

The Equilibrium Order of Games in the Best-of-Three Contests.

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We study a best-of-three all-pay contest with two teams in which the first team to win two games wins the contest. Each team has a value of winning the contest as well as a value of winning a single game. It is assumed that a team's value of winning a game in its home field is higher than its value of winning a game away from home. The stronger team (the team with the higher value of winning the contest) plays twice at its home field and once away from it. We analyze the equilibrium order of games according to which no one has an incentive to switch to a different order of games, since switching would not yield a higher expected payoff.

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1. Introduction

Multi-stage tournaments can be organized in several ways. One way is in the form of a best-of- t contest, where the player that first accumulates $\frac{t+1}{2}$ single victories wins the overall contest. For example, the final playoff of the National Hockey League (NHL) is based on a form of the best-of-seven games. Another example is the quarterfinal round of the Euroleague, the primary Europe-wide club competition in basketball, which was originally based on the best-of-three format¹. The same format is used in the conference finals and semi-finals of the Women's National Basketball League (WNBA) and also in women's and most men's tennis tournaments (see Malueg and Yates (2010)). Multi-stage contests are common not only in sport, but also in other areas, such as R&D competitions (see Harris and Vickers (1987)) and political races (see Klump and Polborn (2006)).

In this paper we deal with best-of-three contests. One of the central issues in these contests is the ordering of games when teams play home and away games, since there are several options. For example, until 2009, the games in the playoff of the WNBA were ordered such that the stronger team played away from home in the first game and then played the second and (if needed) the third game at its home field. In 2010, the league switched to another order, where the stronger team would play the first and (if needed) the third game at home. Another example of a contest in which the order of games was switched is the Euroleague. In the beginning of 90s, the order of games in the quarterfinal round of the Euroleague was the same as it had been in the WNBA until 2009. However, starting from the 2004-05 season, in the quarterfinal round, the stronger

¹ Starting from the 2008-09 season, the quarterfinal round has been based on the form of a best-of-five contest.

team played at its home field in the first game and (if needed) in the third game, whereas the second game was played at the weaker team's home field.

We present a model of a best-of-three contest between two teams, where the team's abilities (values) are common knowledge and each game among the teams is modeled as an all-pay auction where teams simultaneously submit their efforts and the team with the higher effort in a single game wins that game. We consider heterogeneous teams that have asymmetric values of winning the contest. We assume that it does matter whether a team is playing at its home field or at the away from home field. One reason for this assumption is that there is a fixed cost, such as a travelling cost or accommodation cost that occurs only in the away from home games. Another reason can be attributed to the widely acknowledged sporting phenomenon known as the "home advantage"². Courneya and Carron (1992) defined the home advantage as 'the consistent finding that home teams in sports competitions win over 50% of the games played under a balanced home and away schedule'.

Previously, Konrad and Kovenock (2009) and Sela (2011) investigated the best-of- t contests with two players³ ($t \geq 3$), and assumed that there is not only a prize for winning the entire contest, but also a value for winning (Konrad and Kovenock (2009)) or losing (Sela (2011)) in each component contest. Accordingly, we assume that the value of winning in the single game is higher if a team wins in its home field in front of its own audience than if it wins away from home. This assumption correlates with the "home advantage" phenomenon, since, as a result of this assumption there is a higher

² The impact of the home advantage in sports has received attention in the research literature for many years. Terry, Walrond, & Carron (1998) found that in generally, athletes report more positive psychological states when they play at home rather than if they play away from home. Nevill and Holder (1999) found that the home team winning percentage was 68.3% over 40,493 soccer matches.

³ See also Malueg and Yates (2010) who studied best-of-three contests.

probability of winning a single game at home rather than winning a single game away from home.

We analyze the equilibrium order of games in the best-of-three all-pay auction, according to which no one has an incentive to switch to a different order, as switching would not yield a higher expected payoff. We show that in the equilibrium order the weaker team plays at its home field in the first stage and the stronger team plays at its home field in the second stage and if needed in the third one.

