

# Contests with Group-Specific Public Goods and Complementarities in Efforts

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## Abstract

Usually, groups increase their productivity by the specialization of their group members. In these cases, group output is no longer simply a sum of individual outputs, which may have an effect on the incentives to voluntarily contribute to the group's output. We analyze this situation in a contest environment with group-specific public goods that allow for different degrees of complementarity between group members' efforts. More specifically, we use a Tullock contest-success function and a CES-impact function. We show that in equilibrium the degree of complementarity is irrelevant if group members are equally efficient and have an identical valuation of the public good. If group members are heterogenous, higher complementarity of a group also leads to higher similarity in group members' efforts. However, this will decrease the winning probability of the group.

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# 1 Introduction

In many economic situations like R&D races, military conflicts, lobbying, or sports, groups compete for economic rents that are group-specific public goods. Usually, in all these examples, efforts of different group members are to some extent complementary. In R&D races, where teams of researchers develop new technologies, the whole project is often divided into different, more or less complementary sub-projects that are carried out by different researchers. In military conflicts the armed forces are highly specialized and often divided into complementary units. The same is true for the standard lobbying case if representatives of different firms or organizations lobbying for the same policy differ in qualifications and specialize accordingly. In sports contests, team members are usually specialized with respect to qualifications that complement each other in a non-additive way.

This list of examples could be arbitrarily extended because the mere idea of specialization implies that there is a certain degree of complementarity in team or group production. Individuals differ in talents, qualifications, and affections such that we can expect that individuals in a group or team will specialize to increase overall productivity such that we can expect a certain degree of complementarity between the efforts of the group members. Alchian and Demsetz (1972) see the non-additivity as constitutive for group or team production (pp. 777): *“Resource owners increase productivity through cooperative specialization. [...] With team production it is difficult, solely by observing total output, to either define or determine each individual’s contribution to this output of the cooperating inputs. The output is yielded by a team, by definition, and it is not a sum of separable outputs of each of its members. [...] Usual explanations of the gains from cooperative behavior rely on exchange and production in accord with the comparative advantage specialization principle with separable additive production. However [...] there is a source of gain from cooperative activity involving working as a team, wherein individual cooperating inputs do not yield identifiable, separate products which can be summed to measure the total output.”*

Despite the growing interest in the influence of heterogeneity within and between groups,<sup>1</sup> with only a few exceptions the literature on group contests<sup>2</sup> has focused

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<sup>1</sup>See Esteban & Ray, 2008, 2010.

<sup>2</sup>The literature on contests between groups has recently been surveyed by Corchón, 2007, Section

attention on situations where the effort levels of group members are perfect substitutes. This case is an important starting point for the analysis of group contests, however, if complementarities are the rule rather than the exception, it is important to understand how the degree of complementarity between individual efforts influences behavior in and the outcome of the contest. In this paper we analyze a situation where efforts of group members can be imperfect substitutes. In order to capture this idea, we assume that individual efforts  $x_i^k$  are mapped onto group impact<sup>3</sup> (which itself is the input in the contest success function) by means of a CES-impact function,  $g_i \cdot (\sum a_i^k \cdot (x_i^k)^{\gamma_i})^{1/\gamma_i}$ , with variable elasticity of substitution  $1/(1 - \gamma_i)$ , ranging from perfect complements ( $\gamma_i \rightarrow -\infty$ ) to perfect substitutes ( $\gamma_i \rightarrow 1$ ).<sup>4</sup> The contest is of the Tullock type<sup>5</sup>, and the rent is a group-specific public good (i.e. nonrival in consumption).

If groups instead of individuals compete in a contest, the well-known free-rider problem among group members exists. Every individual bears the full costs of its investments, whereas the benefits partly spill over to the rest of the group (Katz, Nitzan, & Rosenberg, 1990; Esteban & Ray, 2001; Epstein & Mealem, 2009; Nitzan & Ueda, 2009). Depending on the sharing rule applied, this problem may also exist for a private good (Nitzan, 1991a, 1991b; Esteban & Ray, 2001; Nitzan & Ueda, 2009). In the recent literature, Baik (2008), Epstein and Mealem (2009), and Lee (2008) have presented contest models with group-specific public goods. A major result in Baik (2008) is that in a model with linear effort costs and additively linear impact functions only those group members with the highest valuation of the rent make positive investments in the contest.<sup>6</sup> In his model, efforts of group members are perfect substitutes and therefore the optimality conditions given by the first-order conditions cannot hold for different valuations. With several group members having

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4.2, Garfinkel & Skaperdas, 2007, Section 7, and Konrad, 2009, Chapters 5.5 and 7. An exception is Münster, 2007 who focusses on CES-impact functions for private goods.

<sup>3</sup>The term ‘impact function’ is defined and discussed in Münster (2009).

<sup>4</sup>Cornes & Hartley, 2007 have analyzed voluntary public-goods provision with CES production functions. They focus attention to the characterization of equilibria of such games. In our model we use a slightly more general CES function in a  $n$ -group contest.

<sup>5</sup>Münster (2009) provides an axiomatic foundation for the Tullock function for group contests.

<sup>6</sup>This result has, of course, a counterpart in the literature on the private provision of public goods where it follows as a special case of the seminal contribution by Bergstrom, Blume, and Varian (1986).

the maximal valuation among the group, there exist multiple equilibria, since the first order condition only defines the total effort spent by the group. Epstein and Mealem (2009) stick to the assumption of additive separability of individual effort in the group-production functions but introduce decreasing returns to investment. Using a technology that fulfills standard “Inada” conditions they show that every individual makes positive investments. Their model is isomorphic to a model with linear impact functions and in which individuals face strictly convex costs. In this sense, effort levels are no longer perfect substitutes, but the impact function is still additively separable. Lee (2008) focuses attention on weakest-link impact functions. The perfect complementarity of efforts creates a coordination problem between group members which gives rise to multiple equilibria, and the equilibrium with highest efforts is determined by the valuation of the player with minimum valuation within each group. Hence, the models of Baik (2008) and Lee (2008) represent the “polar” cases with respect to the elasticity of substitution between group members.<sup>7</sup>

Our model generalizes these results. It turns out that the equilibrium of our model is unique for all values of  $\gamma_i \in \{(-\infty, 0), (0, 1)\}$  ( $\gamma_i = 0$  can only be covered by a limit result). If there is no within-group heterogeneity with respect to valuations of the prize  $v_i^k$  and efficiency  $a_i^k$  of each group member and all groups have the same size and the same sum of efficiency parameters of the group members  $\sum a_i^k$ , the equilibrium is independent of the elasticity of substitution. This result is a useful starting point because it shows that the elasticity of substitution *per se* has no impact on behavior in the contest, contrary to the cursory idea that increasing the degree of complementarity between group-members’ efforts may help to internalize the existing free-rider problem.<sup>8</sup> As a convenient side effect, this independence shows that the standard results on group contests are robust with respect to variations in the elasticity of substitution under these assumptions. The comparative-static analysis of the paper reveals that this effect is even more pronounced in the general case: A larger degree complementary within a single group reduces its winning probability. The intuition for this result is as follows. It is true that a larger degree of complementarity brings the effort levels of the group members closer together. Free-riding

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<sup>7</sup>One might argue that the best-shot technology where only the maximum effort counts is even more extreme, but it is relatively obvious that as for the case of perfect substitutes the equilibrium with maximum effort is also determined by the players with maximum valuations.

