Belief Precision and Effort Incentives in Promotion Contests*

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Abstract

This short paper analyzes contests in which the agent with the highest perceived ability wins a prize. We show that each agent's equilibrium effort is non-monotonic in the precision of the prior beliefs about his ability. If little (much) is known about an agent's ability, his effort incentives are increasing (decreasing) in the degree of precision of the beliefs about his ability. The findings have implications for the design of promotion and incentive schemes in organizations.

Keywords: incentives, reputation, promotion contests, career concerns

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1 Introduction

Following seminal papers by Fama (1980) and Holmström (1982), a sizable theoretical and empirical literature has emphasized the importance of implicit incentives due to career concerns.[1] White-collar employees in particular often exert effort in an attempt to build a reputation for high ability. One key prediction of career concerns models is that an agent's implicit effort incentives are weaker the more is already known about his ability.[2] Intuitively, more precise beliefs are less responsive to new performance observations, which weakens implicit effort incentives.

In career concerns models, each agent's future payoff depends on the *absolute* level of his perceived ability only. The aim of this paper is to analyze how belief precision affects effort incentives when *relative* perceived abilities determine payoffs instead. The motivation is that the number of promotions to better paid positions, one of the most common rewards for high perceived ability, is limited in many organizations. Each employee must hence be assessed relative to his peers competing for the same promotion. Moreover, outside firms often learn less about an agent's ability than his present employer, so that the agent's market salary will be insensitive with respect to changes in his perceived ability that do not alter the (externally observable) promotion decision.[3]

The incentives associated with the rivalry for obtaining a promotion were first explored in Lazear and Rosen's (1981) seminal paper on tournaments.[4] In the canonical model, a group of agents compete for a fixed set of prizes awarded on the basis of relative performances, as in sports settings. However, since an important function of promotions is to sort employees by ability, most actual promotion decisions are based on ability assessments rather than objective performance measurements in a fixed time period. Whenever employees' prior reputations differ due to disparities in their past professional and educational achievements,

[1] See, for example, Gibbons and Murphy (1992) and Chevalier and Ellison (1999).

[2] See Holmström (1982), Dewatripont, Jewitt and Tirole (1999a, 1999b), and Casas-Arce (2009), among others. One exception is Martinez (2009) who finds that the opposite may hold in a carrer concerns model with job assignments where ability evolves over time.

[3] See Waldman (1984), Bernhardt (1995), Zabojnik and Bernhardt (2001), and Ghosh and Waldman (2010) for the theory of promotions as signals and how the salary premium associated with a promotion can arise endogenously in a competitive labor market.

[4] See Lazear (1995) and Prendergast (1999) for overviews.

for example, the employee with the best recent performance need not be the one whose perceived ability is the highest.[5] The principal in our model will therefore base her decision on the agents' relative perceived abilities instead of their relative performances.

More specifically, the model in this paper combines a rank-order tournament as in Lazear and Rosen (1981) with learning about abilities as in Holmström (1982). Two (heterogeneous) agents compete for a fixed prize awarded to the agent with the highest perceived (posterior) ability. The principal uses performances to learn about abilities. Agents have incentives to exert unobservable effort to jam the performance signal in an attempt to influence the principal's decision.

This way of modeling promotion contests yields new insights into the relation between belief precision and effort incentives. We find that each agent's equilibrium effort is non-monotonic in the precision of the prior beliefs about his ability. If little is known about an agent's ability, then an increase in the precision of beliefs about his ability (keeping the mean belief unchanged) increases his equilibrium effort. Once the beliefs about an agent's ability are sufficiently precise, the agent's equilibrium effort level decreases as beliefs about his ability become even more precise.

Each agent's effort incentive also depends on the precision of the beliefs about his opponent's ability. This relation is always positive if the agents' expected abilities are sufficiently similar prior to the contest; otherwise, the agent's equilibrium effort is decreasing in the precision of beliefs about his opponent's ability if and only if this precision exceeds a certain threshold.

