# Efficient Emissions Reduction*

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**Abstract**

We propose a simple mechanism capable of achieving international agreement on the reduction of harmful emissions to their efficient level. It employs a contest creating incentives among participating nations to simultaneously exert efficient productive and efficient abatement efforts. Participation in the most stylised formulation of the scheme is voluntary and individually rational. All rules are mutually agreeable and are unanimously adopted if proposed. The scheme balances its budget and requires no principal. The mechanism provides a benchmark result for the cost of the implementation of these desirable properties. In a more realistic setup which could potentially inform policy decisions, we discuss participation enforcement through punishment clauses, exclusive trade agreements and environmental standards and show that they are effectively discouraging free-riding. (JEL *C7, D7, H4, Q5.* Keywords: *Climate policy, Contests, Efficiency.*)

## 1 Introduction

The disappointing 2009 Copenhagen Accord has highlighted the international impasse in preventing further global warming.[1] Yet immediate action seems to be called for: Recent research reports rapid declines in ice mass balance from both Greenland and Antarctica with a projected sea-level rise of one to two metres by 2100.[2] Since an estimated 160 million people live currently in locations that lie less than one metre above sea level, this will substantially impact the world economy. This paper studies and answers two questions arising in this context: i) How to provide incentives to reduce harmful emissions to their socially efficient levels while not infracting upon productive efficiency? ii) How much would achieving efficiency cost in terms of GDP?

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[1] The UN draft decision is available at `http://unfccc.int/resource/docs/2009/cop15/eng/l07.pdf`.

[2] See, for instance, Dasgupta, Laplante, Meisner, Wheeler, and Yan (2009) or Allison, Bindoff, Bindschadler, Cox, de Noblet, England, Francis, Gruber, Haywood, Karoly, Kaser, Quéré, Lenton, Mann, McNeil, Pitman, Rahmstorf, Rignot, Schellnhuber, Schneider, Sherwood, Somerville, K.Steffen, Steig, Visbeck, and Weaver (2009). Mitrovica, Gomez, and Clark (2009) predict less uniform sea level changes with a rise of up to six metres at some coastal sites in the northern hemisphere.

Our answers involve a stylised contest among nations rewarding the countries with the highest abatement efforts with some share of global output. This contest provides incentives for the efficient provision of both productive and reductive efforts. Participation in the simplest formulation of the scheme is individually rational, that is, players find it more profitable to participate in the reductive contest than to stay out and free-ride on the agreement. In this formulation, no enforcing sanctions are required, as the players first commit their output share in order to participate in the contest and only then choose their efforts. Under a more realistic, gradual negotiations policy the threat of punitive action against agreement deserters—in the form of punishment clauses, exclusive trade agreements and environmental standards—is shown to avert free-riding.

The main type of emissions we model are greenhouse gases. These are widely seen as the main contributing factor to global warming. Emitted by one country, they are disseminated around the globe regardless of where they were produced. A reduction of their emission benefits all countries but the costs of such reductions are carried individually. This generates a classic free rider problem in which each country would like the threat of global warming removed but none is ready to pay the cost.[3] An example for how we think about productive and reductive (or abatement) efforts is the (simultaneous) investment decision into a power plant's generating capacity and emissions filters.[4]

There are at least three existing approaches to overcome the inefficient emissions abatement problem: command-and-control regulation, quantity-oriented market approaches, and tax-or-pricing regimes. The approach adopted by the 187 signatories of the Kyoto protocol is the quantity-oriented market approach targeting a reduction based on developed countries' emissions in 1990.[5] The treaty, however, failed to obtain ratification by major players including, most prominently, the United States. Moreover, the concern was expressed that developing countries might have ratified the treaty without the intention of keeping emissions in check. This mars the current emissions reduction reality with the dual frustrations of insufficient participation and diluted objectives.