<sup>8</sup>Hirshleifer (1983) argues for the special case of perfect complements (“weakest-link” technology) that the complementarity between group members’ efforts helps solving the free-rider problem.

that is especially pronounced in the boundary case  $\gamma_i = 1$  is therefore mitigated. However, the coordination of efforts is increasingly determined by the group member with the lowest valuation, and it is this latter effect that turns out to be dominant.

An immediate implication of this irrelevance result is the conjecture that the elasticity of substitution may become relevant if there is heterogeneity between group members with respect to the valuations of the rent and/or heterogeneity between groups with respect to group size and/or efficiency. The first channel of influence generalizes the intuition developed by Esteban & Ray, 2010 that within-group heterogeneity is of crucial importance to understand incentives in contests.

A first important result of this general case is the characterization of active and inactive groups. We know from Hillman and Riley (1987) and Stein (2002) (for contests between individuals) that players may prefer not to participate in a contest if valuations are sufficiently heterogenous. This result carries over to the case of group contests. However, if a group is active, every member of this group exerts positive effort. In addition, the equilibrium strategies are continuous (but not necessarily continuously differentiable) in the elasticity of substitution everywhere except at  $\vec{\gamma} = 0$ , because groups that become inactive or active do this in a “smooth” way.

The limit behavior of our model is of particular interest. First, the results for the two limiting cases  $\gamma_i \rightarrow 1$  and  $\gamma_i \rightarrow -\infty$  have been analyzed by Baik (1993, 2008) and Lee (2008). In the former case, equilibrium effort is determined by the players with the maximum valuations, and in the latter case by the players with the minimum valuation in each group, which implies that group size does not matter. However, there are multiple equilibria in both cases. We can show that our model is consistent with both results in the sense that the equilibrium converges to one of the equilibria of the boundary cases. As a consequence, the uniqueness of the equilibrium for all values of  $\vec{\gamma}$  except 1 and  $-\infty$  shows that the multiplicity problems are not robust with respect to small changes in the elasticity of substitution, and that – as in Cornes & Hartley, 2007 – our limit results can be interpreted as an equilibrium-selection mechanism for the boundary cases.

The paper is organized as follows. We introduce the model in Section 2 and start with introductory examples in Section 3. We characterize the simultaneous Nash equilibrium of the general model in Section 4. In subsection 4.1 we will state convergence results for  $\vec{\gamma}$  approaching 1, 0, and  $-\infty$ , and in subsection 4.2 the

comparative-static results are summarized. Section 5 concludes.

## 2 The model

Assume that  $n$  groups compete for a given rent  $R$ .  $m_i$  is the number of individuals in group  $i$  and  $k$  is the index of a generic member of this group. The rent is a group-specific public good that has a value  $v_i^k > 0$  to individual  $k$  of group  $i$ , and we assume the following ordering:  $v_i^{max} \geq \dots \geq v_i^{min}$ .  $p_i$  represents the probability of group  $i = 1, \dots, n$  to win the contest. It is a function of some vector of aggregate group output  $q_1, \dots, q_n$ . We focus on Tullock-form contest-success functions where the winning probability of a group  $i$  is defined as:

$$\mathbf{Assumption 1:} \quad p_i(q_1, \dots, q_n) = \frac{q_i}{\sum_{j=1}^n q_j}, \quad i = 1, \dots, n.$$

The aggregate group output depends on individual effort  $x_i^k$ ,  $Q_i = q_i(x_i^1, \dots, x_i^{m_i})$ ,  $i = 1, \dots, n$ . Following the literature we will call  $q_i(\cdot)$  *impact functions* in the following and make the assumption that they are of the constant elasticity of substitution (CES) type.

$$\mathbf{Assumption 2:} \quad q_i(x_i^1, \dots, x_i^{m_i}) = g_i \cdot \left( \sum_{l=1}^{m_i} a_i^l \cdot (x_i^l)^{\gamma_i} \right)^{1/\gamma_i}, \quad \gamma_i \in (-\infty, 1], \\ i = 1, \dots, n.$$

The function has the usual parameters  $a_i^k$  for the efficiency of an individual's effort and  $g_i$  for the relative strength of the group. Note that we obtain a closed-form solution only if for all  $i$  it holds that  $\gamma_i \neq 0$ . The Cobb-Douglas case  $\gamma_i \rightarrow 0$  will be covered by a limit result.

**Assumption 3:** Individuals are risk neutral, face linear costs, and maximize their net rent.

It follows from Assumptions 1, 2, and 3 that the individual utility functions are as follows:

$$\pi_i^k(x_1^1, \dots, x_n^{m_n}) := \pi_i^k(x_i^k, x_{/x_i^k}) = v_i^k \cdot \frac{g_i \cdot \left( \sum_l a_i^l \cdot (x_i^l)^{\gamma_i} \right)^{1/\gamma_i}}{\sum_j g_j \cdot \left( \sum_l a_j^l \cdot (x_j^l)^{\gamma_j} \right)^{1/\gamma_j}} - x_i^k, \quad (1)$$

where  $x_{/x_i^k}$  refers to the vector  $x_1^1, \dots, x_n^{m_n}$  without  $x_i^k$ . We are looking for a Nash equilibrium of this game where individuals choose their effort  $x_i^k$  simultaneously to maximize their expected utility,

$$x_i^{k*} \in \arg \max_{x_i^k} \pi_i^k(x_i^k, x_{-x_i^k}^*) \quad \forall i, k, \quad (2)$$

where “\*” refers to equilibrium values.

### 3 Introductory examples

In this section we analyze two simple special cases that provide intuition for the relevance of the degree of complementarity in contests. As we will see, the degree of complementarity is only relevant if the valuations between members of the same group differ or if groups differ in size. The examples restrict attention to a contest between two groups, 1 and 2, with  $m_1$  and  $m_2$  members. The valuation of the group members are either  $v_i^{max}$  or  $v_i^{min}$ ,  $v_i^{max} \geq v_i^{min}$ ,  $i = 1, 2$ . The examples are chosen to highlight the central mechanisms of this model, we therefore delegate all technical details about the existence of interior solutions, active and inactive groups and group members, etc. to the next section.

**Example 1:** In this example we restrict attention to groups of equal size  $m_1 = m_2 = m$  with only a single valuation of the members of a given group,  $v_i^{min} = v_i^{max} = v_i$ ,  $i = 1, 2$  and identical technologies with  $a_i^k = 1$ ,  $g_i = 1$  and  $\gamma_i = \gamma$ . We assume that all members of a given group behave identically. Using this fact, it follows that

$$x_1(v_1, v_2) = \frac{v_1^2 \cdot v_2}{m \cdot (v_1 + v_2)^2}, \quad x_2(v_1, v_2) = \frac{v_1 \cdot v_2^2}{m \cdot (v_1 + v_2)^2}$$

in an interior equilibrium. Investments in the contest are independent of  $\gamma$ . This example shows that the elasticity of substitution does not play a role if there is no within-group heterogeneity and groups are of equal size and have the same impact function. The reason for this result is the combination of a constant-return to scale impact function with a contest success function that is homogenous of degree zero. Conversely, it must be either within-group heterogeneity and/or differences in group size and technology that may cause behavioral changes due to changes in  $\gamma$ . The next example shows that this may in fact be the case.