Two effects are responsible for these results. First, as in single-agent learning models, there is a "learning effect": the rate of belief updating about an agent's ability is lower the more precise the prior beliefs about his ability are. Second, the likelihood that the contest outcome is close ex post depends on the precisions of the prior beliefs about both agents' abilities. The sign of this "rivalry effect" is a priori ambiguous. If expected abilities are identical prior to the contest, more precision always raises the anticipated likelihood of a close outcome and therefore strengthens effort incentives. If the prior expected abilities

[5] Note that contests based on relative perceived abilities are generally not equivalent to tournaments with handicaps (as in section IV of Lazear and Rosen, 1981, or in Meyer, 1991, 1992) either, because in the former the principal can update his beliefs about different agents' abilities at different rates.

differ, however, an increase in precision can dampen effort incentives.

The same rivalry effect can arise in tournaments. Rosen (1986) analyzes ladder tournaments in which winning probabilities in each match are a fixed function of abilities and effort levels. In one version of his model, players are ignorant about their own as well as their opponents' abilities. The (common) belief distributions about the abilities of two players meeting for a match, however, are always identical. Assuming there are two possible ability levels, Rosen (1986) shows that uncertainty about ability dampens effort incentives in this context. Our analysis implies that if the prior beliefs about the competitors' abilities differ, uncertainty can have the opposite impact on effort incentives in tournaments.

The next section describes the model. Section 3 contains the main results, and section 4 discusses their implications for the design of promotion and incentive schemes in hierarchies.

2 A Model of Promotion Contests

Consider a one-period game between a principal and two agents $j = 1, 2$. The principal's objective is to select the agent with the highest ability, but the agents' innate ability levels, η_1 and η_2, are unobservable to all parties. We assume that the prior distribution of beliefs about η_j is normal with mean m_j and precision (equal to the inverse of the variance) h_j. The prior distributions of η_1 and η_2 are independent. All parties share the same prior beliefs.

Before the principal selects one of the agents, the agents simultaneously decide how much effort to exert and the principal observes the agents' performances. Agent j's effort $a_j \in [0, \infty)$ is unobservable to the principal and agent $k \neq j$. The cost of effort is an increasing and strictly convex function $c(a_j)$ with $c(0) = c'(0) = 0$ and $\lim_{a_j \to \infty} c'(a_j) = \infty$. Agent j's performance is

$$y_j = \eta_j + a_j + \varepsilon_j,$$

where ε_j is a stochastic noise term. We assume that ε_1 and ε_2 are independently and normally distributed with zero means and precision h_ε.

After observing y_1 and y_2 the principal updates his beliefs and selects agent $j \neq k \in \{1, 2\}$ if and only if

$$E\left[\eta_j \mid y_j\right] > E\left[\eta_k \mid y_k\right]. \tag{1}$$

Agent j maximizes

$$\Pi_j\left(a_j; a_j^e, a_k, a_k^e\right) = P_j\left(a_j; a_j^e, a_k, a_k^e\right) W - c\left(a_j\right),$$

where $P_j\left(a_j; a_j^e, a_k, a_k^e\right)$ is the probability of j's selection as a function of the agents' actual and anticipated (a_1^e and a_2^e) effort levels, and $W > 0$ is the prize that the selected agent wins.

Denote by (a_1^*, a_2^*) the equilibrium effort choices. In equilibrium, each agent's effort choice must be optimal given beliefs and the competing agent's effort choice, and all parties must rationally anticipate the equilibrium effort choices, i.e., $a_j^e = a_j^*$ for $j = 1, 2$.

Given the normality and independence assumptions, the learning process about each agent's skill is well-known. If $a_j^e = a_j^*$, then the posterior distribution of η_j is normal with mean

$$\frac{h_j m_j + h_\varepsilon\left(y_j - a_j^*\right)}{h_j + h_\varepsilon} \tag{2}$$

and precision $h_j + h_\varepsilon$.