For any international environmental agreement the prevention of free-riding is crucial. In the simultaneous negotiations setup, achieving participation in a symmetric equilibrium does not pose problems. In order to discourage free-riding also under a gradual negotiations policy we propose (a combination of) two strategies. We demonstrate that agreement members can deter free-riding by switching to a pre-agreed and inefficient contract which leaves a deserter with less than the participation utility. Alternatively, we explore the ideas of exclusive trade agreements and environmental standards which exclude a deserter from a fraction of trade within the agreement. Both strategies discourage agreement desertion but the second is more useful in encouraging participation of individual nations after the agreement is formed by a core group. In reality, of course, reaching agreement on and committing to the (structuring of the) necessary transfers and the exact specification of the

[3] One may advocate the view that some countries could climatically benefit from warming. Russian President Vladimir Putin, for instance, is reported to have said that climate change might be good for his country as people would no longer need to buy fur coats (Reuters, 2-April-07). The impact on the world economy and consequences in terms of migration, however, make us pessimistic about the likelihood of emerging net beneficiaries.

[4] See, for instance, the investments of Brandon Shores generation station in emissions reduction documented in Maryland Department of Natural Resources (2007). Other examples include design tradeoffs between engine thrust and emissions in Boeing's 787 Dreamliner or Airbus Industry's A380 aircrafts.

[5] For details, see Barrett (1998).

contest presents a formidable task.[6]

Nordhaus (2006), among others, argues that emission fees or taxes are likely to be more efficient than quantitative quotas given the considerable uncertainty on climate change. As alternative mechanism Nordhaus proposes a harmonised carbon tax leading to efficient reduction efforts in the general spirit of Pigouvian taxation. There are many details like the necessary sanctions, taxation location, trade barriers and transfers to developing countries which are subject to negotiation under such a scheme. A system based on the contest approach, by contrast, does not require agreement on multiple tax locations and implementation techniques, may be self-enforcing and thus easier to negotiate while still sharing the main benefits of a taxation approach.

## 1.1 Related literature

The idea that in many circumstances efficient efforts can be induced by awarding a prize on the basis of a rank order among competitors' efforts is due to Lazear and Rosen (1981). This idea has found numerous applications and extensions, for instance in the work of Green and Stokey (1983), Nalebuff and Stiglitz (1983), Dixit (1987), Moldovanu and Sela (2001), or Siegel (2009). To our knowledge, however, Arbatskaya and Mialon (2010) present the only existing model of multi-dimensional efforts. The literature on sabotage, for instance Münster (2007), is concerned with co-linear, ie. wholly destructive effort pairs directed at the opponents. The idea of behind our efficiency result is close to Gershkov, Li, and Schweinzer (2009) who analyse the efficient single-dimensional effort choice in partnership problems. While our setup is more complicated, much of their methodology still applies. Morgan (2000) and Goeree, Maasland, Onderstal, and Turner (2005) are the only existing analyses of public good provision relating to contests that we are aware of. Morgan (2000) studies a lottery which uses proceeds obtained from ticket sales for the provision of a public good. Contrasting with our market design analysis he is not concerned with designing mechanisms to achieve efficiency. Goeree, Maasland, Onderstal, and Turner (2005) derive the optimal fund-raising mechanism among the class of all-pay auctions. A more distantly related paper is Engers and McManus (2007) who analyse and revenue-rank first & second price all-pay auctions. For a detailed survey of the contests literature see the comprehensive Konrad (2008).[7] enforcing monetary discipline among a group of trading nations seems to be similarly applicable.

Our team setup is vindicated by the universally accepted property of international environmental agreements (IEA) to be self-enforcing. Indeed, there is no supranational principal to enforce such arrangements between countries. Nevertheless, self-enforcement is typically not achieved in the IEA literature.[8] The main contributions, including Barrett (1994), have found that IEA are either unlikely to consist of many participants, or if they do, are similarly unlikely to produce substantial benefits. Moreover, Diamantoudi and Sartzetakis (2006) show that no more than four countries will find it

6 For a discussion of some of the involved problems see, for instance, Wagner (2002) or Liverman (2009). For a recent critique from the political science point of view see Biermann, Pattberg, van Asselt, and Zelli (2009) and the references therein.

7 The contest model we develop may be applied to problems other than climate change (in both the global and local varieties). For instance, the problem of controlling nuclear proliferation or

8 See Barrett (2003), Finus (2008), and Guesnerie and Tulkens (2009) for the main results and further discussion.

profitable to form a coalition regardless of the number of countries participating in the negotiations. Kolstad (2007) demonstrates that the size of IEAs decreases as uncertainty grows. Besides, the outcome of such non-cooperative coalition formation games depends on specific membership rules. For example, Carraro, Marchiori, and Oreffice (2009) show that the introduction of a minimum participation rule increases the number of signatories. Chander and Tulkens (2006) show that, typically, the involved contracts are not renegotiation proof.

