novel mechanism for deciding which agents to audit–through a contest–and show how the mechanism allows the decision maker to learn about the types of all agents, even when she only audits some of them.[4]

Section 2 describes the model, and provides some preliminary results about the decision maker's prize allocation at the end of the game. Section **??** considers a contest for attention in a simplified example. In this situation, allocating attention through a standard all-pay auction results always results in the decision maker being able to identify and award the prize to the most-qualified applicant. Section **??** generally defines a contest for attention, and presents the main results of the paper. We show that a decision maker can always allocate attention in such a way that she is able to identify and award prizes to the most-

[3]This is in contrast to Austen-Smith (1998) and Cotton (2010), where politicians announce a take-it-or-leave-it prices for access.

[4]Although we assume that the decision maker is limited in the number of reviews she can conduct, making it costly for the decision maker to conduct each review would produce similar results. As long as the decision maker cares enough about prize-winner qualifications, relative to the cost of reviewing applications, he will prefer the mechanism for granting access and awarding prizes described here to any other mechanism.

qualified applicants, even when there are significant asymmetries between applicants. Section 3.3 discusses the results.

## 2 Setting

An individual decision maker is responsible for splitting a budget between $n$ independent agents. Denote an arbitrary agent by $i$, and a vector of all other agents besides $i$ by the subscript $-i$. The total size of the prize equals 1, and the share assigned to agent $i = \{1, ..., n\}$ equals $p_i$, where $p = (p_1, ..., p_n)$ and $\sum_{i=1}^{n} p_i \leq 1$. (Section 4 allows for non-divisible prizes.)

Agents differ in terms of their qualifications, where $q_i$ denotes the qualifications of agent $i$, and $q = (q_1, ..., q_n)$. When the decision maker knows $q$, he can choose an allocation to maximize his payoff (these payoffs will be described below). However, he is ex ante uncertain about agent qualifications, and is therefore less than fully informed about $q$. Each agent's $q_i$ is the realization of an independent random variable continuously distributed on $\mathrm{R}_+$ according to distribution function $F_i$ and density function $f_i$. The distributions are common knowledge.

Before choosing an allocation $p$, the decision maker can review agent qualifications. If he reviews $i$'s application, he learns $q_i$. Due to time constraints, however, the decision maker is unable to review the qualifications of all agents. Formally, he may review up to $\bar{k}$ applicants, where $\bar{k} \in \{1, ..., n-1\}$. That is, he cannot review all agents. Let $r_i \in \{0, 1\}$ indicate whether the decision maker reviews applicant $i$'s qualifications. Define $\omega \equiv (r_1 q_1, ..., r_n q_n)$, which is the vector of qualifications observed through the review process. We are concerned with whether the decision maker's choice of *how* to allocate her limited attention affects her ability to optimally allocate the prizes.hisThis is equivalent the decision maker having access to a verification service which reveals $q_i$. This verification service may be scarce, so he cannot verify all $n$ qualifications. How does he decide which agent's qualifications to verify?

Prior to the decision maker deciding which agents to review, the agents may make observable payments (e.g., money, time, effort). This means that the decision maker's choice of which agents to review may depend on agent payments. Let $t_i \geq 0$ denote any payment made by agent $i$, where $t = (t_1, ..., t_n)$.

**Agent payoffs.** Agents strictly prefer receiving higher allocations, and they find payments in competition for attention costly. Therefore, agent $i$ earns utility

$$U_i(p_i, t_i) = V_i(p_i) - c_i t_i,$$

where $V_i$ is twice continuously differentiable, $V_i'(p_i) > 0$, and $c_i > 0$. An agent's utility does not directly depend on the payments made or allocation received by others. Furthermore,

$V_i$ is independent of $q_i$. By keeping agent valuations independent of their qualifications, the model is able to highlight how qualifications alone impact one's willingness to contribute. An agent's utility is linear in its payment $t_i$; assuming linearity allows for a closed-form solution for the equilibrium payment function.[5] The functions $U_i$ are common knowledge.[6]

**Decision maker as mechanism designer.** The analysis is concerned with whether the decision maker can allocate her limited attention in such a way that she becomes fully informed about the qualifications of all agents. To focus on this question, we assume that the decision maker's utility, $W$, depends only on allocation shares, $p$, and qualifications, $q$. He does not directly benefit from agent payments.[7]

Let $p^*(q)$ define the decision maker's preferred allocation given $q$, where $p^* = (p_1^*, ..., p_n^*)$. Thus,

$$p^*(q) \equiv \arg\max_p W(p; q) \text{ s.t. } \sum_{i=1}^{N} p_i \leq 1.$$

All else equal, the decision maker prefers to award larger shares of the prize to more-qualified agents. That is, $W$ is such that $\frac{\partial p_i^*}{\partial q_i} > 0$ and $\frac{\partial p_i^*}{\partial q_j} < 0$ for all $j \neq i$ and any possible $q$. Furthermore, we assume that $p_i^*(q) > 0$ for all $i$ and any possible $q$; this condition simplifies the analysis, but is not required for the main results of the paper to hold.[8] When the decision maker knows $q$, he sets $p = p^*$. However, he is ex ante uncertain about $q$, and is therefore uncertain about the identity of $p^*$.

At the end of the game, the decision maker chooses a budget division $p$. Again, to focus the analysis on the contest for attention, we require that the decision maker uses a sequentially rational strategy when selecting $p$. That is, she cannot commit to a mechanism for dividing the budget that she has an incentive to deviate from after observing payment and qualifications. At the end of the game, she chooses a budget allocation that maximizes

[5]Alternatively, $-c_i t_i$ may be replaced by a function $C : \mathrm{R}_+ \to \mathrm{R}$. So long as $C$ is strictly decreasing in $t_i$, agent equilibrium payment functions are strictly increasing in $q_i$. Therefore, the main result of the analysis holds and the decision maker learns the qualifications of all agents even when he only grants access to some of the agents.