We now turn to j's effort decision. From (1) and (2) it follows that, if $a_k = a_k^*$ and $a_i^e = a_i^*$ for all $i = 1, 2$, j's winning probability as a function of a_j is:

$$\begin{aligned} P_j(a_j; a_j^*, a_k^*, a_k^*) &= \Pr\left\{\frac{h_j m_j + h_\varepsilon(\eta_j + a_j + \varepsilon_j - a_j^*)}{h_j + h_\varepsilon} > \frac{h_k m_k + h_\varepsilon(\eta_k + \varepsilon_k)}{h_k + h_\varepsilon}\right\} \\ &= \Pr\left\{\frac{h_\varepsilon}{h_j + h_\varepsilon}(a_j - a_j^*) > \frac{h_k m_k + h_\varepsilon(\eta_k + \varepsilon_k)}{h_k + h_\varepsilon} - \frac{h_j m_j + h_\varepsilon(\eta_j + \varepsilon_j)}{h_j + h_\varepsilon}\right\}. \end{aligned}$$

Define the random variable

$$\zeta_j \equiv \frac{h_k m_k + h_\varepsilon(\eta_k + \varepsilon_k)}{h_k + h_\varepsilon} - \frac{h_j m_j + h_\varepsilon(\eta_j + \varepsilon_j)}{h_j + h_\varepsilon}.$$

The independence and normality assumptions imply that the *prior* distribution of ζ_i is normal with mean

$$z_j \equiv m_k - m_j \tag{3}$$

and variance[6]

$$\sigma^2 = \left(\frac{h_\varepsilon}{h_1 + h_\varepsilon}\right)^2 \left(\frac{1}{h_1} + \frac{1}{h_\varepsilon}\right) + \left(\frac{h_\varepsilon}{h_2 + h_\varepsilon}\right)^2 \left(\frac{1}{h_2} + \frac{1}{h_\varepsilon}\right) \tag{4}$$

[6] Since the prior distributions of ζ_1 and ζ_2 have the same variance, we can simply denote this variance by σ^2, not using any subscript.

We denote this distribution by $\varphi_j(\cdot)$ with c.d.f. $\Phi_j(\cdot)$. Note that $\varphi_1(-z) = \varphi_2(z)$ for all $z \in \mathbb{R}$, and denote by

$$\varphi(0) = \varphi_1(0) = \varphi_2(0)$$

the prior density of identical posterior reputations (provided $a_i = a_i^*$ for $i = 1, 2$).

The probability that j wins given $a_k = a_k^*$ is then

$$P_j(a_j; a_j^*, a_k^*, a_k^*) = \Pr\left\{\zeta_j < \frac{h_\varepsilon}{h_j + h_\varepsilon}(a_j - a_j^*)\right\} = \Phi_j\left(\frac{h_\varepsilon}{h_j + h_\varepsilon}(a_j - a_j^*)\right). \tag{5}$$

The marginal impact of a_j on j's expected payoff given $a_k = a_k^*$ is

$$\frac{\partial \Pi_j(a_j; a_j^*, a_k^*, a_k^*)}{\partial a_j} = \varphi_j\left(\frac{h_\varepsilon}{h_j + h_\varepsilon}(a_j - a_j^*)\right)\frac{h_\varepsilon}{h_j + h_\varepsilon}W - c'(a_j).$$

The first-order conditions for an equilibrium with $a_1^*, a_2^* > 0$ are thus[7]

$$c'(a_1^*) = \varphi(0)\frac{h_\varepsilon}{h_1 + h_\varepsilon}W, \tag{6}$$

$$c'(a_2^*) = \varphi(0)\frac{h_\varepsilon}{h_2 + h_\varepsilon}W. \tag{7}$$

The assumptions on $c(\cdot)$ imply that each first-order condition has a unique and strictly positive solution.

3 Belief Precision and Effort Incentives

The career concerns literature predicts that the more is known about an agent's ability, the weaker are his incentives to exert effort in order to affect his perceived ability. We are interested in whether the same is true in our contest where only *relative* perceived abilities matter. From the first-order conditions in (6) and (7) it is apparent that $a_1^* > a_2^*$ if and only if $h_1 < h_2$: the agent about about whose ability more is known exerts less effort. As we

[7] The second-order equilibrium conditions are

$$\varphi_j'(0)\left(\frac{h_\varepsilon}{h_j + h_\varepsilon}\right)^2 W_j < c''(a_j^*) \text{ for all } j \neq k \in \{1, 2\}.$$

If $m_j = m_k$, then $\varphi_j'(0) = 0$ for $j = 1, 2$, so the second-order conditions always hold in that case. In what follows, we will assume that the second-order conditions are satisfied.

will show next, however, improvements in the precision of beliefs about one or both agents' abilities can nonetheless strengthen effort incentives. When comparing two contests, say contest A and contest B, that are the same except that h_1 is higher in contest B than in contest A, then it is possible that the equilibrium effort levels of both agents are higher in contest B than in contest A.