The implication of this equilibrium order is that in the best-of-three contest, teams agree about the order of the games. This is important, since sports contests involve at least two different entities, which, among other things, must agree on the rules of the game, the time of the game and the venue of the game (see Noll (2003)). Several studies have dealt with the best order of actions in sports. For example, Apesteguia and Palacios-Huerta (2010) found a systematic first-mover advantage in the soccer penalty shoot-out. In another work, Page and Page (2007) underlined the second leg home advantage for European soccer cup competitions. In addition, Magnus and Klaassen (1999) showed that there is an advantage to serve first in the first set in tennis.

The paper is organized as follows: Section 2 describes the model. Section 3 presents the equilibrium behavior in all three possible orders of the games. Section 4 presents the equilibrium order of games, and Section 5 summarizes with conclusions.

2. The Model

We consider a sequential all-pay auction (contest) with two teams denoted by $i = 1, 2$ and three stages (games) denoted by $t = 1, 2, 3$. The teams compete in sequential

games and the first to win two games wins the contest. Team i 's value of winning the contest (or the type of the team) is V_i . Valuations are common knowledge. We assume that $V_1 > V_2$. Each team exerts an effort of x_i^t in stage t . These efforts are submitted simultaneously, and the team with the higher effort in stage t wins the game in that stage. Each team has a linear cost function $C(x_i) = x_i$.

The teams also have a value of winning a single game. It is assumed that a team's value of winning a game in its home field (k_H) is higher than its value of winning a game away from home (k_A). The stronger team (team 1) plays two games at its home field and the weaker team (team 2) plays only once at its home field. We further assume that $V_1 + k_A > V_2 + k_H$, which means that the home advantage cannot change the types' order. Furthermore, teams have a fixed cost (FC) that occurs only at the games away from home. This FC is relatively small such that $2V_1 - 2V_2 + 2k_H - 2k_A > FC$ and $k_A > FC$.

3. The Equilibrium

Three different orders of games can occur in the best-of-three contest. The first one (Option A) is where the weaker team (team 2) plays the first two games away from home and the third game (if needed) at its home field. This order is denoted by (1-1-2). The second possible order (Option B), is where the weaker team (team 2) plays at its home field in the first stage and games 2 and 3 (if needed) are played away from home. This order is denoted by (2-1-1). And the last one (Option C), is where the weaker team (team 2) plays at its home field in the second stage. This order is denoted by (1-2-1).

In order to analyze the subgame-perfect equilibrium of the best-of-three all-pay contest, we begin with the last stage of the contest and go backwards to the previous

stages. In our model we need the following simple condition in order to prevent trivial games:

$$2V_2 + FC + 2k_A > V_1 + 2k_H \quad (1)$$

Condition (1) is also necessary, since otherwise, by the analysis below it would be verified that team 2 will not have any incentive to compete in the first stage.

3.1 Option A (1-1-2)

In this option the weak team (team 2) plays the first two games away from home and the third game (if needed) are played at its home field.

3.1.1 Stage 3 (team 2 is the host)

The teams compete in the last stage only if each team won one of the previous games. Therefore, the payoff of team 1 if it wins the game in stage 3 is $V_1 + k_A$ and if it loses, it is 0. Similarly, if team 2 wins the game in this stage, its payoff is $V_2 + k_H$, since team 2 is playing at home. If team 2 loses, its payoff is 0. Since the game in this stage is at team 2's home field, team 1 has a fixed cost of $FC > 0$.

According to Hillman and Riley (1989) and Baye, Kovenock and de Vries (1996), since $V_1 + k_A > V_2 + k_H$, there is always a mixed-strategy equilibrium in which the teams randomize on the interval $[0, V_2 + k_H]$ according to their effort cumulative distribution functions $F_i^3, i = 1, 2$. These functions are given by:

$$\begin{aligned} (V_1 + k_A)F_2(x_1) - x_1 - FC &= V_1 - V_2 - k_H + k_A - FC \\ (V_2 + k_H)F_1(x_2) - x_2 &= 0 \end{aligned} \quad (2)$$

3.1.2. Stage 2 (team 1 is the host)

Assume first that team 1 won the game in stage 1. By (2), team 2's expected payoff in the next stage is 0. Thus, if team 2 wins, its payoff is only k_A . But if team 2 loses in this

stage, its payoff is 0. In addition, since team 2 is playing away from home in this stage, team 2 has a fixed cost of $FC > 0$. Similarly, if team 1 wins in this stage, it wins the contest, and its payoff is $V_1 + k_H$, but if team 1 loses in this stage, by (2) its expected payoff in the next stage is $V_1 - V_2 - k_H + k_A - FC$.