[6]Most auction models assume that valuations or costs are unknown. This paper makes the alternative assumption that $V_i$ and $c_i$ are known, but that agents have private information about their own qualifications. This is consistent with an example in which a politician knows whether an interest group is rich or poor and he knows how desirable the group would find a change in policy or increased funding, but the politician doesn't know whether the interest group can make a strong argument that the change in policy or increase in funding will benefit the politician's constituents as a whole.

[7]If the decision maker also cares about payments, the main result of the analysis will still hold: the decision maker *can* allocate attention in such a way that he becomes fully informed about agent qualifications. However, when she cares about payments, she may not prefer such a mechanism to one that maximizes total payments, for example.

[8]This assumption holds when, for example, $\left.\frac{\partial W}{\partial p_i}\right|_{p_i=0} = \infty$. Various forms of $W$ meet the requirements described in the paper, including $W(p; q) = \sum_{i=1}^{N} q_i \ln p_i$ and $W(p; q) = \prod_{i=1}^{N} p_i^{q_i}$.

her expected utility given her beliefs about $q$. We denote the decision maker's updated beliefs about agent qualifications by $\mu$.[9] Given that the decision maker's payoffs do not directly depend on $t$ or $\omega$, the contributions and revealed qualifications can only influence $p$ by influencing the decision maker's beliefs $\mu$. If the decision maker's beliefs place probability 1 on the true state of the world $q$, then he will choose the optimal allocation, $p^*$.

At the beginning of the game, the decision maker commits to a set of rules for determining which agents to review. For all sets of payments, $t$, these rules identify a subset of agents to be reviewed.

The analysis focuses on "Contests for Attention" in which agents provide payments in competition for a limited number of reviews. A contest for attention mechanism $\Gamma = (k, \theta)$ defines the number of agents to be reviewed $k \in \{0, 1, ..., \bar{k}\}$, and a vector of scoring functions $\theta = \{\theta_1, ..., \theta_n\}$. Function $\theta_i : \mathrm{R}_+ \to \mathrm{R}_+$ maps agent $i$'s payment $t_i$ onto the real line. Each $\theta_i$ is a strictly-increasing, continuous function in $t_i$, with $\theta_i(0) = 0$. The decision maker reviews the qualifications of $i$ if there exists fewer than $k$ other agents with $\theta_j(t_j) > \theta_i(t_i)$. Tied agents receive access with equal probability.

When $\theta_i(t_i) = t_i$ for each $i$, $\Gamma$ is a standard all-pay auction for $k$ identical prizes. The agents pay bids, and the $k$ highest bidders win reviews. A more general function $\theta_i$ allows the review strategy to take into account individual agent, including differences in $v_i$, $c_i$, $p_i^*$, and $F_i$. That is, the model allows for the handicapping of bid functions in the all-pay auctions.[10] This is an all-pay contest in that an agent pays its bid, $t_i$, regardless of whether it wins.

**Game order.** The game takes place as follows.

1. The decision maker commits to a set of rules used to select applicants to review, $\Gamma = (k, \theta)$.

2. Given $\Gamma$ and their privately observed qualifications, each applicant simultaneously chooses a payment $t_i \geq 0$ in competition for attention.

3. The decision maker reviews agents according to $\Gamma$, fully learning the qualifications of those she reviews. He then allocates the budget to maximize his expected utility given his beliefs about qualifications.

**Note on our assumptions.** Here we make explicit two assumptions from the description of the game above. The assumptions are intended to focus the analysis on awarding

[9] Formally, function $\mu(\cdot \mid t, \omega)$ defines the decision maker's beliefs about the state of the world given payments $t$ and revealed qualifications $\omega$. These updated beliefs may be fully represented by a vector of updated density functions, $\left\{\hat{f}_i(\cdot \mid t, \omega)\right\}_{i=1}^{n}$, where $\mu(\hat{q} \mid t, \omega) \equiv \prod_{i=1}^{n} \hat{f}_i(\hat{q}_i \mid t, \omega)$.

[10] The paper allows for a "handicap auction" mechanism in which all payers' bids may not be treated equally. See for example Feess et al. (2008) and Siegel (2010).

the prize based on qualifications. First, we assume that the decision maker is only uncertain about agent qualifications; not their costs or valuations. This assumption works well in certain cases, such as when a politician knows the value of a policy choice or pork transfer to an industry, but remains uncertain about how his options affect the welfare of the median voter. It also works well when the value of a contract is the same for all applicants, but applicant experience is initially unknown. In other cases, it is less appropriate. For these cases, it would be straightforward, but cumbersome, to allow for multi-dimensional uncertainty. Doing so, will add a dimension of uncertainty that is not overcome by the competition for access mechanism. That is, using a competition for access mechanism to award access will result in a better-informed decision maker; although, he will still have some uncertainty about the optimal allocation.

Second, we assume that agent valuations are independent of their qualifications. If, alternatively, agent valuations of the prize are increasing in their qualifications, then the decision maker can optimally allocate the prizes by selling them through an auction. We are interested in cases where this well known result does not apply. We could also assume that agent valuations are imperfectly correlated with qualifications; in which case the analysis becomes more cumbersome, but our results continue to hold. That is, a competition for access mechanism will result in a "better-informed" decision maker when there is uncertainty about both qualifications and another characteristic; this is in contrast to the "fully-informed" decision maker when only qualifications are unknown.

**Solution concept.** For the majority of the analysis, we take the decision maker's behavior as given and focus on the agent strategies that constitute a Bayesian Nash Equilibrium of the contest for attention. Let function $T_i$ denote agent $i$'s payment strategy in the contest for attention, where $T_i(q_i)$ is $i$'s payment when it has qualifications $q_i$. The vector of payment strategies of all agents $T = \{T_1, ..., T_n\}$ constitutes an equilibrium of the contest for attention if no agent $i$ has an incentive to deviate from $T_i$ given contest mechanism $\Gamma$ and the strategies of the other agents.

Combined with the decision maker's strategies and beliefs, the agents' equilibrium strategies constitute a Perfect Bayesian Equilibrium of the game.

# 3 Analysis

We show that there always exists a contest for attention such that the decision maker becomes fully informed about the qualifications of all agents, even though he only reviews the qualifications of some agents. Such a contest sets contest score functions $\theta$ that fully accounts for agent asymmetries when comparing their payments. In equilibrium of the full-information contest for attention, the most qualified agents win attention. Following the full-information contest for attention, the decision maker can identify and implement $p^*$.