We first state the formal results and then discuss the effects at play:

Proposition 1 *Let $j \neq k \in \{1,2\}$.*

(i) *For any $(m_1, m_2, h_k, h_\varepsilon)$, there exists a threshold $\widehat{h}_j > 0$ such that*

$$\frac{da_j^*}{dh_j} \gtreqqless 0 \text{ if and only if } h_j \lesseqqgtr \widehat{h}_j.$$

Moreover, $\lim_{h_j \to 0} \frac{da_j^*}{dh_j} = \infty$, *and* $\lim_{h_j \to \infty} a_j^* = 0$.

(ii) *If*

$$(m_1 - m_2)^2 \leq \left(\frac{h_\varepsilon}{h_j + h_\varepsilon}\right)^2 \left(\frac{1}{h_j} + \frac{1}{h_\varepsilon}\right), \tag{8}$$

then

$$\frac{da_j^*}{dh_k} > 0 \text{ for all } h_k.$$

Otherwise, for any $(m_1, m_2, h_j, h_\varepsilon)$, there exists a threshold $\widehat{h}_k > 0$ such that

$$\frac{da_j^*}{dh_k} \gtreqqless 0 \text{ if and only if } h_k \lesseqqgtr \widehat{h}_k.$$

By making the distributions φ_1 and φ_2 more precise (i.e., by lowering σ^2), an increase in either h_1 or h_2 has an impact on $\varphi(0)$, the prior density of identical posterior reputations, which appears in both first-order conditions (equations (6) and (7)). The sign of this "rivalry effect" is a priori ambiguous. A marginal change in effort is more (less) likely to alter the contest outcome if posterior reputations are similar with a higher (lower) probability. Hence, an increase (decrease) in $\varphi(0)$ raises (lowers) effort incentives. Given our normality assumptions, a marginal increase in either h_1 or h_2 increases $\varphi(0)$ if and only if

$$\sigma(h_j, h_k, h_\varepsilon) > |m_1 - m_2|.$$

Since σ is strictly decreasing in h_1 and h_2, the marginal impact of h_k on $\varphi(0)$ is positive for *all* h_k if and only if $|m_1 - m_2| \leq \lim_{h_k \to \infty} \sigma$, which is equivalent to condition (8) in part (ii) of the proposition.

Intuitively, the rivalry effect is positive whenever more precise information about (one's own or the rival's) ability increases the likelihood of the race being close. Conditional on mean reputations being similar, more uncertainty (about either player's ability) dampens effort incentives by reducing the return to effort. If one agent has a considerably lower mean reputation than his rival, however, then an increase in uncertainty (again about either player's ability) strengthens effort incentives by raising the likelihood that the weaker rival can beat the stronger rival. The same effects would arise in a tournament model when each agent is ignorant about his own and his opponent's ability.

The rivalry effect explains the results in part (ii) of Proposition 1.[8] An increase in h_j has an additional effect on the first-order condition that determines a_j^*, however. It lowers the rate $\frac{h_\varepsilon}{h_j + h_\varepsilon}$ at which a marginal increase in y_j (given beliefs) improves j's posterior reputation (see (2)). This is a standard effect in learning models, which ceteris paribus predicts that higher precision leads to lower effort.

The results in part (i) of the proposition stem from the combination of this learning effect and the rivalry effect. If the principal's information about i's ability is sufficiently good (high h_i), then the negative learning effect always dominates. As the rate of belief updating about i's ability converges to zero, i's effort incentives completely vanish. When there is sufficient uncertainty about i's ability (h_i close to zero), on the other hand, the rivalry effect is positive and dominates the negative learning effect. To see this, note that the rivalry effect goes to infinity as h_i approaches zero, while the learning effect remains finite. Figure 1 illustrates Proposition 1(i).