**Example 2:** In this example we assume again  $m_1 = m_2 = m$  and for all  $i$  and  $k$  that  $a_i^k = 1$  and  $\gamma_i = \gamma$ . However, we allow for heterogenous valuations within groups:  $v_1^{min} = v_2^{min} = v^{min} \leq v^{max} = v_1^{max} = v_2^{max}$ . The population is divided into  $m^{min} = m^{max} = m/2$  for both groups. One gets the following symmetric equilibrium:

$$\begin{aligned} x^{max}(v^{min}, v^{max}, m, \gamma) &= \frac{v^{max}}{2 \cdot m \cdot \left( \left( \frac{v^{min}}{v^{max}} \right)^{\frac{\gamma}{1-\gamma}} + 1 \right)}, \\ x^{min}(v^{min}, v^{min}, m, \gamma) &= \frac{v^{min}}{2 \cdot m \cdot \left( \left( \frac{v^{max}}{v^{min}} \right)^{\frac{\gamma}{1-\gamma}} + 1 \right)}. \end{aligned} \quad (3)$$

As expected,  $\gamma$  may influence the outcome of the game if differences among the valuations of the rent among the group members exist.

## 4 The general case

We now turn to the analysis of the general case. In order to have a lean notation, let  $y_i^k = (x_i^k)^{\gamma_i}$  and  $Y_i = (\sum_l a_i^l \cdot y_i^l)$ . Further,  $Q = \sum_j Q_j = \sum_j g_j \cdot Y_j^{\frac{1}{\gamma_j}} = g_i \cdot Y_i^{\frac{1}{\gamma_i}} + \sum_{j \neq i} g_j Y_j^{\frac{1}{\gamma_j}} = Q_i + Q_{/i}$  in the following. Also, let  $\vec{\gamma}$  denote the vector of all  $\gamma_i$ . While deriving the equilibrium strategies, we will omit the parameters of these functions for better readability (e.g  $y_i^k$  instead of  $y_i^k(\gamma_i, x_i^k)$ ).

Hillman and Riley (1987) and Stein (2002) have shown that groups/individuals may prefer to stay inactive if the size of all groups is equal to 1. Baik (2008) has shown that only group members with maximum valuation participate in a contest. Hence, it is possible that some individuals and/or groups will stay inactive in our setup. We therefore start with an analysis of active individuals and groups.

**Definition 1:** An individual  $k$  of group  $i$  is said to participate if  $x_i^k > 0$ . A group  $i$  is said to participate if there exists some  $k$  such that  $x_i^k > 0$ . A group is said to fully participate if  $\forall k : x_i^k > 0$ .

**Lemma 1:** Let  $\gamma_i \neq 0$  for all  $i$ . In a Nash equilibrium of a contest fulfilling Assumptions 1, 2, and 3 if a group participates, it fully participates.

The proof of this as well as the next Lemma can be found in the appendix. Lemma 1 implies that in order to determine whether an individual participates, it is sufficient to determine whether its group participates. Let  $V_i(\gamma_i) \equiv g_i \cdot \left( \sum_l a_i^l \cdot (a_i^l \cdot v_i^l)^{\frac{\gamma_i}{1-\gamma_i}} \right)^{\frac{1-\gamma_i}{\gamma_i}}$ . Without loss of generality, suppose the groups are ordered

such that  $V_i(\gamma_i) \geq V_{i+1}(\gamma_{i+1})$ .  $Q_i^*(\vec{\gamma})$  and  $Q^*(\vec{\gamma})$  shall denote  $Q_i$  and  $Q$  in equilibrium. The following Lemma determines the groups that participate in equilibrium.

**Lemma 2:** a) Let  $\gamma_j \neq 0$  for all  $j$ . There exist best response strategies of the members of a group, if and only if the following group best response function is fulfilled:

$$\hat{Q}_i(\vec{\gamma}, Q_{/i}) = \max\left(0, \sqrt{Q_{/i} \cdot V_i(\vec{\gamma})} - Q_{/i}\right). \quad (4)$$

b) Groups  $1 \dots n^*(\vec{\gamma})$  participate, where  $n^*(\vec{\gamma}) \equiv \arg \max_i i$  such that  $V_i(\vec{\gamma}) > Q^*(\vec{\gamma})$ .

c) If the Nash equilibrium is unique,  $Q_i^*(\vec{\gamma})$  and  $Q^*(\vec{\gamma})$  are continuous functions in all arguments if for all  $j$ :  $\gamma_j \neq 0$ .

Lemma 2.c is useful for the comparative-static analysis. Given that the number and identity of active groups depends on  $\vec{\gamma}$ , it is *a priori* not clear that aggregate effort and indirect utilities are continuous in  $\vec{\gamma}$ . The Lemma reveals that continuity is in fact guaranteed except at  $\gamma_j = 0$ . The economic intuition is as follows: Assume that  $\hat{\gamma}_j$  is a point where a formerly active group becomes inactive or a formerly inactive group becomes active. The aggregate group effort of the active group is continuously reduced to zero as  $\gamma_j$  approaches  $\hat{\gamma}_j$ , and the formerly inactive group continuously increases its effort from 0 as  $\gamma_j$  increases from  $\hat{\gamma}_j$ . Hence, there is a “smooth” fade out or fade in of groups at those points.

The following proposition characterizes the unique Nash equilibrium of the game. For readability, the strategies  $x_i^k$  are defined as functions of  $Q^*(\vec{\gamma})$  and  $V_i(\gamma_i)$ .

**Proposition 1.** Assume that for all  $j$ ,  $\gamma_j \neq 0$ . The unique Nash equilibrium of the game characterized by Assumptions 1,2, and 3 is given by strategies  $x_i^{k*}(\vec{\gamma})$  that fulfill

$$x_i^{k*}(\vec{\gamma}) = \begin{cases} Q^*(\vec{\gamma}) \cdot \left(1 - \frac{Q^*(\vec{\gamma})}{V_i(\gamma_i)}\right) \cdot (g_i)^{\frac{\gamma_i}{1-\gamma_i}} \cdot \frac{(a_i^k \cdot v_i^k)^{\frac{1}{1-\gamma_i}}}{V_i(\gamma_i)^{\frac{1}{1-\gamma_i}}}, & V_i(\gamma_i) > Q^*(\vec{\gamma}) \\ 0, & V_i(\gamma_i) \leq Q^*(\vec{\gamma}) \end{cases}, \quad (5)$$

where  $Q^*(\vec{\gamma}) = \frac{n^*(\vec{\gamma})-1}{\sum_{i=1}^{n^*(\vec{\gamma})} V_i(\gamma_i)^{-1}}$ ,  $n^*(\vec{\gamma})$  is defined in Lemma 2.a, and groups are ordered such that  $V_i(\gamma_i) \geq V_{i+1}(\gamma_{i+1})$ .

*Proof.* To obtain  $Q^*(\vec{\gamma})$  we sum (4) over all  $i \leq n^*(\vec{\gamma})$ :

$$Q^*(\vec{\gamma}) = \frac{n^*(\vec{\gamma}) - 1}{\sum_{i=1}^{n^*(\vec{\gamma})} V_i(\gamma_i)^{-1}}. \quad (6)$$

With an explicit solution for  $Q^*(\vec{\gamma})$ , we can now determine individual expenditures  $x_i^{k*}(\vec{\gamma})$  by solving equation (4) using (6). The participation condition of a group is given by Lemma 2, while Lemma 1 ensures that there does not exist an incentive for any group member to deviate to  $x_i^k = 0$ . It was further shown that the first-order conditions return local maxima. Since the system of equations given by the first-order conditions of the participating groups has a unique solution this is indeed the unique Nash equilibrium.  $\square$

A focal special case has no intra-group heterogeneity  $v_i^k = v_i \forall k \forall i$  and  $a_i^k = a_i \forall k \forall i$ . The following corollary of Proposition 1 can then be established.