## 3.1 Fully informed decision maker

The analysis shows that there always exists a contest for attention mechanism such that agents' equilibrium payments are strictly increasing in their qualifications. Proposition 1 shows that this is a sufficient condition to guarantee a fully-informed decision maker in equilibrium.

**Proposition 1** *Suppose the decision maker awards access through contest* $\Gamma$*. If there exists a contest equilibrium* $T$ *such that for each* $i$*,* $T_i'(q_i) > 0$ *for all* $q_i \in \mathrm{R}_+$*, then* $\Gamma$ *always results decision maker's beliefs putting probability* $1$ *on the true state of the world.*

If all equilibrium payment functions are strictly increasing in qualifications, then the contest equilibrium is a separating equilibrium in which an agent's payment fully reveals its qualifications to the decision maker. The decision maker correctly infers an agent's qualifications by observing its payment.

The strict monotonicity of $T_i$ means that the function is invertible. We define $Q_i \equiv T_i^{-1}$. In equilibrium, when the decision maker observes payment $t_i$ from an agent that contributes according to $T_i$, the decision maker believes that $i$ has qualifications $Q_i(t_i)$. The decision maker acts as if $q_i = Q_i(t_i)$ unless he reviews $i$ and directly observes a different $q_i$. Formally, beliefs are such that $\mu(Q_i(t_i)) = 1$ when $r_i = 0$, and $\mu(q_i) = 1$ when $r_i = 1$. In equilibrium, these beliefs are correct.

A fully-informed decision maker is certain about the identity of the most-qualified agents. That is, a fully-informed decision maker is a sufficient condition for implementation of the decision maker's ideal budget allocation $p^*$.

## 3.2 Main Result

The main result establishes that there always exists a competition for access mechanism that results in a fully informed decision maker. The analysis holds for any $k \in \{1, ..., \bar{k}\}$; therefore, such a full revelation mechanism exists even when the decision maker can give access to a very small number of agents, including $K = 1$.

**Proposition 2** *For each* $k \in \{1, ..., K\}$*, there always exists a contest for attention* $\hat{\Gamma} = \{k, \hat{\theta}\}$ *with access equilibrium* $\hat{T}$ *such that* $\hat{T}_i = \hat{\theta}_i^{-1}$ *for all* $i = 1, ..., n$*. Such a contest always results in a fully-informed decision maker who chooses allocation* $p^*$*.*

In addition to establishing that a full-information contest always exists, the proposition also establishes that in equilibrium, $\hat{T}_i(q_i) = \hat{\theta}_i^{-1}(t_i)$ for all $q_i$. Where $\hat{Q}_i(t_i) \equiv \hat{T}_i^{-1}(q_i)$, this condition means $\hat{\theta}_i = \hat{Q}_i$. That is, the decision maker reviews the $k$ agents who's payments signal the highest qualifications, and in equilibrium, these are the most-qualified agents. This does not mean that the most qualified agents necessarily submit the highest payments to the decision maker in the competition for access game. Rather, it means that allocation of access takes into account differences in agent characteristics (e.g., valuation, wealth, qualification distribution) such that when competing for attention no agent is at an advantage or disadvantage compared to other agents with similar qualifications.

The proof to Proposition 2 in the appendix derives $\hat{\theta}$ and $\hat{T}$. In the following equation for $\hat{T}$, the function $\Omega_{ik}(q_i, q_{-i}) = 1$ if fewer than $k$ other agents have qualifications greater than $q_i$; otherwise, $\Omega_{ik}(q_i, q_{-i}) = 0$.

$$\hat{T}_i(q_i) = \frac{1}{c_i} \int_0^{q_i} \int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \left[ \left(1 - \Omega_{ik}(y; \hat{q}_{-i})\right) \times V_i'\left(p_i^*\left(y, \hat{q}_{-i}\right)\right) \times \frac{\partial p_i^*\left(y, \hat{q}_{-i}\right)}{\partial q_i} \right] d\hat{q}_{-i} dy. \quad (1)$$

For each $i$, $\hat{\theta}_i(t_i) = \hat{T}_i^{-1}(q_i)$. Note that $\hat{T}_i$ is invertible since it is strictly increasing in $q_i$. The derivative of $\hat{T}_i$ with respect to $q_i$ is

$$\hat{T}_i'(q_i) = \frac{1}{c_i} \int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \left[ \left(1 - \Omega_{ik}(q_i; \hat{q}_{-i})\right) \times V_i'\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) \times \frac{\partial p_i^*\left(q_i, \hat{q}_{-i}\right)}{\partial q_i} \right] d\hat{q}_{-i}, \quad (2)$$

which is strictly positive.[11] If agent $i$ receives access under $\Gamma$, the decision maker observes $q_i$ directly. If $i$ does not receive attention, the decision maker believes that $q_i = \hat{Q}_i(t_i)$, as is consistent with the Perfect Bayesian Equilibrium of the game.

## 3.3 Discussion of Main Result

In equilibrium, independent of other heterogeneity, the decision maker reviews the $k$ agents with the highest qualifications and then chooses his fully informed allocation. Since a relatively rich agent (e.g., one with a low $c_i$) finds any payment less costly, its equilibrium payment function is steeper than an otherwise similar poor agent. In equilibrium, a rich agent contributes more than an otherwise similar poor agent with the same qualifications, but the rich agent is not more likely to win attention since the access mechanism accounts for its higher ability to pay. Similarly, in equilibrium, differences in agent valuations (i.e., $V_i$), qualification distributions (i.e., $F_i$), or politician biases (i.e., $p_i^*$) do not give agents an

[11] It must be the case that $\Omega_{ik} = 0$ for some agents and $\Omega_{ik} = 1$ for others. This is guaranteed by $k \in \{1, ..., K\}$.

advantage or disadvantage when trying to secure attention, as $\hat{\Gamma}$ accounts for this heterogeneity.