Thus, since $V_2 + 2k_H - 2k_A + FC > 0$ we obtain that teams 1 and 2 randomize on the interval $[0, k_A]$ according to their effort cumulative distribution functions $F_i^2, i = 1, 2$ which are given by:

$$(V_1 + k_H)F_2(x_1) + (V_1 - V_2 - k_H + k_A - FC)(1 - F_2(x_1)) - x_1 = V_1 + k_H - k_A \quad (3)$$

$$k_A F_1(x_2) - x_2 - FC = -FC$$

Assume now that team 2 won the game in stage 1. By (2), team 1's expected payoff in the next stage is $V_1 - V_2 - k_H + k_A - FC$. Thus, if team 1 wins in this stage, its payoff is $(V_1 - V_2 - k_H + k_A - FC) + k_H = V_1 - V_2 + k_A - FC$, but if it loses, its payoff is 0. If team 2 wins, its payoff will be $V_2 + k_A$ and if it loses, by (2) its expected payoff is 0. In addition, since the game in this stage is at team 1's home field, team 2 has a fixed cost of $FC > 0$. Thus, since assumption (1) implies that $2V_2 + FC + 2k_A > V_1 + 2k_H$, we obtain that teams 1 and 2 randomize on the interval $[0, V_1 - V_2 + k_A - FC]$ according to their effort cumulative distribution functions $F_i^2, i = 1, 2$ which are given by:

$$(V_1 - V_2 + k_A - FC)F_2(x_1) - x_1 = 0 \quad (4)$$

$$(V_2 + k_A)F_1(x_2) - x_2 - FC = 2V_2 - V_1$$

3.1.3 Stage 1 (team 1 is the host)

By (3), team 1's expected payoff in the next stage is $V_1 + k_H - k_A$. Thus, if team 1 wins, its payoff is $(V_1 + k_H - k_A) + k_H = V_1 + 2k_H - k_A$, but if it loses, by (4) its expected payoff in the next stage is 0. Similarly, by (4) team 2's expected payoff in the next stage

is $2V_2 - V_1$. Thus, if team 2 wins, its payoff is $(2V_2 - V_1) + k_A$. If team 2 loses, by (3) its expected payoff in the next stage is $(-FC)$. In addition, since the game in this stage is at team 1's home field, team 2 has a fixed cost of $FC > 0$. Thus since $2V_1 - 2V_2 + 2k_H - 2k_A - FC > 0$, we obtain that teams 1 and 2 randomize on the interval $[0, 2V_2 - V_1 + FC]$ according to their effort cumulative distribution functions $F_i^1, i = 1, 2$ which are given by:

$$\begin{aligned} (V_1 + 2k_H - k_A)F_2(x_1) - x_1 &= 2V_1 - 2V_2 + 2k_H - 2k_A - FC \\ (2V_2 - V_1 + k_A)F_1(x_2) + (-FC)(1 - F_1(x_2)) - x_2 - FC &= -2FC \end{aligned} \tag{5}$$

The above analysis of the subgame perfect equilibrium of the best-of-three all-pay auction for Option A implies that:

Lemma 1: *Team 1's expected payoff in Option A is $2V_1 - 2V_2 + 2k_H - 2k_A - FC$, whereas team 2's expected payoff in Option A is $(-2FC)$.*

3.2 Option B (2-1-1)

In this option, the weak team (team 2) plays the first game at its home field and the second and third games (if needed) are played at team's 1 home field.