Comparative statics results with respect to the other main model parameters are as expected. Evidently, $\frac{da_j^*}{dW} > 0$: the higher the prize, the stronger are effort incentives. It is also straightforward to show that a_j^* is increasing in m_j as long as $m_j < m_k$ but decreasing in m_j for $m_j > m_k$. Intuitively, the marginal return to effort is higher in a closer race, i.e.,

[8] The comparative statics results with respect to h_ε are qualitatively similar. If $m_1 = m_2$, then $\frac{da_j}{dh_\varepsilon} > 0$ for all h_ε. If $|m_1 - m_2| > \lim_{h_\varepsilon \to \infty} \sigma = 0$, then there exists a threshold $\widehat{h}_\varepsilon > 0$ such that $\frac{da_j}{dh_\varepsilon} \gtreqless 0$ if and only if $h_\varepsilon < \widehat{h}_\varepsilon$.

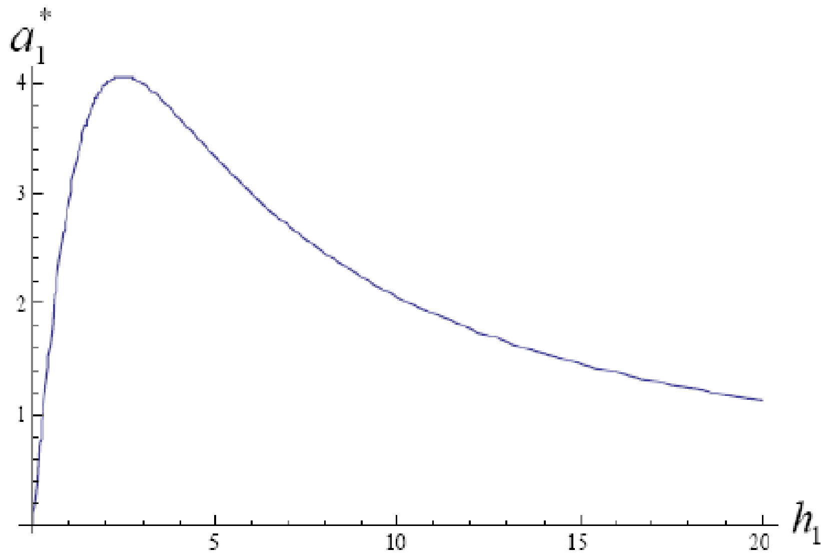

Figure 1: Agent 1's equilibrium effort if $c(a) = \frac{a^2}{2}$, $m_1 = m_2 = 1$, $h_2 = 2$, $h_\varepsilon = 1$, $W = 10$.

for m_j closer to m_k.[9]

4 Implications

Employers typically have more information about the abilities of employees with a longer "time on the job." This is one reason why career concerns models imply that it is easier to motivate junior employees (the other reason being that tenure is often correlated with age and younger workers have longer careers ahead of them).[10] Recent hires have an incentive to work hard in order to build a reputation for high ability, while high-powered incentive contracts are needed to motivate more senior employees (see Gibbons and Murphy, 1992).

Combining these insights about the attractiveness of implicit versus explicit incentives with our results on the non-monotonicity of effort incentives in contests based on relative

[9] Miklós-Thal and Ullrich (2010) provide empirical evidence for this prediction.

[10] A negative relation between information precision and the return to effort due to a shorter time until retirement could be added to our model by letting i's prize (if he wins) be decreasing in h_i. This would introduce a third and negative effect of h_i on a_i^*. As long as the marginal impact of h_i on i's prize is finite, however, a_i^* would still be increasing in h_i for h_i close enough to zero.

abilities yields several implications. It suggests that at low levels of the organizational hierarchy, promotion and salary decisions on the basis of individual ability assessments are preferable. While junior employees have strong incentives to prove themselves, relative ability assessments are not very effective at inducing effort when little is known about abilities. At intermediate levels of the hierarchy, where employees know more about each other's abilities, relative ability assessments have better incentive properties. Explicit incentives are necessary at the top of the hierarchy, occupied by senior employees of well-known abilities. Most law and consulting firms are structured along these lines. Upon graduation young lawyers typically spend six to ten years working as an associate. Promotions from junior to senior associate positions are mainly based on individual ability assessments. After reaching the senior associate level, lawyers are either promoted to partner or leave the firm. At this stage, they are in a race against each other to win what has been called the "promotion-to-partner tournament" (see Galanter and Palay, 1991). Once partners, lawyers face strong explicit effort incentives through their partnership profit-sharing agreements. To explore these issues in more depth, it would be interesting to study a multi-period model that incorporates relative versus absolute ability assessments at different levels of the organizational hierarchy.