**Importance of review process.** An agent can "exaggerate" its qualifications by paying $t_i > \hat{T}_i(q_i)$. However, doing so also increases the probability that the agent wins attention and its true qualifications are discovered directly. In equilibrium, the slope of an agent's payment function must be steep enough such that any expected benefit from exaggerating its qualifications is offset by the monetary costs of doing so. At the same time, the slope must be moderate enough that the agent doesn't want to pay less than its equilibrium payment. This implies Eq. 2, which is formally derived in proof to Proposition 2 in the appendix.

The main result requires that the decision maker review at least one agent; that is, it only holds for $k \in \{1, ..., \bar{k}\}$. It is the increase in the probability of being reviewed that keeps an agent from paying more than $\hat{T}_i(q_i)$. Without this disincentive to overpay the strict monotonicity of the equilibrium payment functions breaks down.

## 3.4 Homogeneous Agents

Section 3.2 solves for a full revelation contest for attention in for the model with potentially heterogeneous agents. This section considers the special case in which agents are completely homogeneous, only differing in terms of their qualifications. In this game, $c_i$, $V_i$, $p_i^*$, and $F_i$ are identical for all agents.

When agents are homogeneous, the full-information contest described in Proposition 2 simplifies to a standard all-pay auction in which the agents that submit the highest payments receive attention. Therefore, the decision maker becomes fully informed about agent qualifications if he uses a standard all-pay auction to select which agents to review.

**Proposition 3** *In the game with homogeneous agents, f or each $k \in \{1, ..., \bar{k}\}$, allocating access to the $k$ agents that submit the highest payments in a standard all-pay auction results in a fully-informed decision maker who implements $p^*$.*

## 3.5 A More General Result

Section 3.2 describes a specific contest for attention that always results in full information about agent qualifications. Although the described full information contest is intuitively appealing (e.g., it always awards access to the most qualified agents), it is not unique.

There exists other contests for attention that satisfy Proposition 1, resulting in the decision maker being fully informed about agent qualifications. Proposition 4 provides a more general result.

**Proposition 4** *Let $Z$ denote any vector of functions $\{Z_1, ..., Z_n\}$ where for each $i$, $Z_i : \mathrm{R}_+ \to \mathrm{R}_+$, $Z_i(0) = 0$, $Z_i'(x) > 0$ for all $x \in \mathrm{R}_+$, and there exists at least $k$ other $j \neq i$ such that $\lim_{x\to\infty} Z_i(x) \geq \lim_{x\to\infty} Z_j(x)$. Then, there exists a contest for attention mechanism $\tilde{\Gamma} = \{k, \tilde{\theta}\}$ with equilibrium $\tilde{T} = \{\tilde{T}_1, ..., \tilde{T}_n\}$ such that for each $i$ and all $q_i \in \mathrm{R}_+$, $\tilde{\theta}_i(\tilde{T}_i(q_i)) = Z_i(q_i)$. Such a mechanism results in a fully informed decision maker who implements $p^*$.*

The function $Z_i$ describes the relationship between agent $i$'s qualifications and the probability that it receives attention in equilibrium. The conditions imposed on $Z$ by Proposition 4 imply that an agent's probability of receiving attention is strictly increasing in its qualifications, an agent with the lowest possible qualifications never receives attentions,[12] and for any $q_i > 0$ the agent wins attention with probability $\tilde{\theta}_i(q_i) \in (0, 1)$. The requirement that for each agent there must exist at least $k$ other $j \neq i$ such that $\lim_{x\to\infty} Z_i(x) \geq \lim_{x\to\infty} Z_j(x)$ results in separating equilibrium over all possible $q$.[13]

# 4 Non-Divisible Prizes

This section shows that the main result of the paper continues to hold when prizes are non-divisible. Here, the decision maker must award $m \in \{1, ..., \bar{k}\}$ identical prizes to agents. An agent may either receive one prize, or not win a prize; agents may not receive multiple prizes or fractions of prizes. The decision maker's utility is such that he strictly prefers to award a prize to each of the $m$ most-qualified agents. The concept of qualifications and their distribution is unchanged from the earlier analysis. Let $v_i > 0$ denote the value of receiving a prize to agent $i$, where $v = \{v_1, ..., v_n\}$ is common knowledge.

In this alternative framework, the decision maker may still design a full information contest for attention to allocate attention, similar to the one described by Proposition 2.

**Proposition 5** *When the decision maker must award $m \in \{1, ..., \bar{k}\}$ non-divisible prizes, there always exists a contest for attention $\breve{\Gamma} = \{k, \breve{\theta}\}$ with access equilibrium $\breve{T}$ such that $k = m$ and $\breve{T}_i = \breve{\theta}_i^{-1}$ for all $i$. Such a contest always results in a fully-informed decision maker who awards prizes to the most-qualified agents.*

The proposition shows that one can always design a contest for attention such that the decision maker becomes fully informed about the qualifications of all agents (by observing

[12] Technically, an agent with the lowest qualifications may receive attention if enough other agents also have the lowest qualifications. This happens with probability 0.

[13] If this condition is not met for agent $i$, then there exists a cut point for $q_i$ such that any $q_i$ greater than the cut point results in the agent winning attention for sure, and there is pooling amongst any agent-type with $q_i$ greater than the cut point.

their payments). The decision maker is therefore certain to award the prizes to the most-qualified agents.