3.2.1 Stage 3 (team 1 is the host)

The teams compete in the last stage only if each team won one of the previous games. Therefore, the expected payoff of team 1 if it wins the game in stage 3 is $V_1 + k_H$ and if it loses, it is 0. Similarly, if team 2 wins the game in this stage, its expected payoff is $V_2 + k_A$ since team 1 is playing at its home field. If team 2 loses its expected value is 0. Since the game in this stage is at team 1's home field, team 2 has a fixed cost of $FC > 0$.

Since $V_1 + k_H > V_2 + k_A$, there is always a mixed-strategy equilibrium in which the teams randomize on the interval $[0, V_2 + k_A]$ according to their effort cumulative distribution functions $F_i^3, i = 1, 2$ which are given by:

$$\begin{aligned} (V_1 + k_H)F_2(x_1) - x_1 &= V_1 - V_2 + k_H - k_A \\ (V_2 + k_A)F_1(x_2) - x_2 - FC &= -FC \end{aligned} \tag{6}$$

3.2.2. Stage 2 (team 1 is the host)

Assume first that team 1 won the game in stage 1. By (6), team 2's expected payoff in the next stage is $(-FC)$. Thus, if team 2 wins, its payoff is $k_A - FC$. But if team 2 loses in this stage, its payoff is 0. In addition, since team 2 is playing away from home in this stage, team 2 has a fixed cost of $FC > 0$. Similarly, if team 1 wins, it wins the contest, and its payoff is $V_1 + k_H$. But if team 1 loses in this stage, by (6) its expected payoff in the next stage is $V_1 - V_2 + k_H - k_A$.

Thus, since $k_A - FC > 0$ we obtain that teams 1 and 2 randomize on the interval $[0, k_A - FC]$ according to their effort cumulative distribution functions $F_i^2, i = 1, 2$ which are given by:

$$\begin{aligned} (V_1 + k_H)F_2(x_1) + (V_1 - V_2 + k_H - k_A)(1 - F_2(x_1)) - x_1 &= V_1 + k_H - k_A + FC \\ (-FC + k_A)F_1(x_2) - x_2 - FC &= -FC \end{aligned} \tag{7}$$

Assume now that team 2 won the game in stage 1. By (6), team 1's expected payoff in the next stage is $V_1 - V_2 + k_H - k_A$. Thus, if team 1 wins in this stage its payoff is $(V_1 - V_2 + k_H - k_A) + k_H = V_1 - V_2 + 2k_H - k_A$, but if it loses, its payoff is 0. If team 2 wins, its value will be $V_2 + k_A$ and if it loses, by (6) its expected value will be $(-FC)$. In addition, since the game in this stage is at team 1's home field, team 2 has a fixed cost of $FC > 0$. Thus, since assumption (1) implies that $2V_2 + FC + 2k_A > V_1 + 2k_H$, we obtain

that teams 1 and 2 randomize on the interval $[0, V_1 - V_2 + 2k_H - k_A]$ according to their effort cumulative distribution functions $F_i^2, i = 1, 2$ which are given by:

$$\begin{aligned} (V_1 - V_2 + 2k_H - k_A)F_2(x_1) - x_1 &= 0 \\ (V_2 + k_A)F_1(x_2) + (-FC)(1 - F_1(x_2)) - x_2 - FC &= 2V_2 - V_1 + 2k_A - 2k_H - FC \end{aligned} \quad (8)$$

3.2.3 Stage 1 (team 2 is the host)

By (7), team 1's expected payoff in the next stage is $V_1 + k_H - k_A + FC$. Thus, if team 1 wins, its payoff is $(V_1 + k_H - k_A + FC) + k_A = V_1 + k_H + FC$, but if it loses, by (8) its expected payoff in the next stage is 0. Similarly, by (8) team 2's expected payoff in the next stage is $2V_2 - V_1 + 2k_A - 2k_H - FC$. Thus, if team 2 wins, its payoff is $(2V_2 - V_1 + 2k_A - 2k_H - FC) + k_H = 2V_2 - V_1 + 2k_A - k_H - FC$. If team 2 loses, by (7) its expected payoff in the next stage is $(-FC)$. In addition, since the game in this stage is at team's 2 home field, team 1 has a fixed cost of $FC > 0$. Thus, since $2V_1 - 2V_2 + 2k_H - 2k_A + FC > 0$, we obtain that teams 1 and 2 randomize on the interval $[0, 2V_2 - V_1 + 2k_A - k_H - FC]$ according to their effort cumulative distribution functions $F_i^1, i = 1, 2$ which are given by:

$$\begin{aligned} (V_1 + k_H + FC)F_2(x_1) - x_1 - FC &= 2V_1 - 2V_2 + 2k_H - 2k_A \\ (2V_2 - V_1 + 2k_A - k_H - FC)F_1(x_2) + (-FC)(1 - F_1(x_2)) - x_2 &= -FC \end{aligned} \quad (9)$$

The above analysis of the subgame perfect equilibrium of the best-of-three all-pay auction for Option B implies that:

Lemma 2: *Team 1's expected payoff in Option B is $2V_1 - 2V_2 + 2k_H - 2k_A$, whereas team 2's expected payoff in Option B is $(-FC)$.*

3.3 Option C (1-2-1)

In this option the weaker team (team 2) plays the game at its home field in the second stage, whereas the games in stages 1 and 3 (if needed) are played at team's 1 home field.

3.3.1 Stage 3 (team 1 is the host)

The teams compete in the last stage only if each team won one of the previous games. Similar to Option B, there is always a mixed-strategy equilibrium in which the teams randomize on the interval $[0, V_2 + k_A]$ according to their effort cumulative distribution functions $F_i^3, i=1,2$ which are given by:

$$\begin{aligned} (V_1 + k_H)F_2(x_1) - x_1 &= V_1 - V_2 + k_H - k_A \\ (V_2 + k_A)F_1(x_2) - x_2 - FC &= -FC \end{aligned} \tag{10}$$

3.3.2. Stage 2 (team 2 is the host)

Assume first that team 1 won the game in stage 1. By (10), team 2's expected payoff in the next stage is $(-FC)$. Thus, if team 2 wins, its payoff is $k_H - FC$. But if team 2 loses in this stage, its payoff is 0. Similarly, if team 1 wins in this stage, it wins the contest, and its payoff is $V_1 + k_A$. But if team 1 loses in this stage, by (10) its expected payoff in the next stage is $V_1 - V_2 + k_H - k_A$. In addition, since team 1 is playing away from home in this stage, team 1 has a fixed cost of $FC > 0$.

Thus, since $k_H - FC > 0$ we obtain that teams 1 and 2 randomize on the interval $[0, k_H - FC]$ according to their effort cumulative distribution functions $F_i^2, i=1,2$ which are given by:

$$\begin{aligned} (V_1 + k_A)F_2(x_1) + (V_1 - V_2 + k_H - k_A)(1 - F_2(x_1)) - x_1 - FC &= V_1 + k_A - k_H \\ (-FC + k_H)F_1(x_2) - x_2 &= 0 \end{aligned} \tag{11}$$

Assume now that team 2 won the game in stage 1. By (10), team 1's expected payoff in the next stage is $V_1 - V_2 + k_H - k_A$. Thus, if team 1 wins in this stage its payoff is $(V_1 - V_2 + k_H - k_A) + k_A = V_1 - V_2 + k_H$, but if it loses, its payoff is 0. In addition, since the game in this stage is at team 2's home field, team 1 has a fixed cost of $FC > 0$. If team 2 wins, its payoff is $V_2 + k_H$ but if it loses, by (10) its expected payoff is $(-FC)$. Thus, since $2V_2 + FC > V_1$, we obtain that teams 1 and 2 randomize on the interval $[0, V_1 - V_2 + k_H]$ according to their effort cumulative distribution functions $F_i^2, i = 1, 2$ which are given by:

$$\begin{aligned} (V_1 - V_2 + k_H)F_2(x_1) - x_1 - FC &= -FC \\ (V_2 + k_H)F_1(x_2) + (-FC)(1 - F_1(x_2)) - x_2 &= 2V_2 - V_1 \end{aligned} \tag{12}$$