**Corollary 1:** *Suppose for all groups  $i$  and all individuals  $k$ , it holds that  $a_i^k = a_i$  and  $v_i^k = v_i$  and further for all other groups  $j$  it holds that  $a_i \cdot m_i = a_j \cdot m_j$ . Then the equilibrium efforts are independent of  $\vec{\gamma}$ .*

*Proof.* Inserting the above values for every individual  $l$   $a_i^l = a_i$  and  $v_i^l = v_i$  and setting for all other groups  $j$   $a_j \cdot m_j = a_i \cdot m_i$  into (5) directly yields the result.  $\square$

The corollary shows that  $\vec{\gamma}$  is only relevant if there is either heterogeneity with respect to valuations within groups and/or heterogeneity with respect to group size. In all other cases equilibrium behavior does not depend on  $\vec{\gamma}$ . This finding implies that an increase in complementarity between group members' effort *per se* has no effect on the within-group free-rider problem, as could have been conjectured from Hirshleifer (1983). A further implication of the corollary is that the results on group contests that have been derived in the literature for the case of perfect substitutes or perfect complements carry over to arbitrary elasticities of substitution if homogenous groups differ only in their valuations of the rent and their group efficiency parameter  $g_i$ .

## 4.1 Convergence Results

Before we move on to the core convergence results with respect to  $\vec{\gamma}$  and the comparative statics of the model, let us first note that the winning probability of group  $i$  takes the form:

$$\frac{Q_i^*(\vec{\gamma})}{Q^*(\vec{\gamma})} = \left(1 - \frac{Q^*(\vec{\gamma})}{V_i(\gamma_i)}\right), \quad (7)$$

which can be derived from (4). We will now state convergence results where for all groups  $j$ ,  $\gamma_j$  approaches 1, 0, and  $-\infty$ .

**Proposition 2.** For  $\gamma_i \rightarrow 1$ , we get  $\frac{x_i^{k*}}{X_i^*} = 0$  if  $\exists a_i^l v_i^l > a_i^k v_i^k$  and  $\frac{1}{\#\{l: a_i^l v_i^l = a_i^k v_i^k\}}$  otherwise.

*Proof.* It is straightforward to derive the following equation from (5):

$$\frac{x_i^{k*}(\vec{\gamma})}{X_i^*(\vec{\gamma})} = \frac{(a_i^k \cdot v_i^k)^{\frac{1}{1-\gamma}}}{\sum_l (a_i^l v_i^l)^{\frac{1}{1-\gamma}}} \quad (8)$$

For the limit it then holds:

$$\begin{aligned} \lim_{\gamma_i \rightarrow 1} \frac{(a_i^k \cdot v_i^k)^{\frac{1}{1-\gamma_i}}}{\sum_l (a_i^l \cdot v_i^l)^{\frac{1}{1-\gamma_i}}} &= \lim_{\gamma_i \rightarrow 1} \left( \sum_l \left( \frac{a_i^l \cdot v_i^l}{a_i^k \cdot v_i^k} \right)^{\frac{1}{1-\gamma_i}} \right)^{-1} \\ &= \begin{cases} 0, & \exists a_i^l \cdot v_i^l > a_i^k \cdot v_i^k \\ \frac{1}{\#\{l: a_i^l \cdot v_i^l = a_i^k \cdot v_i^k\}}, & \text{else} \end{cases}. \end{aligned} \quad (9)$$

□

Proposition 2 shows that for  $\gamma_i$  increasing towards one, the group members with lower valuations will decrease their efforts towards zero, and only the group members with the highest valuations contribute. If there is more than one individual with the highest valuation, we converge to an equilibrium where those individuals contribute equally.<sup>9</sup>

Next we will analyze the other boundary case when all  $\gamma_j$  approach  $-\infty$ . In order to have a lean notation we denote  $\gamma_j = \gamma$  and  $\lim_{\gamma \rightarrow -\infty} f(\gamma)$  by  $f(-\infty)$  for all functions  $f(\cdot)$ :

**Proposition 3.** For  $\gamma \rightarrow -\infty$ , we obtain:

a)  $\lim_{\gamma_i \rightarrow -\infty} V_i(\gamma_i) = \frac{g_i}{m_i} HM(v_i^1, \dots, v_i^{m_i})$

b)  $\lim_{\gamma \rightarrow -\infty} \frac{x_i^{k*}(\gamma)}{X_i^*(\gamma)} = \frac{1}{m_i}$

c)  $\lim_{\gamma \rightarrow -\infty} Q^*(\gamma) = \frac{n^*(-\infty)}{\sum_j \sum_l 1/(v_j^l \cdot g_i)}$

d)  $x_i^{k*}$  is independent of  $a_j^l \forall j, l$

where  $HM(v_i^1, \dots, v_i^{m_i}) = \frac{m_i}{\sum_l \frac{1}{v_i^l}}$  is the harmonic mean of the valuations within the group.

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<sup>9</sup>In this latter case we get multiple equilibria if  $\gamma_i = 1$  with the property that the sum of contributions is always identical (Baik, 2008). In this sense, our convergence result can be interpreted as an equilibrium-selection mechanism by focussing on the equal-contribution equilibrium.

The results follow directly from the determination of the limit of (5).

Since relative strength of groups is determined by  $V_i$ , the limit behavior of  $V_i$  is of course of great interest. From Proposition 3 b) we see that the distribution and level of relative strengths  $a_i^k$  of each group member have no effect on  $V_i$ . The irrelevance of  $x_i^k$  is further shown by part d) of the proposition, where we see that even equilibrium efforts  $x_i^{k*}$  are unaffected by  $a_i^k$ . This was to be expected, since under perfect complements in fact all inputs are crucial for the level of  $q_i$ . Proposition 3 b) shows that (as expected given the results by Lee (2008)) all group members participate with equal amounts. In this sense, for  $\gamma$  near  $-\infty$ , we obtain similar results as for a  $\min(\dots)$  impact function. However, this function creates multiple equilibria with an associated equilibrium-selection problem. Given the uniqueness of equilibria for all finite  $\bar{\gamma}$ , our limit result can be interpreted as an equilibrium-selection mechanism where individual contributions depend on the harmonic mean of the valuations.

Next we look at the limit behavior for  $\gamma \rightarrow 0$ . It turns out that we have to consider  $\gamma \rightarrow 0^+$  and  $\gamma \rightarrow 0^-$  separately because the problem may not be continuous at this point.

**Proposition 4.** *At  $\gamma_i = 0$ ,  $V_i(\gamma_i)$  is discontinuous if  $\sum_l a_i^l \neq 1$ .*

$$\lim_{\gamma_i \rightarrow 0^+} V_i = \begin{cases} \infty, & \sum a_i^k > 1 \\ \prod (a_i^k \cdot v_i^k)^{a_i^k}, & \sum a_i^k = 1 \\ 0, & \sum a_i^k < 1 \end{cases} \quad (10)$$

$$\lim_{\gamma_i \rightarrow 0^-} V_i = \begin{cases} 0, & \sum a_i^k > 1 \\ \prod (a_i^k \cdot v_i^k)^{a_i^k}, & \sum a_i^k = 1 \\ \infty, & \sum a_i^k < 1 \end{cases} \quad (11)$$

Since the winning probability, the equilibrium efforts, and impacts are all functions of all  $V_i$ , it follows that these values will in general also be discontinuous in  $\gamma_i$ . In particular, the winning probability and the participation condition of group  $i$  are increasing functions of  $V_i$ . For  $\gamma \rightarrow 0^+$  the group with the strictly highest  $\sum a_i^k$  will therefore win with probability one while for  $\gamma \rightarrow 0^-$  the group with the strictly lowest  $\sum a_i^k$  will win with probability one. Only if all groups have  $\sum a_i^k = 1$ , these effects do not occur and we obtain for  $V_i$  the  $a_i^k$ -weighted geometric mean of  $v_i^k \cdot a_i^k$ .