The proof to Proposition 5 in the appendix derives the $\breve{\theta}$ and $\breve{T}$ that satisfy Proposition 5. In the following equation for $\breve{T}_i$, the function $\Phi_{i,k}(q_i)$ denotes the ex ante probability that fewer than $k$ other agents have qualifications greater than $q_i$. Given the characteristics of the qualification distributions $F = \{F_1, ..., F_N\}$, the function $\Phi_{i,k}$ is strictly increasing in $q_i$, and differentiable.

$$\breve{T}_i(q_i) = \frac{1}{c_i}\Phi_{i,k}(q_i)v_i. \tag{3}$$

For each $i$, $\breve{\theta}(t_i) = \breve{T}_i^{-1}(q_i)$. Note that $\breve{T}_i$ is invertible since it is strictly increasing in $q_i$. The derivative of $\breve{T}_i$ with respect to $q_i$ is

$$\breve{T}_i'(q_i) = \frac{1}{c_i}\Phi_{i,k}'(q_i)v_i, \tag{4}$$

which is strictly positive. Although the analysis behind Eq. 3 and 4 is similar to the case when prizes are divisible, the intuition differs. When prizes were divisible, the slope of the payment function (Eq. 2) reflects the disincentive necessary to prevent agents from over representing their qualifications. When prizes are non-divisible, and the decision maker awards attention through the mechanism described in Proposition 5, agents would never overrepresent their qualifications. This is because in equilibrium an agent always receives attention and discloses its true qualifications before receiving a prize. Exaggerating one's qualifications increases the agent's payment without increasing the likelihood it receives a prize. Here the slope of $\breve{T}_i$ is just steep enough that agents never want to *underrepresent* their qualifications.

# 5 Conclusion

The analysis shows that the way in which a decision maker allocates his limited attention can affect his information. If he allocates attention through a contest mechanism, he becomes fully informed about the qualifications of all agents, even though he only reviews some of the agents. Such mechanisms always exist, including one in which the decision maker awards access to the agents whose contributions signal the highest qualifications. The analysis holds for various prize structures, including when the decision maker splits a divisible resource (e.g., budget) between agents, and when the decision maker chooses which agents receive one of a limited number of prizes (e.g., jobs).

This paper contributes to the literature in at least three important ways. First, we show that even when the decision maker cannot give access to all privately informed agents, he can still become fully informed about agent qualifications if agents compete for attention. Second, it contributes to the literature on auctions and competitions, as the contests for attention studied in this paper are a variation of a handicap all-pay auction. Third, the paper makes a contribution to the applied fields addressed by the model. For example, the results provide an insight into political lobbying. They suggest that campaign contributions may play a beneficial role in a decision making process. By observing a special interest group's willingness to pay in an effort to gain attention, a politician can become better informed about the merits of the group's position. Similarly, the results concerning non-divisible prizes provide insights into the labor market where job applicants put in costly effort in an effort to get noticed and get invited for an interview.

Although the framework is already quite general, there is room to make it even more so. For example, the model assumes that only agent qualifications are unobserved; all other characteristics, including valuations and cost parameters, are common knowledge. This assumption makes the analysis tractable, and may be reasonable for the case when agents are well established special interest groups. However, it is less realistic when agents are job applicants, for example. It would be interesting to consider how well the results hold up when there are multiple dimensions of uncertainty. We expect that both more general evidentiary structures and multidimensional uncertainty would mean the decision maker cannot become fully informed about agents who do not receive attention, although competition for attention should still help the decision maker become *better informed* compared to if he assigned access randomly or based on some ex ante observed agent characteristic. Its less clear if the contests for attention found here would continue to be optimal from the perspective of the decision maker. These are questions for future research.

# 6 Appendix

## 6.1 Proofs

**Proof of Proposition 1.** Given that $T_i$ is strictly monotonic, there exists a one-to-one mapping between agent $i$'s qualifications $q_i$ and its contribution $t_i = T_i(q_i)$. Furthermore, it implies that $T_i$ is invertible; let $Q_i(t_i) \equiv T_i^{-1}(q_i)$. The rational decision maker, upon seeing the agent's contribution will correctly infer that $q_i = Q_i(t_i)$. This is true for all $i$, and in equilibrium the decision maker is fully informed. ■

**Proof of Proposition 2.** In this proof, we walk through the derivation of the vector of strictly increasing functions $\hat{T}$ such that if the decision maker awards access according to mechanism $\hat{\Gamma} = \{k, \hat{\theta}\}$ (where $\hat{\theta}_i^{-1}(t_i) = \hat{T}_i(q_i)$) then for each $i$, making payments according to $\hat{T}_i$ is a best response when all other agents play according to $\hat{T}_{-i}$. Therefore, $\hat{T}$ constitutes an access equilibrium of mechanism $\hat{\Gamma}$. The derivation of $\hat{T}$ relies only on the initial conditions of the model, and such a $\hat{T}$ and $\hat{\Gamma}$ will always exist. Furthermore, the solution for $\hat{T}$ meets the requirements of Lemma **??**, and therefore $\hat{\Gamma}$ is a full revelation mechanism.

Now, for the derivation of $\hat{T}$ and $\hat{\Gamma}$. Let $\hat{T} = \{\hat{T}_1, ..., \hat{T}_N\}$ denote a set of payment functions, where for each $i$, the function $\hat{T}_i$ is differentiable and strictly increasing in $q_i$. (Later, we show that such conditions are met in equilibrium.) Since $\hat{T}_i$ is strictly increasing, it is invertible. Define $\hat{Q}_i \equiv \hat{T}_i^{-1}$, and let $\hat{Q} = \{\hat{Q}_1, ..., \hat{Q}_N\}$. If an agent contributes according to $\hat{T}_i$, then the agent's qualifications are given by $\hat{Q}_i(t_i)$. Suppose that $\hat{T}$ is the access equilibrium of a competition for access mechanism $\hat{\Gamma} = \{k, \hat{\theta}\}$ that always awards access to the most qualified agents. Since $\hat{\Gamma}$ always awards access to the agents with the highest $q_i$, then it must be that for all $i$ and $j$, $\hat{\theta}_i(\hat{T}_i(q_i)) > \hat{\theta}_j(\hat{T}_j(q_j))$ if and only if $q_i > q_j$. This will be the case if, for all agents, $\hat{\theta}_i(t_i) = \hat{Q}_i(t_i)$. When $\hat{\theta}_i(t_i) = \hat{Q}_i(t_i)$ for all $i$, the $k$ agents that signal the highest qualifications receive access, and in equilibrium these $k$ agents with the highest signals are the most qualified agents. Below, the analysis solves for $\hat{T}$ such that these conditions are satisfied.