A Appendix

Proof of Proposition 1: Let $\sigma(h_1, h_2, h_\varepsilon)$, equal to the square root of σ^2 defined in (4), denote the standard deviation of the distributions $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$. Making use of the normality of $\varphi_j(\cdot)$, the first-order condition defining a_j^* can be rewritten as

$$\frac{1}{\sqrt{2\pi}\sigma(h_j, h_k, h_\varepsilon)} \exp\left(-\frac{(m_k - m_j)^2}{2\sigma^2(h_j, h_k, h_\varepsilon)}\right) \frac{h_\varepsilon}{h_j + h_\varepsilon} W_j = c'\left(a_j^*\right). \tag{9}$$

Applying the implicit function theorem to (9) and rearranging terms yields

$$\frac{da_j^*}{dh_j} = \underbrace{\frac{\exp\left(-\frac{(m_k-m_j)^2}{2\sigma^2(h_j,h_k,h_\varepsilon)}\right)}{\sqrt{2\pi}\sigma(h_j, h_k, h_\varepsilon)} \frac{W_j \frac{h_\varepsilon}{h_j+h_\varepsilon}}{c''\left(a_j^*\right)}}_{>0} \times \left[-\frac{\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_j}}{\sigma(h_j, h_k, h_\varepsilon)}\left(1 - \frac{(m_k - m_j)^2}{\sigma^2(h_j, h_k, h_\varepsilon)}\right) \underbrace{-\frac{1}{h_j + h_\varepsilon}}_{<0}\right]. \tag{10}$$

It is easy to see from (4) that $\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_j} < 0$. This implies that whenever $(m_k - m_j)^2 \geq \sigma^2(h_j, h_k, h_\varepsilon)$, then $\frac{da_j^*}{dh_j} < 0$. However, the sign of $\frac{da_j^*}{dh_j}$ is ambiguous if $(m_k - m_j)^2 < \sigma^2(h_j, h_k, h_\varepsilon)$.

We proceed by examining the limit values of $\frac{da_j^*}{dh_j}$. First, since $\lim_{h_j\to\infty} \frac{\partial P_j\left(a_j;a_j^*,a_k^*,a_k^*\right)}{\partial a_j} = 0$,

$$\lim_{h_j\to\infty} \frac{da_j^*}{dh_j} = 0.$$

Second, we show that $\lim_{h_j\to 0} \frac{da_j^*}{dh_j} = \infty$. Using $\lim_{h_j\to\infty} \sigma = \infty$, the limit of (10) can be simplified as follows:

$$\lim_{h_j\to 0} \frac{da_j^*}{dh_j} = \frac{W_j}{\sqrt{2\pi}c''\left(a_j^*\right)} \lim_{h_j\to 0} \left(-\frac{\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_j}}{\sigma^2(h_j, h_k, h_\varepsilon)}\right).$$

Using the expression for σ^2 in (4) and further simplifying leads to

$$\begin{aligned}\lim_{h_j\to 0}\left(-\frac{\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_j}}{\sigma^2(h_j, h_k, h_\varepsilon)}\right) &= \lim_{h_j\to 0} \frac{\frac{h_\varepsilon(2h_j+h_\varepsilon)}{2h_j^2(h_j+h_\varepsilon)^2}}{\left(\frac{h_\varepsilon[h_j(h_j+h_\varepsilon)+h_k(h_k+h_\varepsilon)]}{h_j h_k(h_j+h_\varepsilon)(h_k+h_\varepsilon)}\right)^{\frac{3}{2}}} \\ &= \lim_{h_j\to 0} \frac{\frac{1}{2}\left(\frac{h_\varepsilon}{h_j(h_j+h_\varepsilon)}\right)^2}{\left(\frac{h_\varepsilon}{h_j(h_j+h_\varepsilon)}\right)^{\frac{3}{2}}} = \lim_{h_j\to 0} \frac{1}{2}\left(\frac{h_\varepsilon}{h_j(h_j + h_\varepsilon)}\right)^{\frac{1}{2}} = \infty.\end{aligned}$$