For recent contributions to the literature on IEA-membership dynamics see Rubio and Ulph (2007) and Harstad (2010). Particularly referring to Climate Change agreements, Harstad (2010) shows how short term agreements may have adverse effects on countries' investments in green technology. Indeed, as Buchholz and Konrad (1994) and Beccherle and Tirole (2010) point out, anticipating negotiations can decrease the level of R&D and green investments. Our mechanism does not generate such disincentives. Moreover, as it implements single-stage (eg. annual) efficiency, an efficient dynamic extension is trivial.

Barrett (2006) studies an alternative to the Kyoto protocol in proposing a system of two treaties, one promoting cooperative 'breakthrough' R&D investments and the other encouraging collective adoption of new technologies emerging from this R&D activity. This solution improves participation but may not be cost-effective. Carbone, Helm, and Rutherford (2006) argue that even if countries pursue only their self-interest, an international system of trading permits can achieve substantial emission reduction. Our analysis shows, however, that it is unlikely to achieve efficiency. A recent paper which does achieve efficient resource allocations in important cases is Ogawa and Wildasin (2009). They use increased local taxation to encourage the migration of polluters (and pollution) to neighbouring constituencies to achieve efficiency. Many environmental papers employ contests to model lobbying activities; see, for example, Hurley and Shogren (1997) or Heayes (1997) and the references therein. The only environmental contest modelling abatement incentives we are aware of is Dijkstra (2007). He is interested in the time (in)consistency of environmental policy under imperfect government commitment and is not concerned with implementing efficiency.

We depart from the existing literature in two key aspects. First, we design an incentive mechanism which is—in its most stylised form—self-enforcing and induces full participation in a balanced-budget team setup. Second, our scheme is taking into account both productive and reductive efforts. Our key theoretical result is, of course, to obtain efficiency for a broad class of model specifications. Our main contribution is, however, to provide a first benchmark result of the cost for implementing the first-best solution: efficiency in both production and abatement efforts, no distortions, balancing the budget while being widely self-enforcing, and no hold-up problems.[9] Even if the highly stylised mechanism we discuss may seem difficult to implement directly, it delivers new and significant insights on the cost of abatement and on enforcement policies. Following the model definition in section 2, we present the idea of our mechanism through an illustrative example in section 3.1. Although highly stylised, this simple example conveys much of the intuition of the general results presented in section 4. Participation, the asymmetric case, an alternative family of success functions, exclusive

[9] Hold-up problems may arise if countries who already invested in green technology (and thus have lower emissions reduction costs) are asked to bear the lion's share of the burden of collective emissions reduction. As Harstad (2010) points out, this creates disincentives to invest in green technology.

trade agreements, and comparative statics are examined in section 5. All proofs are in the appendix.

## 2 The symmetric model

There is a set $\mathcal{N}$ of $n \geq 2$ risk-neutral players. These players are symmetric in the basic model.[10] Each player $i \in \mathcal{N}$ exerts efforts along two non-verifiable dimensions: productive effort $e_i \in [0, \infty]$ and reductive (abatement) effort $f_i \in [0, \infty]$. We denote the full vectors by $\mathbf{e} = e_1, \ldots, e_n$ and $\mathbf{f} = f_1, \ldots, f_n$, respectively. The combined efforts cost $c(e_i, f_i)$ is assumed to be strictly convex in either argument, additively separable and costless for zero effort in either component. Productive efforts generate strictly concave individual gains of $y(e_i)$ and cause strictly convex global emissions of $m(\max\{0, \sum_h e_h - \sum_h f_h\})$—only depending on the difference between global productive and reductive efforts—of which player $i$ suffers a known share $s_i$.[11] As there is no productive element to $f$, reductive efforts are modelled as 'end of pipe' technology. We interpret the shares $s_i$ as physical pollution and assume that $\sum_h s_h = 1$ to introduce a public bad team problem.[12]

As means to alleviate this problem we use an incentive system which (partially) ranks individual reductive efforts and awards the top-ranked players prizes. The total prize pool is taken to be the sum of fraction $(1 - \alpha)$ of each participant's individual output $y(e_i)$. Thus the mechanism's budget balances by definition. From this total prize pool, a fraction $\beta^1$ is awarded to the winner, $\beta^2$ to the player coming second, and so on, with $\sum_h \beta^h = 1$.