Since $\hat{T}$ satisfies the requirements of Proposition 1, $\hat{\Gamma}$ is a full revelation mechanism. The decision maker rationally puts probability 1 on $q_i = \hat{Q}_i(t_i)$ when agent $i$ does not receive attention. If agent $i$ does receive attention, the decision maker learns $q_i$ directly. Let $\Omega_{ik}\left(\theta_i(t_i); \{\theta_j(t_j)\}_{\forall j \neq i}\right) \in \{0, 1\}$ indicate whether fewer than $k$ other agents have $\theta_j(t_j) > \theta_i(t_i)$. Therefore, $\Omega_{ik}\left(\theta_i(t_i); \{\theta_j(\hat{T}_j(\hat{q}_j))\}_{\forall j \neq i}\right) = 1$ if agent $i$ receives attention in state $\hat{q}$ given $i$'s own payment $t_i$.[14]

To derive the equilibrium payment function $\hat{T}_i$, the analysis considers the payment of agent $i$ taking as given that all other agents make payments according to $\hat{T}$. Agent $i$ chooses $t_i$ to maximizes his expected utility:

$$\begin{aligned} \int_{\hat{q}_{-i}} \pi_i(\hat{q}_{-i}) \Big[ & \Omega_{ik}(\hat{Q}_i(t_i); \{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j \neq i}) \times V_i\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) \\ & + \Big(1 - \Omega_{ik}(\hat{Q}_i(t_i); \{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j \neq i})\Big) \times V_i(p_i^*(\hat{Q}_i(t_i), \hat{q}_{-i}))\Big] d\hat{q}_{-i} - c_i\ t_i. \end{aligned} \tag{5}$$

Since the decision maker expects that he is fully informed, he chooses policy according to $p^*(q)$; for each $i$ plugging in $q_i = \hat{Q}_i(t_i)$ when he only observes agent $i$'s payment (i.e., when $i$ does not receive attention), and plugging in $q_i = q_i$ when $i$ has attention. When $\Omega_{ik} = 1$ the agent receives attention, and the decision maker awards $i$ allocation $p_i^*\left(q_i, q_{-i}\right)$ based on his true qualifications, as revealed through the review process. When $\Omega_{ik} = 0$ the agent does not receive attention, and the decision maker awards $i$ allocation $p_i^*(\hat{Q}_i(t_i), q_{-i})$ based on the equilibrium qualifications that correspond to payment $t_i$.

[14]It is straightforward, but unnecessary for the analysis to derive $\Omega_{ik}$ from the qualification distributions $\{F_j\}_{j=1}^N$. We leave that exercise to the reader.

First order conditions for the agent's problem are given by

$$\int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \Bigg[ \left(1 - \Omega_{ik}(\hat{Q}_i(t_i); \{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j \neq i})\right) \times V_i' \left(p_i^* \left(\hat{Q}_i(t_i), \hat{q}_{-i}\right)\right) \times \left.\frac{\partial p_i^*(q_i, \hat{q}_{-i})}{\partial q_i}\right|_{q_i = \hat{Q}_i(t_i)} \times \frac{\partial \hat{Q}_i(t_i)}{\partial t_i} + \left.\frac{\partial \Omega_{ik}(q_i; \{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j \neq i})}{\partial q_i}\right|_{q_i = \hat{Q}_i(t_i)} \times \frac{\partial \hat{Q}_i(t_i)}{\partial t_i} \times \left[V_i\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) - V_i(p_i^*(\hat{Q}_i(t_i), \hat{q}_{-i}))\right] \Bigg] d\hat{q}_{-i} - c_i = 0. \tag{6}$$

The first row of notation in the above condition represents the marginal impact that a change in $t_i$ has on the decision maker's beliefs about $q_i$ and the resulting change in agent policy utility when $i$ does *not* receive attention. The second row represents the marginal impact of a change in $t_i$ on on the probability the agent wins attention.

The strict monotonicity of $\hat{T}_i$ (which we establish below) means that in equilibrium, $1/\hat{Q}_i'(t_i) = \hat{T}_i'(q_i)$. Also in equilibrium all agents including $i$ contribute according to their equilibrium payment functions, therefore if $\hat{T}$ is an equilibrium, then $\hat{Q}_i(t_i) = q_i$ and $\hat{Q}_i(\hat{T}_i(q_i)) = q_i$. The first order conditions simplify to Eq. 2 in Section 3.2. The initial conditions regarding $V_i$ and $W$ imply that $\hat{T}_i'(q_i) > 0$ for all $q_i \in \mathrm{R}_+$. Integrating with respect to $q_i$ gives the closed-form solution for the equilibrium payment function, which is given by Eq. 1.

It is possible to verify the concavity of the agents' maximization problem, given the competition for attention mechanism $\hat{\Gamma}$. To do so, plug in $1/\hat{T}_i'(q_i)$ for the first occurrence of $\hat{Q}_i'(t_i)$ in Eq. 6. Then substitute in Eq. 2 for $\hat{T}_i'(q_i)$. Simplifying the expression gives revised first derivative

$$c_i \int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \times \left.\frac{\partial \Omega_{ik}(q_i; \{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j \neq i})}{\partial q_i}\right|_{q_i = \hat{Q}_i(t_i)} \times \frac{\partial \hat{Q}_i(t_i)}{\partial t_i} \times \left[V_i\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) - V_i(p_i^*(\hat{Q}_i(t_i), \hat{q}_{-i}))\right] d\hat{q}_{-i}. \tag{7}$$

Since $\partial p_i^*/\partial q_i > 0$ for all $q_{-i}$, it follows that $p_i^*(q_i, \hat{q}_{-i})$ given any $\hat{q}_i$ iff $q_i > \hat{Q}_i(t_i)$, or equivalently iff $t_i < \hat{T}_i(q_i)$. Thus, $V_i\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) - V_i(p_i^*(\hat{Q}_i(t_i), \hat{q}_{-i})) > 0$ for all $\hat{q}_i$ iff $t_i < \hat{T}_i(q_i)$. Given this holds for all $\hat{q}_{-i}$, $t_i < \hat{T}_i(q_i)$ is a necessary and sufficient condition for Eq. 7 to be positive. It is negative iff $t_i > \hat{T}_i(q_i)$. Thus, under mechanism $\hat{\Gamma}$, the agent maximization problem achieves its maximum at $t_i = \hat{T}_i(q_i)$.