3.3.3 Stage 1 (team 1 is the host)

By (11), team 1's expected payoff in the next stage is $V_1 + k_A - k_H$. Thus, if team 1 wins its payoff is $(V_1 + k_A - k_H) + k_H = V_1 + k_A$, but if it loses, by (12) its expected payoff in the next stage is $(-FC)$. Similarly, by (12) team 2's expected payoff in the next stage is $2V_2 - V_1$. Thus, if team 2 wins, its payoff is $(2V_2 - V_1) + k_A = 2V_2 - V_1 + k_A$. But if team 2 loses, by (11) its expected payoff in the next stage is 0. In addition, since the game in this stage is at team's 1 home field, team 2 has a fixed cost of $FC > 0$. Thus, we obtain that teams 1 and 2 randomize on the interval $[0, 2V_2 - V_1 + k_A]$ according to their effort cumulative distribution functions $F_i^1, i = 1, 2$ which are given by:

$$\begin{aligned} (V_1 + k_A)F_2(x_1) + (-FC)(1 - F_2(x_1)) - x_1 &= 2V_1 - 2V_2 \\ (2V_2 - V_1 + k_A)F_1(x_2) - x_2 - FC &= -FC \end{aligned} \tag{13}$$

The above analysis of the subgame perfect equilibrium of the best-of-three all-pay auction in Option C implies that:

Lemma 3: *Team 1's expected payoff in Option C is $2V_1 - 2V_2$, whereas team 2's expected payoff in Option C is $(-FC)$.*

4. Results

In this section we find the equilibrium order of games in the best-of-three all-pay auction. A comparison of Lemmas 1, 2 and 3 implies team 1's preference which is given by:

Proposition 1: *The stronger team will prefer to play the second and third games at its home field according to Option B, rather than according to Options A and C.*

Likewise, team 2's preference is:

Proposition 2: *The weaker team will refuse to play its first and second games away from home (Option A), while it will be indifferent to Options B and C.*

The intersection of Propositions 1 and 2 yields the following equilibrium order of games:

Theorem 1: *Consider two asymmetric teams, $V_1 > V_2$, who compete in the best-of-three all-pay contest. Then, the equilibrium order of games, according to which no one has an incentive to switch to a different order of games since switching would not yield a higher expected payoff, is the order where the weaker team plays at its home field in the first stage and the stronger team plays at its home field in the second and if needed the third stage (Option B).*

According to Theorem 1, since there is an equilibrium order of games, we can conclude that the best-of-three all-pay contest is not a zero-sum game with respect to the teams' expected payoffs. On the other hand, it is clear that the best-of-three all-pay contest is a zero-sum game with respect to the teams' probabilities of winning. Thus, for each option of the game orders (A, B or C), one of the teams has an incentive to switch

to a different order of games, since by switching, this team will increase its probability of winning the contest.

In the Appendix we calculate the teams' probabilities of winning the contest in all the options (A, B, C) of the game orders. We use these calculations in the following example to demonstrate the conflict between a team's expected payoff and its probability of winning.

Example 1: Consider two teams 1 and 2 where team 1's value of winning the contest is $V_1 = 100$ and team 2's is $V_2 = 95$. The value of winning a single game in the home field for both teams is $k_H = 30$, and the value of winning a single game in the away field is $k_A = 28$. The fixed cost in the away from home game is $FC = 13$.

By the teams' probabilities of winning we obtain that Team 1's probability to win the contest in Option A is 0.499 (team 2's probability to win is 0.501). Team 1's probability to win the contest in Option B is 0.606 and team 1's probability to win the contest in Option C is 0.590 (see Appendix).

On the other hand, by Lemmas 1,2 and 3, the expected payoff of team 1 in Option A is 1, (when the expected payoff of team 2 in that option is (-26)), in Option B it is 14 (when the expected payoff of team 2 is (-13)), and in Option C it is 10 (when the expected payoff of team 2 is (-13)).