We assume that some noisy (partial) ranking of the players' reductive efforts is observe- and verifiable. It gives player $i$'s probability $p_i^h(\mathbf{f})$ of being awarded prize $h$ as a function of the imperfectly monitored reductive efforts of all participants. We assume that $p_i(\mathbf{f})$ is strictly increasing in $f_i$, strictly decreasing in all other arguments, equal to ${}^1/_n$ for identical arguments, twice continuously differentiable, and zero for $f_i = 0$ with at least one $f_{j \neq i} > 0$, $j \in \mathcal{N}$.[13]

A (subgame perfect) equilibrium in this contest game consists of two elements: a pair of sharing rules $(\alpha, \beta)$ specifying the prizes in the reduction tournament and a pair of efforts $(e, f)$ determining output and the winning probabilities. Since we are implementing efficiency we are looking for a symmetric equilibrium in pure strategies.

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[10] Proposition 4 generalises the model to the asymmetric case. The workings of the asymmetric model are illustrated in several examples in subsections 5.3 and 5.4.

[11] Requiring non-negative differences in the damage function $m(\cdot)$ ensures that reductive efforts cannot substitute productive efforts. Since this requirement is fulfilled for most of our analysis, we redefine $m := m(\max\{0, \sum_h e_h - \sum_h f_h\})$ and only make the non-negative argument explicit when necessary.

[12] In principle, our results also apply to the more general case of $0 < S = \sum_h s_h < n$ in which the individual shares $s_i$ could be interpreted as eg. perceived pollution. Depending on this precise interpretation the planner's objective (1) may then change to $\sum_i (y(ei) - c(e_i, f_i)) - Sm(\sum_i (e_i - f_i))$.

[13] Since this contest success function is general, the reductive efforts determining the contest outcome can be easily normalised with respect to, for instance, the individual (perceived) emission consumption share $s_i$. As usual, this ranking technology can be interpreted as monitoring technology, ie. the slope of the function can be determined, eg. by the frequency of inspections. From a design point of view, the underlying assumption is that higher monitoring precision comes at a higher cost; infinite precision is not attainable.

## 2.1 Timing

Since the players' expected payoffs are symmetric, we can think of a simple proposal game in which the design parameters $\langle \alpha, \beta, p(\mathbf{f}) \rangle$ are proposed by one player and the game is played iff all others simultaneously agree to the proposed parameters. The equilibrium of this game is subgame perfect. In order to allow for more realistic, gradual negotiations, our design is slightly more involved: we propose a two-stage mechanism at the first stage of which an arbitrary player (called player 1) is chosen to propose the two balanced budget contracts $C = \langle \alpha, \beta, p(\mathbf{f}) \rangle$ and $C' = \langle \alpha', \beta', p(\mathbf{f})' \rangle$. The first contract $C$ is invoked if all players agree to participate in the agreement. It implements efficient efforts and is subgame perfect. The second contract $C'$ is invoked by the agreeing players if at least one player fails to participate and implements inefficient efforts which successfully deter non-agreement.[14]

More precisely, at the first stage of the game, if all players accept $C$, then the contest specified by $C$ is set up, players commit their shares $(1-\alpha)y(e_i)$ and the game proceeds to the next stage. If at least one player rejects, the agreeing players form a residual agreement, implement $C'$ and again proceed to the second stage. If less than two players agree to setting up the mechanism $(C, C')$, then the game ends and each player obtains their individual utility without agreement. At the second stage, conditional on the formation of an unanimous agreement, players choose their efforts simultaneously to maximise own expected utilities. The noisy ranking of reductive efforts specifies a winner, second, etc, and final output realises. The prize pool is then redistributed among the participants: the winner obtains the share $(1-\alpha)\beta^1$ of total output, the player coming second gets $(1-\alpha)\beta^2$, and so on. Similarly, if at least one player decides not to participate, then the contest specified by $C'$ is implemented by the agreeing players.