When an agent increases his payment, doing so signals higher qualifications, but it also increases the probability that the agent wins attention (in which case, the decision maker ignores the signal and depends instead on the qualifications revealed through the review process). When the equilibrium condition given by expression 2 holds, the monetary costs of "exaggerating"' one's qualifications (i.e., signaling higher qualifications than one actually has) is greater than the expected benefit of doing so. Therefore, an agent does not have an incentive to contribute more than $\hat{T}_i(q_i)$. Similarly, he also prefers to contribute $\hat{T}_i(q_i)$ to any lower $t_i$. This follows because agent expected utility is strictly increasing in all $t_i < \hat{T}_i(q_i)$. This also rules out the possibility that an agent prefers not to participate. Not participating is equivalent to setting $t_i = 0$, which results in $p_i = 0$ for sure. When each agent $i$'s payment strategy $\hat{T}_i$ is given by equation 1, no agent has an incentive to deviate from playing their respective strategy. The set of payment strategies $\hat{T}$ for all agents constitutes an attention equilibrium under competition for attention mechanism $\hat{\Gamma} = \{k, \hat{Q}\}$.

■

**Proof of Proposition 3.** Consider equations 1 and 2. When agents are homogeneous, the right hand sides of both expressions are independent of an agent's identity. (Note that if $F_i$ is the same for all agents, then $\pi_i$ and $\Omega_{ik}$ will be as well.) Therefore, the equilibrium payment functions $T$ will also be identical across all agents. Since $T$ is identical for all agents, the one-to-one mapping between agent qualifications and payments is also the same across agents. This means that in equilibrium agent $i$ submits a higher payment than agent $j$ if and only if $q_i > q_j$. Lemma **??** implies that the all-pay auction mechanism is a full revelation mechanism in this setting. ■

**Proof of Proposition 4.** The proof to Proposition 4 follows the same method as the proof to Proposition 2. Therefore, we do not provide as many details or discussion here as we do in the earlier proof. Here, we assume an arbitrary vector of function $Z$ that meet the conditions in Proposition 2, then using the same method as in the proof to Proposition 2, we solve for $\tilde{\theta}$ and $\tilde{T}$. Such $\tilde{\theta}$ and $\tilde{T}$ may always be found (and therefore they always exist). It is straightforward to show that for each $i$, $\tilde{T}_i$ is strictly increasing in $q_i$; therefore the conditions of Lemma **??** are met and $\tilde{\Gamma} = \{k, \tilde{\theta}\}$ is a full revelation mechanism.

To derive the equilibrium payment function $\tilde{T}_i$, the analysis considers the payment of agent $i$ taking as given that all other agents make payments according to $\tilde{T}_{-i}$. Agent $i$ chooses $t_i$ to maximizes his expected utility:

$$\int_{\hat{q}_{-i}} \pi_i(\hat{q}_{-i}) \Big[\Omega_{ik}(\tilde{\theta}_i(t_i); \{Z_j(\hat{q}_j)\}_{\forall j \neq i}) \times V_i\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) + \Big(1 - \Omega_{ik}(\tilde{\theta}_i(t_i); \{Z_j(\hat{q}_j)\}_{\forall j \neq i})\Big) \times V_i(p_i^*(\tilde{Q}_i(t_i), \hat{q}_{-i}))\Big] d\hat{q}_{-i} - c_i\ t_i.$$

This is a more general version of Eq. 5 in the earlier proof. First order conditions for the agent's problem are given by

$$\int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \Bigg[ \Big(1 - \Omega_{ik}(\tilde{\theta}_i(t_i); \{Z_j(\hat{q}_j)\}_{\forall j \neq i})\Big) \times V_i'\left(p_i^*(\tilde{Q}_i(t_i), \hat{q}_{-i})\right) \times \frac{\partial p_i^*(q_i, \hat{q}_{-i})}{\partial q_i}\Big|_{q_i = \tilde{Q}_i(t_i)} \times \frac{\partial \tilde{Q}_i(t_i)}{\partial t_i} + \frac{\partial \Omega_{ik}(q_i; \{Z_j(\hat{q}_j)\}_{\forall j \neq i})}{\partial q_i}\Big|_{q_i = \tilde{\theta}_i(t_i)} \times \frac{\partial \tilde{Q}_i(t_i)}{\partial t_i} \times \Big[V_i\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) - V_i(p_i^*(\tilde{Q}_i(t_i), \hat{q}_{-i}))\Big]\Bigg] d\hat{q}_{-i} - c_i = 0.$$

The strict monotonicity of $\tilde{T}_i$ means that in equilibrium, $1/\tilde{Q}_i'(t_i) = \tilde{T}_i'(q_i)$. Also in equilibrium all agents including $i$ contribute according to their equilibrium payment functions, therefore $\tilde{Q}_i(t_i) = q_i$. Additionally, $\tilde{\theta}_i(\hat{T}_i(q_i)) = Z_i(q_i)$. The first order conditions simplify to

$$\tilde{T}_i'(q_i) = \frac{1}{c_i} \int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \left[ (1 - \Omega_{ik}(Z_i(q_i); \{Z_j(\hat{q}_j)\}_{\forall j \neq i})) \times V_i'\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) \times \frac{\partial p_i^*\left(q_i, \hat{q}_{-i}\right)}{\partial q_i} \right] d\hat{q}_{-i}.$$

Given the conditions imposed by the proposition on $Z$, this expression is strictly positive. Therefore, the requirements of Lemma **??** are met, and $\tilde{\Gamma}$ is a full revelation mechanism. It is straightforward to integrate with respect to $q_i$ in order to solve for $\tilde{T}_i$.

$$\tilde{T}_i(q_i) = \frac{1}{c_i} \int_0^{q_i} \int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \left[ (1 - \Omega_{ik}(y; \hat{q}_{-i})) \times V_i'\left(p_i^*\left(Z_i(y), \{Z_j(\hat{q}_j)\}_{\forall j \neq i}\right)\right) \times \frac{\partial p_i^*\left(q_i, \hat{q}_{-i}\right)}{\partial q_i}\Big|_{q_i = y} \right] d\hat{q}_{-i} dy.$$