We can see that the weaker team (team 2) has the highest probability to win the contest in Option A, and it is even higher than the stronger team's. However, team 2 will reject this order, since its expected payoff is the lowest among all the other options. One possible explanation for this paradoxical phenomenon is the existence of a fixed cost in the away from home games. In multi-stage contests, the influence of a fixed cost on a teams' behavior appears to be quite complex, since it has different effects on the teams' expected payoffs and their probability of winning, such that the teams may have

different preferences about the order of games regarding the expected payoff on the one hand and the probability of winning on the other.

5. Concluding Remarks

We studied the best-of-three contest between two heterogeneous teams, when the stronger team plays at its home field twice. Each game was modeled as an all-pay auction. We showed that if a team has a fixed cost that occurs only in the away from home games, and the value of winning a single game at home is higher than the value of winning a single game away from home, there is only one order of games that is acceptable to both teams. Furthermore, none of the teams has an incentive to switch to a different order of games with regard to their expected payoff. In this equilibrium order the weaker team plays at its home field in the first stage and then plays two games at the stronger team's field.

This work also demonstrates the key role of a fixed cost for the analysis of multi-stage contests. As we showed, the analytical result of a contest with and without fixed costs could be completely different.

Appendix

By the analysis of the subgame-perfect equilibrium, we obtain that in the best-of-three all-pay auction, team 1 wins the contest if it wins the first two games, or if it loses the first game and then wins in the second and in the third games or if it wins in the first and the third games and loses in the second game. In the following, we calculate the teams' probabilities of winning in all these options. Note that if team 1's probability to win the contest is p_1 , then team 2's probability to win the contest is equal to $1 - p_1$ in all the options.

Team 1's probability of winning the contest in Option A (1-1-2)

1. Team 1 wins the first two games, which happens with the probability

$$\left(1 - \frac{2V_2 - V_1 + k_A + FC}{2(V_1 + 2k_H - k_A)}\right) \left(1 - \frac{k_A}{2(V_2 + 2k_H + FC - k_A)}\right)$$

2. Team 1 loses the first game and wins in games 2 and 3, which happens with the probability

$$\left(\frac{2V_2 - V_1 + k_A + FC}{2(V_1 + 2k_H - k_A)}\right) \left(\frac{V_1 - V_2 + k_A - FC}{2(V_2 + k_A)}\right) \left(1 - \frac{V_2 + k_H}{2(V_1 + k_A)}\right)$$

3. Team 1 wins in games 1 and 3 and loses in game 2, which happens with the probability

$$\left(1 - \frac{2V_2 - V_1 + k_A + FC}{2(V_1 + 2k_H - k_A)}\right) \left(\frac{k_A}{2(V_2 + 2k_H + FC - k_A)}\right) \left(1 - \frac{V_2 + k_H}{2(V_1 + k_A)}\right)$$

Hence, team 1's probability to win the contest is:

$$\begin{aligned} p_1^A &= \left(1 - \frac{2V_2 - V_1 + k_A + FC}{2(V_1 + 2k_H - k_A)}\right) \left(1 - \frac{k_A}{2(V_2 + 2k_H + FC - k_A)}\right) \\ (14) &+ \left(\frac{2V_2 - V_1 + k_A + FC}{2(V_1 + 2k_H - k_A)}\right) \left(\frac{V_1 - V_2 + k_A - FC}{2(V_2 + k_A)}\right) \left(1 - \frac{V_2 + k_H}{2(V_1 + k_A)}\right) \\ &+ \left(1 - \frac{2V_2 - V_1 + k_A + FC}{2(V_1 + 2k_H - k_A)}\right) \left(\frac{k_A}{2(V_2 + 2k_H + FC - k_A)}\right) \left(1 - \frac{V_2 + k_H}{2(V_1 + k_A)}\right) \end{aligned}$$

Team 1's probability of winning the contest in Option B (2-1-1)