Since $c'' > 0$, the latter implies that

$$\lim_{h_j \to 0} \frac{da_j^*}{dh_j} = \infty.$$

It remains to show that there exists a *unique* $\widehat{h}_j$ such that $\frac{da_j^*}{dh_j} > 0$ if and only if $h_j < \widehat{h}_j$. Since $c'' > 0$, $\frac{da_j^*}{dh_j}$ has the sign of the sum between square brackets in (10). Using the expression for σ in (4) and simplifying, we find that

$$\begin{aligned} & -\frac{\frac{\partial \sigma(h_j,h_k,h_\varepsilon)}{\partial h_j}}{\sigma(h_j,h_k,h_\varepsilon)} \left(1 - \frac{(m_k - m_j)^2}{\sigma^2(h_j,h_k,h_\varepsilon)}\right) - \frac{1}{h_j + h_\varepsilon} \\ & = \frac{h_k(2h_j + h_\varepsilon)(h_k + h_\varepsilon)\left(1 - \frac{(m_k-m_j)^2}{\sigma^2(h_j,h_k,h_\varepsilon)}\right) - 2h_j\left[h_j(h_j+h_\varepsilon) + h_k(h_k+h_\varepsilon)\right]}{2h_j(h_j+h_\varepsilon)\left[h_j(h_j+h_\varepsilon)+h_k(h_k+h_\varepsilon)\right]}. \end{aligned} \tag{11}$$

$\frac{da_j^*}{dh_j}$ has the same sign as the numerator in (11). The partial derivative of the numerator in (11) with respect to h_j is

$$\begin{aligned} & -2h_k(h_k+h_\varepsilon)\frac{(m_k-m_j)^2}{\sigma^2(h_j,h_k,h_\varepsilon)} + h_k(2h_j+h_\varepsilon)(h_k+h_\varepsilon)\frac{(m_k-m_j)^2}{\sigma^2(h_j,h_k,h_\varepsilon)}\underbrace{\frac{\partial\sigma^2}{\partial h_j}}_{<0} \\ & -2h_j(h_j+h_\varepsilon) - 2h_j(2h_j+h_\varepsilon), \end{aligned}$$

which is clearly negative. Together with the limit values $\lim_{h_j\to 0}\frac{da_j^*}{dh_j} = \infty$ and $\lim_{h_j\to\infty}\frac{da_j^*}{dh_j} = 0$, this implies that there exists a unique $\widehat{h}_j > 0$ such that $\frac{da_j^*}{dh_j} \gtreqless 0$ if and only if $h_j \lesseqgtr \widehat{h}_j$.

Finally,

$$\frac{da_j^*}{dh_k} = \underbrace{\frac{\exp\left(-\frac{(m_k-m_j)^2}{2\sigma^2(h_j,h_k,h_\varepsilon)}\right)}{\sqrt{2\pi}\sigma(h_j,h_k,h_\varepsilon)}\frac{W_j\frac{h_\varepsilon}{h_j+h_\varepsilon}}{c''(a_j^*)}}_{>0}\underbrace{\left(-\frac{\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_k}}{\sigma(h_j,h_k,h_\varepsilon)}\right)}_{>0}\left(1-\frac{(m_k-m_j)^2}{\sigma^2(h_j,h_k,h_\varepsilon)}\right). \tag{12}$$

From (12) it is easy to see that $\frac{da_j^*}{dh_k} > 0$ if and only if $|m_k - m_j| < \sigma(h_j,h_k,h_\varepsilon)$. Since $\lim_{h_k\to 0}\sigma^2 = \infty$ and $\sigma^2(h_j,h_k,h_\varepsilon)$ is strictly decreasing in h_k (see (4)), this implies that $\frac{da_j^*}{dh_k} > 0$ for all h_k if and only if $|m_k - m_j| \le \lim_{h_k\to\infty}\sigma(h_j,h_k,h_\varepsilon) = \left(\frac{h_\varepsilon}{h_j+h_\varepsilon}\right)^2\left(\frac{1}{h_j}+\frac{1}{h_\varepsilon}\right)$. Otherwise, there exists an $\widehat{h}_k > 0$ such that $\frac{da_j^*}{dh_k} \gtreqless 0$ if and only if $h_k \lesseqgtr \widehat{h}_k$. QED

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