To obtain a proper intuition for the behavior near  $\gamma = 0$ , it is helpful to show an example.

**Example 3:** We assume that  $v_1 = v_2$  but allow for differences in group size with  $m_i > 1$ . Further, we fix  $a_i^k = 1$ ,  $g_i = g_j = 1$ , and  $\gamma_i = \gamma$ . Therefore, we are always in a situation with  $\sum a_i^k = m_i > 1$ . In this case, (5) implies

$$x_1(m_1, m_2, \gamma, v) = \frac{v \cdot m_1^{\frac{1-2\gamma}{\gamma}} \cdot m_2^{\frac{1-\gamma}{\gamma}}}{\left(m_1^{\frac{1-\gamma}{\gamma}} + m_2^{\frac{1-\gamma}{\gamma}}\right)^2}, \quad x_2(m_1, m_2, \gamma, v) = \frac{v \cdot m_1^{\frac{1-\gamma}{\gamma}} \cdot m_2^{\frac{1-2\gamma}{\gamma}}}{\left(m_1^{\frac{1-\gamma}{\gamma}} + m_2^{\frac{1-\gamma}{\gamma}}\right)^2}, \quad (12)$$

if all members of the same group behave identically. In this case, individual efforts depend on the size of the groups. Coming back to Example 2, (12) can be used to determine that the values of the impact functions are

$$q_1(m_1, m_2, \gamma, v) = v \cdot \frac{m_1^{\frac{1-\gamma}{\gamma}} \cdot m_2^{\frac{1-\gamma}{\gamma}}}{\left(m_1^{\frac{1-\gamma}{\gamma}} + m_2^{\frac{1-\gamma}{\gamma}}\right)^2}, \quad q_2(m_1, m_2, \gamma, v) = v \cdot \frac{m_1^{\frac{1-\gamma}{\gamma}} \cdot m_2^{\frac{1-\gamma}{\gamma}}}{\left(m_1^{\frac{1-\gamma}{\gamma}} + m_2^{\frac{1-\gamma}{\gamma}}\right)^2},$$

which in turn can be used to determine the equilibrium winning probabilities:

$$p_1(m_1, m_2, \gamma) = \frac{m_2^{\frac{\gamma-1}{\gamma}}}{m_1^{\frac{\gamma-1}{\gamma}} + m_2^{\frac{\gamma-1}{\gamma}}}, \quad p_2(m_1, m_2, \gamma) = \frac{m_1^{\frac{\gamma-1}{\gamma}}}{m_1^{\frac{\gamma-1}{\gamma}} + m_2^{\frac{\gamma-1}{\gamma}}}. \quad (13)$$

The limit behavior of these probabilities is

$$\lim_{\gamma \rightarrow 0^-} p_1(m_1, m_2, \gamma) = \begin{cases} 1, & m_1 < m_2 \\ 0, & m_1 > m_2 \end{cases},$$

$$\lim_{\gamma \rightarrow 0^+} p_1(m_1, m_2, \gamma) = \begin{cases} 0, & m_1 < m_2 \\ 1, & m_1 > m_2 \end{cases},$$

and analogously for  $p_2(m_1, m_2, \gamma)$ . Figure 1 shows  $p_1(m_1, m_2, \gamma)$  (dashed line) and  $p_2(m_1, m_2, \gamma)$  (solid line) for the case  $m_1 > m_2$ . We will focus on  $p_1(m_1, m_2, \gamma)$  in the following. The graph starts at 0.5 at  $\gamma = 1$ . This is the well-known case where group size has no impact on the winning probability (Baik, 2008).  $p_1(m_1, m_2, \gamma)$  steadily rises to 1 as  $\gamma$  converges to 0. At this point it jumps to 0 and increases to 0.5 again as  $\gamma$  converges to  $-\infty$ . In this case, group-size again does not matter because only the minimum contribution counts (Lee, 2008). As evident from the left panel of Figure 2, for the smaller group the efforts are larger over the whole range of  $\gamma$ . Therefore, the changes in the winning probability at  $\gamma = 0$  are due to a changing productivity

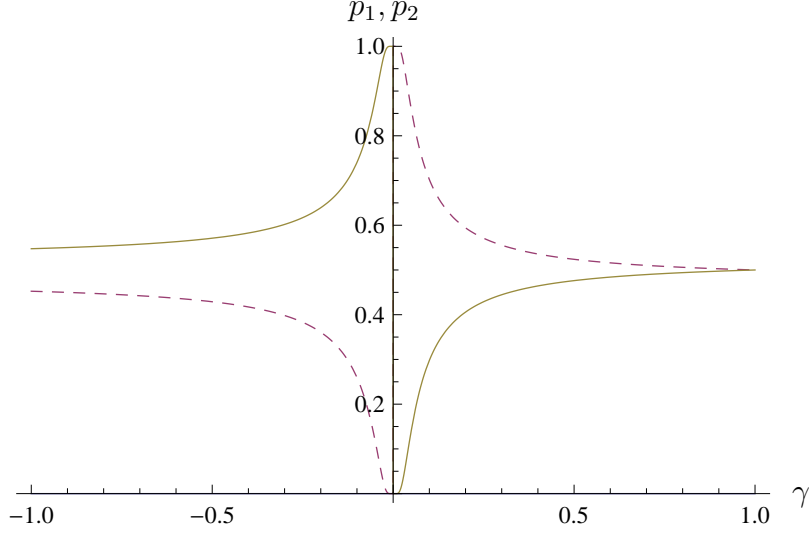


Figure 1: Equilibrium probabilities for different values of  $\gamma$  ( $m_1 = 11$ ,  $m_2 = 10$ ).

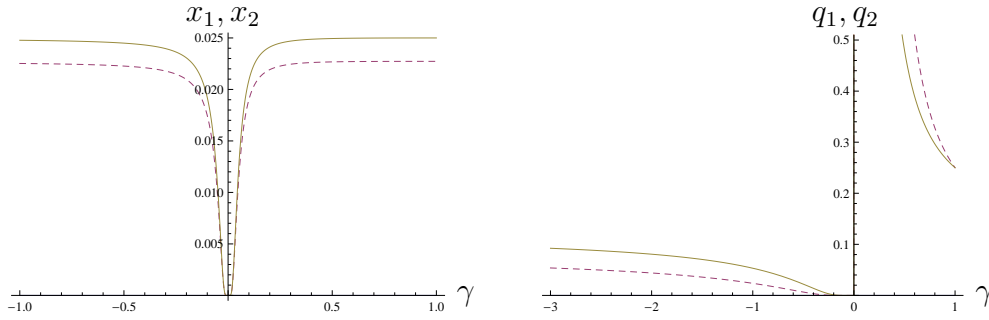


Figure 2: Effort levels (left) and impacts (right) for different values of  $\gamma$ .

of the larger and the smaller group with  $\gamma$ . This is evident from the right panel of Figure 2, where the impact of group 2 is consistently higher than those of group 1 for  $\gamma < 0$  and vice versa for  $\gamma > 0$ . The driving force behind these results is thus the CES function which for  $\sum a_i^k \neq 1$  changes not only the degree of complementarity but also the efficiency with  $\gamma$  as visible when inserting  $x_i^k = x_i$  and  $\gamma_i = \gamma$  into the impact function:

$$q_i(x_i, \dots, x_i) = g_i \cdot x_i \cdot \left( \sum_{k=1}^{m_i} a_i^k \right)^{1/\gamma}. \quad (14)$$

Whenever  $\sum a_i^k > 1$ , the function becomes infinitely large for  $\gamma \rightarrow 0^+$  and infinitely small for  $\gamma \rightarrow 0^-$ . The rate of convergence depends on the sum of all  $a_i^k$ , which was smaller for group 2 in the above case. Therefore it had a disadvantage for positive

$\gamma$  and an advantage for negative  $\gamma$ . This suggest to generally impose  $\sum a_i^k = 1$  to model differences in efficiency between group members and use the parameter  $g_i$  for differences in efficiency between groups. Only then the comparative statics with respect to  $\vec{\gamma}$  will capture solely the effect of different degrees of substitution and no productivity effects.