■

**Proof of Proposition 5.** The proof to Proposition 5 follows the same method as the proof to Proposition 2. Therefore, we do not provide as many details or discussion here as we do in the earlier proof. Consider the expected payoff function for agent $i$:

$$EU_i = \begin{cases} \Phi_{i,k}(q_i)v_i - c_i t_i & \text{if } \breve{Q}_i(t_i) \geq q_i \\ \Phi_{i,k}(\breve{Q}_i(t_i))v_i - c_i t_i & \text{if } \breve{Q}_i(t_i) \leq q_i. \end{cases}$$

If agent $i$ over represents its qualifications, it increases the probability it receives attention, but does not increase the probability it receives a prize. This is because when an agent receives attention the decision maker becomes fully informed about its qualifications, and the decision maker will award it the prize only if fewer than $k$ other agents disclosed or signaled higher qualifications than $q_i$. If agent $i$ under represents its qualifications, it only wins attention and can disclose its true qualifications if fewer than $k$ other agents have actual qualifications above $\breve{Q}_i(t_i)$. If it doesn't win attention, it will not win a prize. If it does win attention, it will win a prize (i.e., if $Q_i(t_i) < q_i$ is one of the $k$ highest qualifications, so will be $q_i$).

It should be clear from the expected payoff function that an agent will never pay $t_i > \breve{T}_i(q_i)$. The analysis can therefore solve for the equilibirium payment function by focusing on the case when $\breve{Q}_i(t_i) \leq q_i$. First order conditions are $\Phi'_{i,k}(\breve{Q}_i(t_i))\breve{Q}'_i(t_i)v_i - c_i = 0$. In equilibrium, $\breve{Q}_i(t_i) = q_i$, and given the strict monotonicity of $\breve{T}_i$, $\breve{Q}'_i(t_i) = 1/\breve{T}'_i(q_i)$. Therefore, the first order conditions may be rewritten as Eq. 4, which gives the slope of the equilibrium payment function. Inverting Eq. 4 with respect to $q_i$ gives Eq. 3, the equilibrium payment function. Note that the slope of the payment function is strictly increasing in $q_i$, satisfying the requirement that for all agents, $\breve{T}_i$ is strictly increasing in $q_i$. ■

# References


**Austen-Smith, David**, "Allocating Access for Information and Contributions," *Journal of Law, Economics, and Organization*, Fall 1998, *14* (2), 277–303.

**Baron, David P. and David Besanko**, "Regulation, Asymmetric Information, and Auditing," *RAND Journal of Economics*, 1984, *15* (4), 447–470.

**Baye, Michael R., Dan Kovenock, and Casper G. de Vries**, "Rigging the Lobbying Process: An Application of the All-Pay Auction," *American Economic Review*, 1993, *83* (1), 289–94.

**Baye, Michael R, Dan Kovenock, and Casper G. de Vries**, "The All-Pay Auction with Complete Information," *Economic Theory*, 1996, *8* (2), 291–305.

**Border, Kim C. and Joel Sobel**, "Samurai Accountant: A Theory of Auditing and Plunder," *Review of Economic Studies*, 1987, *54* (4), 525–540.

**Bull, Jesse and Joel Watson**, "Evidence disclosure and verifiability," *Journal of Economic Theory*, 2004, *118*, 1–31.

— **and** — , "Hard evidence and mechanism design," *Games and Economic Behavior*, 2007, *58*, 75–93.

**Cotton, Christopher**, "Should We Tax or Cap Political Contributions? A Lobbying Model With Policy Favors and Access," *Journal of Public Economics*, 2009, *93* (7/8), 831–842.

— , "Pay-to-Play Politics: Informational lobbying and campaign finance reform when contributions buy access," 2010. University of Miami Working Paper, WP2010-22.

**Eso, Peter and Balazs Szentes**, "Optimal Information Disclosure in Auctions and the Handicap Auction," *Review of Economic Studies*, 2007, *74*, 705–731.

**Feess, Eberhard, Gerd Muehlheusser, and Markus Walzl**, "Unfair Contests," *Journal of Economics*, 2008, *93* (3), 267–291.

**Fudenberg, Drew and Jean Tirole**, "Perfect Bayesian Equilibrium and Sequential Equilibrium," *Journal of Economic Theory*, 1991, *53*, 236–260.

**Fullerton, Richard L. and R. Preston McAfee**, "Auctioning Entry into Tournaments," *Journal of Political Economy*, 1999, *107* (3), 573–603.

**Hillman, Arye L. and John G. Riley**, "Politically Contestable Rents and Transfers," *Economics & Politics*, 1989, *1* (1), 17–39.

**Khalil, Fahad and Jacques Lawarree**, "Catching the Agent on the Wrong Foot: Ex Post Choice of Monitoring," *Journal of Public Economics*, 2001, *82*, 327–347.

**Kirkegaard, Rene**, "Favoritism in Asymmetric Contests: Head Starts and Handicaps," May 2010. working paper.

**Kofman, Alfredo and Jacques Lawarree**, "Collusion in Hierarchical Agency," *Econometrica*, 1993, *61* (3), 629–656.

**Leslie, Derek**, "Are Delays in Acadmic Publishing Necessary?," *American Economic Review*, 2005, *95* (1), 407–413.

**Lipman, Barton L. and Duane J. Seppi**, "Robust Inference in Communication Games with Partial Provability," *Journal of Economic Theory*, 1995, *66*, 370–405.

**Milgrom, Paul**, "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 1981, *12*, 380–391.

**Mookherjee, Dilip and Ivan Png**, "Optimal Auditing, Insurance, and Redistribution," *Quarterly Journal of Economics*, 1989, *104* (2), 399–415.

**Siegel, Ron**, "All-Pay Contests," *Econometrica*, 2009, *77* (1), 71–92.

_ , "Head Starts in All-Pay Contests," 2010. working paper.

**Townsend, Robert M.**, "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory*, 1979, *21* (2), 265–293.