1. Team 1 wins the first two games, which happens with the probability

$$\left(1 - \frac{2V_2 - V_1 + 2k_A - k_H}{2(V_1 + k_H + FC)}\right) \left(1 - \frac{k_A - FC}{2(V_2 + k_A)}\right)$$

2. Team 1 loses the first game and wins in games 2 and 3, which happens with the probability

$$\left(\frac{2V_2 - V_1 + 2k_A - k_H}{2(V_1 + k_H + FC)} \right) \left(\frac{V_1 - V_2 + 2k_H - k_A}{2(V_2 + k_A + FC)} \right) \left(1 - \frac{V_2 + k_A}{2(V_1 + k_H)} \right)$$

3. Team 1 wins in games 1 and 3 and loses in game 2, which happens with the probability

$$\left(1 - \frac{2V_2 - V_1 + 2k_A - k_H}{2(V_1 + k_H + FC)} \right) \left(\frac{k_A - FC}{2(V_2 + k_A)} \right) \left(1 - \frac{V_2 + k_A}{2(V_1 + k_H)} \right)$$

Hence, team 1's probability to win the contest is:

$$\begin{aligned} p_1^B &= \left(1 - \frac{2V_2 - V_1 + 2k_A - k_H}{2(V_1 + k_H + FC)} \right) \left(1 - \frac{k_A - FC}{2(V_2 + k_A)} \right) \\ (15) &+ \left(\frac{2V_2 - V_1 + 2k_A - k_H}{2(V_1 + k_H + FC)} \right) \left(\frac{V_1 - V_2 + 2k_H - k_A}{2(V_2 + k_A + FC)} \right) \left(1 - \frac{V_2 + k_A}{2(V_1 + k_H)} \right) \\ &+ \left(1 - \frac{2V_2 - V_1 + 2k_A - k_H}{2(V_1 + k_H + FC)} \right) \left(\frac{k_A - FC}{2(V_2 + k_A)} \right) \left(1 - \frac{V_2 + k_A}{2(V_1 + k_H)} \right) \end{aligned}$$

Team 1's probability of winning the contest in Option C (1-2-1)

1. Team 1 wins the first two games, which happens with the probability

$$\left(1 - \frac{2V_2 - V_1 + k_A}{2(V_1 + k_A + FC)} \right) \left(1 - \frac{k_H - FC}{2(V_2 + 2k_A - k_H)} \right)$$

2. Team 1 loses the first game and wins in games 2 and 3, which happens with the probability

$$\left(\frac{2V_2 - V_1 + k_A}{2(V_1 + k_A + FC)} \right) \left(\frac{V_1 - V_2 + k_H}{2(V_2 + k_H + FC)} \right) \left(1 - \frac{V_2 + k_A}{2(V_1 + k_H)} \right)$$

3. Team 1 wins in games 1 and 3 and loses in game 2, which happens with the probability

$$\left(1 - \frac{2V_2 - V_1 + k_A}{2(V_1 + k_A + FC)} \right) \left(\frac{k_H - FC}{2(V_2 + 2k_A - k_H)} \right) \left(1 - \frac{V_2 + k_A}{2(V_1 + k_H)} \right)$$

Hence, team 1's probability to win the contest is:

$$\begin{aligned}
P_1^C &= \left(1 - \frac{2V_2 - V_1 + k_A}{2(V_1 + k_A + FC)}\right) \left(1 - \frac{k_H - FC}{2(V_2 + 2k_A - k_H)}\right) \\
(16) &+ \left(\frac{2V_2 - V_1 + k_A}{2(V_1 + k_A + FC)}\right) \left(\frac{V_1 - V_2 + k_H}{2(V_2 + k_H + FC)}\right) \left(1 - \frac{V_2 + k_A}{2(V_1 + k_H)}\right) \\
&+ \left(1 - \frac{2V_2 - V_1 + k_A}{2(V_1 + k_A + FC)}\right) \left(\frac{k_H - FC}{2(V_2 + 2k_A - k_H)}\right) \left(1 - \frac{V_2 + k_A}{2(V_1 + k_H)}\right)
\end{aligned}$$

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