## 4.2 Comparative statics

We now turn to the comparative-static analysis of the influence of the elasticity of substitution on the behavior in the contest using the approach developed by Cornes and Hartley (2005). Most interestingly, individual valuations in relation to the valuations of the other group members define the individuals' share of the amount of effort spent by the group,  $x_i^{k*}/X_i^*$ . The valuation of other groups have no effect on these shares. As was to be expected, a larger elasticity of substitution  $\gamma_i$  increases *ceteris paribus* the dispersion of these shares, since the exponent discriminates more strongly between differences in valuations. The next proposition states the effect of  $\gamma_i$  on the individual shares.

**Proposition 5.** *The share of an individual of its group's effort,  $\frac{x_i^k}{X_i}$ , increases (decreases) strictly in the elasticity of substitution among efforts if the valuation times the efficiency  $a_i^k \cdot v_i^k$  of the individual is strictly larger (smaller) than the share-weighted geometric mean of the group members' valuation times efficiency  $\prod_l (a_i^l \cdot v_i^l)^{\left(\frac{x_i^l}{X_i}\right)}$ .*

*Proof.* Taking the derivative of (8) with respect to  $\gamma_i$  yields

$$\frac{\partial \frac{x_i^k}{X_i}}{\partial \gamma_i} = \frac{(a_i^k \cdot v_i^k)^{\frac{1}{1-\gamma_i}}}{\sum_l (a_i^l \cdot v_i^l)^{\frac{1}{1-\gamma_i}}} \frac{1}{(1-\gamma_i)^2} \left( \ln(a_i^k \cdot v_i^k) - \frac{\sum_l (a_i^l \cdot v_i^l)^{\frac{1}{1-\gamma_i}} \ln(a_i^l \cdot v_i^l)}{\sum_l (a_i^l \cdot v_i^l)^{\frac{1}{1-\gamma_i}}} \right). \quad (15)$$

The RHS of the above equation is positive whenever the term in brackets is positive. Setting  $\ln(a_i^k \cdot v_i^k) \geq \sum_l (a_i^l \cdot v_i^l)^{\frac{1}{1-\gamma_i}} \ln(a_i^l \cdot v_i^l) / \sum_l (a_i^l \cdot v_i^l)^{\frac{1}{1-\gamma_i}}$  and rearranging yields the condition:

$$\frac{\partial \frac{x_i^k}{X_i}}{\partial \gamma_i} \geq 0 \quad \Leftrightarrow \quad a_i^k \cdot v_i^k \geq \prod_l (a_i^l \cdot v_i^l)^{\left( \frac{(a_i^l \cdot v_i^l)^{\frac{1}{1-\gamma_i}}}{\sum_s (a_i^s \cdot v_i^s)^{\frac{1}{1-\gamma_i}}} \right)}. \quad (16)$$

□

The proposition implies that for all group members with a valuation above the weighted geometric mean, the share of total group effort increases with  $\gamma_i$ . The result shows that the dispersion of valuations plays a crucial role for the comparative-static effects of  $\gamma_i$ .

A second interesting question may be whether the winning probability of groups can be increased by a higher degree of complementarity of efforts. The intuition behind this may be twofold: First, with higher complementarity, the free-rider problem is solved better, such that also individuals with low valuations participate. Second, there often exist gains from specialization. While the latter intuition is induced by the technology itself, which is exogenous in our model, the first intuition can be examined through comparative statics of the model.

**Proposition 6.** *Suppose  $\sum_k a_i^k = 1$ . Then the winning probability of a group  $i$  is weakly increasing in  $\gamma_i$  and strictly increasing whenever there exist two group members  $k$  and  $l$  such that  $a_i^k \cdot v_i^k \neq a_i^l \cdot v_i^l$ .*

*Proof.* By equation (7) the winning probability of group  $i$  is strictly increasing in  $V_i(\gamma_i)$ .  $V_i(\gamma_i)$  has under the assumption of  $\sum_k a_i^k = 1$  the structure of an  $a_i^k$  weighted power mean of the  $a_i^k \cdot v_i^k$  values of the group members. By the weighted power mean inequality we know that  $V_i(\gamma_i)$  is strictly increasing in  $\vec{\gamma}$  whenever there exist two individuals with  $a_i^k \cdot v_i^k \neq a_i^l \cdot v_i^l$ . Whenever all individuals have the same  $a_i^k \cdot v_i^k$ ,  $V_i(\gamma_i) = g_i \cdot a_i^k \cdot v_i^k$  and is therefore independent of  $\gamma_i$ .  $\square$

This result contradicts the common intuition that higher complementarity leads to a better solution of the free-rider problem and thus a better performance of the group. The result shows exactly the opposite: heterogenous groups with higher complementarity tend to perform worse than similar groups with low complementarity. The intuition behind this is that a lower  $\gamma_i$  puts more emphasis on the lower values of  $x_i^k$ , so the lower  $\gamma_i$ , the more the equilibrium will reflect the optimal  $q_i$  of lower valuation group members. This has an important implication for the provision of public goods by groups in general: highly complementary technologies will only be used if there are sufficient gains of specialization coming with them. While higher complementarity solves the free-rider problem, it solves it in the worst possible way: by reducing the incentives of high valuation individuals more than increasing the incentives of low valuation individuals. The effect on the expected payoff of the group members is ambiguous however. Inserting (5) into (2) we obtain:

$$\pi_i^k = p_i \left( v_i^k - Q^*(\vec{\gamma})(g_i)^{\frac{\gamma_i}{1-\gamma_i}} \frac{(a_i^k \cdot v_i^k)^{\frac{1}{1-\gamma_i}}}{V_i(\gamma_i)^{\frac{1}{1-\gamma_i}}} \right). \quad (17)$$

As we know from Proposition 6,  $p_i$  is increasing in  $\gamma_i$ . However, also  $Q^*(\vec{\gamma})$  is increasing in  $\gamma_i$  and for sufficiently high  $a_i^k v_i^k$  the term  $(g_i)^{\frac{\gamma_i}{1-\gamma_i}} \cdot \frac{(a_i^k \cdot v_i^k)^{\frac{1}{1-\gamma_i}}}{V_i(\gamma_i)^{\frac{1}{1-\gamma_i}}}$  must be increasing as well as we know from Proposition 5. Therefore, for the group members with the highest  $a_i^k \cdot v_i^k$ , expected utility may be decreasing in  $\gamma_i$ . However, it is also clear that the group members of the lowest  $a_i^k \cdot v_i^k$  will always improve their expected payoff by lower complementarity, since they will strictly reduce their effort and the group has a higher winning probability. The optimal  $\gamma_i$  a utilitarian planner who maximizes the sum of the group members' expected payoffs would impose is ambiguous: a lower  $\gamma_i$  may induce individuals with a lower valuation  $v_i^k$  but higher efficiency  $a_i^k$  to exert higher effort. If the highest type has a high valuation but a low efficiency, this may lead to efficiency gains. To get a better intuition for this result we return to Example 2.

**Example 2 continued:** Since we are only interested in the effects of higher complementarity for one group, let the aggregate of the valuations of the first group be  $V_1(\gamma_1) = 10$ . Since this is the only way in which parameters from the first group enter the decision problem of the second, no more information about group one would be necessary. One could for example think of a group of a single individual with  $v_1^1 = 10$ ,  $a_1^k = 1$ , and  $g_1 = 1$ . For the second group, assume two individuals with efficiency parameters  $a_2^1 = 0.2$  and  $a_2^2 = 0.8$ . Thus,  $\sum_l a_2^l = 1$  and comparative statics over  $\gamma_2$  contain no effects from changes in productivity. Further, let valuations be heterogenous such that  $v_2^1 = 30$  and  $v_2^2 = 5$ . Finally, the efficiency parameter of the group is  $g_2 = 1$ .

From the fact that  $v_2^1 \cdot a_2^1 = 6 > 4 = v_2^2 \cdot a_2^2$ , we know that for  $\gamma_2 = 1$  only the first individual will participate and for  $\gamma_2 \rightarrow -\infty$ , both individuals will participate. Proposition 6 tells us that the winning probability will decrease with lower values of  $\gamma_2$ .

From Figure 3 we can see how this translates into our example. The effort level of individual 2 (with high efficiency and low valuation, dashed line) slowly increases as we reduce  $\gamma_2$ , while the effort of individual 1 (solid line) falls. Both converge as  $\gamma_2 \rightarrow \infty$ . We also see that the winning probability is falling with lower values of  $\gamma_2$ , as expected. The free-rider problem is thus solved with lower  $\gamma_2$ , but in a way such

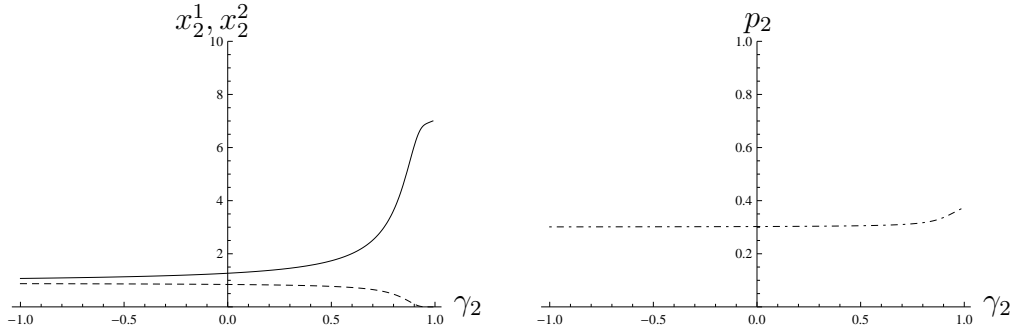


Figure 3: Effort levels and winning probability for different values of  $\gamma_2$ .

that the overall winning probability of the group is decreased. The more interesting result is, however, how this translates into the expected utility of the individuals.

In Figure 4 we see the expected utility of individuals 1 and 2 (again, represented by solid and dashed lines) and the aggregate expected utility (dotdashed line). The expected utility of individual 2 is of course rising in  $\gamma_2$  (falling with higher complementarity), since in the case of perfect substitutes, i.e.  $\gamma_2 = 1$ , individual 2 can fully free ride. The change of expected utility of individual 1 is ambiguous with respect to changes in  $\gamma_2$ . For very high values of  $\gamma_2$ , it is also increasing with  $\gamma_2$ , while for low values it is decreasing in  $\gamma_2$ . Aggregate expected utility is mainly influenced by individual 1 and thus total expected utility of the group members behaves similarly. The result is driven by the fact that the efficiency of individual 2 is much higher than that of individual 1 and at the same time the valuation of individual 1 is much higher than that of individual 2. In the perfect-substitutes case  $\gamma_2 = 1$ , only the less efficient individual 1 contributes effort and individual 2 takes a free ride. As we move away from this case, individual 2's incentives to provide effort increase only slowly. Due to the complementarity, individual 1 incurs very high losses in these cases. Reducing  $\gamma_2$  even further provides however much stronger incentives for individual 2. Individual 1 can thus reduce its effort further and in turn gain utility from the higher complementarity.

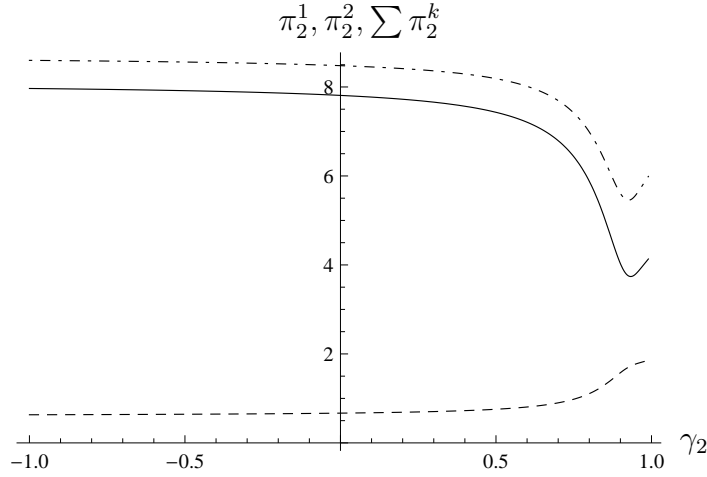


Figure 4: Expected utility for different values of  $\gamma_2$ .

## 5 Concluding Remarks

This paper has started from the observation that group effort can in general not be additively decomposed into some sum (of functions) of individual efforts. The use of a CES-impact function has allowed to identify the main channels of influence of the elasticity of substitution on the behavior in and the outcome of contests. If groups are homogenous (i.e. all group members have the same valuation and efficiency within the group), the elasticity of substitution does not matter. For heterogenous groups, the higher the complementarity of efforts of that group, the lower the divergence of efforts among group members and the lower the winning probability of that group. This contradicts the common intuition that groups can improve their performance by solving the free-rider problem via higher degrees of complementarity of efforts. Only if very high valuation members are also very inefficient at effort production the total expected utility may be higher for higher degrees of complementarity: at high levels of complementarity, highly efficient individuals with low valuations may replace some of the effort that is provided by less efficient group members at low levels of complementarity.

# Appendix A: Proof of Lemma 1

*Proof.* We first check that the interior solution is a local maximum. The first-order condition of the maximization problem (2) can be written as

$$\frac{Q/i Y_i^{\frac{1}{\gamma_i}-1}}{Q^2} = \frac{(y_i^k)^{\frac{1}{\gamma_i}-1}}{v_i^k}. \quad (\text{A.1})$$

The second-order condition is satisfied if

$$\frac{v_i^k \cdot Q/i \cdot Y_i^{\frac{1}{\gamma_i}-2}}{\gamma_i \cdot Q^2} \left( \frac{1 - 2 \cdot \frac{Q_i}{Q}}{\gamma_i} - 1 \right) - \frac{\frac{1}{\gamma_i} - 1}{\gamma_i} \cdot (y_i^k)^{\frac{1}{\gamma_i}-2} < 0. \quad (\text{A.2})$$

Solving the first-order condition for  $v_i^k$  and inserting the expression into the second-order condition we obtain, upon rearranging:

$$\frac{1 - \frac{1}{\gamma_i}}{\gamma_i} \left( 1 - \frac{y_i^k}{Y_i} \right) - 2 \cdot \frac{1}{\gamma_i^2} \cdot \frac{Q_i \cdot y_i^k}{Q \cdot Y_i} < 0, \quad (\text{A.3})$$

which holds for all  $\gamma_i \in (-\infty, 1)$ . Therefore, all solutions of the first-order condition are local maxima taking the other players' strategies as given. The best responses are either given by the solution to the first-order condition, or by a corner solution. From equation (1) it is clear that the only possible corner solutions are non-participation with  $x_i^k = 0$ . We thus need to verify that whenever the best response of one member of the group is given by the solution to the first-order condition, it is not possible for any member of the group to have the best response  $x_i^k = 0$ . First, we will show that whenever there exists a solution of the first-order condition for one individual of a group, it exists for all individuals: From the first-order conditions of two representative group members  $l, k$  we obtain the within-group equilibrium condition:

$$\forall l, k : \frac{(y_i^k)^{\frac{1}{\gamma_i}-1}}{v_i^k} = \frac{(y_i^l)^{\frac{1}{\gamma_i}-1}}{v_i^l} \quad (\text{A.4})$$

for all members  $k, l$  of group  $i$ . Both, the left-hand side (LHS) and right-hand side (RHS) of (A.4) are strictly increasing in  $y_i^k, y_i^l$  if  $\bar{\gamma} \in (0, 1)$ . For  $\bar{\gamma} \in (-\infty, 0)$  both LHS and RHS of (A.4) are strictly decreasing in  $y_i^k, y_i^l$ . Thus, for each  $y_i^k$  there exists a  $y_i^l$  such that the within-group equilibrium condition holds. Since for all group members the LHS of (A.1) is equal, there exists a positive solution to the first-order condition (FOC) for either all group members or none.

Second, we need to show that  $x_i^k = 0$  is not a best response if it is a best response for another individual  $l$  in the group to play  $x_i^l > 0$ . We do so by contradiction:

Obviously, for a corner solution with  $x_i^k = 0$  and  $x_i^l > 0$  the following condition needs to hold:

$$\frac{\partial \pi_i^k}{\partial x_i^k} = \frac{Q/i}{Q^2} \cdot Y_i^{\frac{1}{\gamma_i}-1} \cdot (x_i^k)^{\gamma_i-1} \cdot v_i^k - 1 \Big|_{x_i^k = 0, x_i^l > 0} \leq 0. \quad (\text{A.5})$$

From the fact that there is an individual  $l$  in the group, which participates with strictly positive effort, we know that

$$\frac{\partial \pi_i^l}{\partial x_i^l} = \frac{Q/i}{Q^2} \cdot Y_i^{\frac{1}{\gamma_i}-1} \cdot (x_i^l)^{\gamma_i-1} \cdot v_i^l - 1 \Big|_{x_i^k = 0, x_i^l > 0} = 0. \quad (\text{A.6})$$

Inserting (A.6) into (A.5) yields:

$$\frac{(x_i^l)^{1-\gamma_i}}{v_i^l} - \frac{(x_i^k)^{1-\gamma_i}}{v_i^k} \Big|_{x_i^k = 0, x_i^l > 0} \leq 0 \quad (\text{A.7})$$

from which we obtain by inserting  $x_i^k = 0$ :

$$(x_i^l)^{1-\gamma_i} \Big|_{x_i^l > 0} \leq 0 \quad (\text{A.8})$$

which is a contradiction for all  $\gamma_i < 1$ . Thus there does not exist an equilibrium in which for one player in the group a corner solution at zero effort investments is obtained while for another an interior solution holds.  $\square$

## Appendix B: Proof of Lemma 2

*Proof.* If there exists a solution to the FOC, it is characterized by the following equation, obtained by solving (A.4) for  $y_i^l$  and summing over all  $l$ ,

$$Y_i = y_i^k \cdot \sum_l \left( \frac{v_i^l}{v_i^k} \right)^{\frac{\gamma_i}{1-\gamma_i}}. \quad (\text{B.1})$$

We can now solve equation (A.1) for  $Y_i$  explicitly:

$$Y_i = \left( \sqrt{Q/i \cdot V_i(\gamma_i)} - Q/i \right)^{\gamma_i}. \quad (\text{B.2})$$

Thus, the condition for a strictly interior solution is  $(\sum_l v_i^l \frac{\gamma_i}{1-\gamma_i})^{\frac{1-\gamma_i}{\gamma_i}} > Q/i$ . Note that this condition is the same for all members of a group. In all other cases, we get

$y_i^k = 0$  for  $\gamma_i \in (0, 1)$  and  $y_i^k = \infty$  for  $\gamma_i \in (-\infty, 0)$  as was to be expected and which corresponds to  $x_i^k = 0$ . In these cases we have  $\forall l : y_i^k = y_i^l$  by equation (A.4) and by the definition of  $Q_i$ , we have:  $Q_i = Y_i^{\frac{1}{\gamma_i}} = 0$ . We can write a group best-response function as

$$\hat{Q}_i(\gamma_i, Q_{/i}) = \max \left( 0, \sqrt{Q_{/i} \cdot V_i(\gamma_i)} - Q_{/i} \right). \quad (\text{B.3})$$

establishing part a), since by Lemma 1 either for all group members we obtain an interior solution or for none. Since the best-response function is continuous in  $\gamma_i \neq 0$  and in the strategies of the other groups  $Q_{/i}$ , if a unique Nash equilibrium exists, the equilibrium strategies must also be continuous in  $\gamma_i$ . This establishes part c) of Lemma 2. What remains to be shown is which groups participate in equilibrium. Suppose a group  $\zeta$  participates in equilibrium with strictly positive effort, while a group  $\zeta + 1$  does not participate. Let  $Q_i^*(\vec{\gamma})$  be  $Q_i$  in equilibrium (we ignore here that these are best responses and should thus be functions of  $Q_{/i}^*$ ) and let the other variables introduced above be defined correspondingly in equilibrium. Then by the above condition in equilibrium we have for any given  $\vec{\gamma}$ :

$$\begin{aligned} V_\zeta(\gamma_\zeta) &> Q_{/\zeta}^*(\vec{\gamma}) \\ V_{\zeta+1}(\gamma_{\zeta+1}) &\leq Q_{/\zeta+1}^*(\vec{\gamma}) \end{aligned} \quad (\text{B.4})$$

Since by assumption  $Q_{\zeta+1}^*(\vec{\gamma}) = 0$ , we have  $Q_{/\zeta+1}^*(\vec{\gamma}) = Q^*(\vec{\gamma})$ . Solving (4) for  $Q_{/i}$  tells us that in an equilibrium where group  $\zeta$  participates, the following needs to be true:

$$Q_{/\zeta}^*(\vec{\gamma}) = \frac{Q^*(\vec{\gamma})^2}{V_\zeta(\gamma_\zeta)}. \quad (\text{B.5})$$

We now insert (B.5) into the first equation of (B.4) and the condition  $\hat{Q}_{/\zeta+1} = \hat{Q}$  into the second equation. Thus, the condition (B.4) becomes

$$\begin{aligned} V_\zeta(\gamma_\zeta) &> Q^*(\vec{\gamma}) \\ V_{\zeta+1}(\gamma_{\zeta+1}) &\leq Q^*(\vec{\gamma}) \end{aligned} \quad (\text{B.6})$$

in equilibrium. It follows that  $V_\zeta(\gamma_\zeta) > V_{\zeta+1}(\gamma_{\zeta+1})$ . We can thus order the groups such that  $V_i(\gamma_i) \geq V_{i+1}(\gamma_{i+1})$  and define  $n^*(\vec{\gamma})$  as the group with the highest index number that still participates with strictly positive effort. By (B.6), all groups  $i \leq n^*(\vec{\gamma})$  participate. This establishes part b) of Lemma 2.  $\square$

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