One of the main stumbling blocks for IEAs is the participants' commitment. Countries may sign the agreement but no supranational entity exists which punishes defection. Thus countries can always choose inefficient efforts and free ride once the agreement is signed, hoping that other countries will not. In our setup, this is discouraged as the players who agree to $C$ are able to endogenously punish any desertion by implementing $C'$. It is easy to see that a sufficiently strong punishment contract always exists: setting $C' = \langle \alpha' = 1, \cdot, \cdot \rangle$ replicates the pre-agreement scenario in which all players are worse off than with an agreement. This extreme form of punishment, however, will typically not be necessary. A second-best contract $C'$ will generally be able to implement higher levels of abatement than those realising absent an IEA.[15]

[14] Formally, this second problem is equivalent to allowing a signatory to exit the agreement (ie. renege on his commitments) after the agreement is formed. As pointed out by Chander and Tulkens (2006), this contract will typically not be renegotiation proof and commitment to $C'$ is crucial. We discuss more realistic enforcement measures in section 5.2.

[15] Designing $C'$ just sufficiently bad to serve as a deterrent resembles the idea of $\gamma$-core stability in Chander (2007). An alternative way of deterring this kind of free-riding is to grant most favoured 'green' trading terms only to participating nations. Both ideas are further explored in subsection 5.2.

## 3 Efficiency benchmark

Much of the intuition behind our results can be understood from the simple symmetric two players case on which the body of the paper rests.[16] For this case we write $i = 1, 2$ and $j = 3 - i$. We define the efficient levels of both productive and reductive efforts $(e^*, f^*)$ as those maximising social welfare absent of incentive aspects

$$\max_{(\mathbf{e},\mathbf{f})} u(\mathbf{e}, \mathbf{f}) = 2y(e) - m\,(2e - 2f) - 2c(e, f) \;\Leftrightarrow \begin{cases} y'(e^*) = m'(2e^* - 2f^*) + c_e(e^*, f^*), \\ m'(2e^* - 2f^*) = c_f(e^*, f^*). \end{cases} \tag{1}$$

In the absence of an incentive scheme, a participating player $i = 1, 2$ individually maximises

$$y(e_i) - s_i m(e_i + e_j - f_i - f_j) - c(e_i, f_i) \;\Leftrightarrow \begin{cases} y'(e^*) = s_i m'(2e^* - 2f^*) + c_e(e^*, f^*), \\ s_i m'(2e^* - 2f^*) = c_f(e^*, f^*). \end{cases} \tag{2}$$

where $s_i$ is player $i$'s local share of global emissions. Since $s_i + s_j = 1$, the individual focs in (2) cannot both equal those in (1). This, in a nutshell, is the argument used by Holmström (1982) to show that efficient unverifiable efforts are impossible in a partnership production problem. Introducing an endogenised rank-order emissions reduction reward scheme attaching weights $\beta$ and $(1 - \beta)$ to the players coming first and second, respectively, the individual problem changes to

$$\max_{(e_i, f_i)} \alpha y(e_i) + \sum_h p^h(\mathbf{f})\beta^h(1 - \alpha)\left(y(e_i) + y(e_j)\right) - s_i m(e_i + e_j - f_i - f_j) - c(e_i, f_i), \tag{3}$$

where the prize pool is taken as a share $(1 - \alpha)$ from individual productive output. We define individual rationality as the requirement that the utility from efficient effort provision in (3) exceeds the utility from non-formation of the agreement (2), of free-riding on the others' reductive efforts *within* the agreement and on free-riding on the others' reductive efforts *outside* the agreement.

[16] Proposition 4 together with subsection 5.3 extend this intuition to the full asymmetric setup.

### 3.1 Example of the efficient mechanism

We use a simple symmetric example with quadratic costs and square root production to demonstrate the basic idea of our model.[17] In this framework, substituting into the planner's objective (1) gives

$$\max_{(e,f)} 2e^{1/2} - (2e-2f)^2 - 2(e^2+f^2) \Leftrightarrow \begin{cases} e^* = \dfrac{\left(\frac{3}{5}\right)^{2/3}}{2\times 2^{1/3}} \approx 0.2823, \\ f^* = 5^{-2/3}6^{-1/3} \approx 0.1882. \end{cases} \tag{4}$$

The corresponding individual problem is to

$$\max_{(e_i,f_i)} e_i^{\frac{1}{2}} - s_i(e_i+e_j-f_i-f_j)^2 - (e_i^2+f_i^2) \quad \Leftrightarrow \begin{cases} \hat{e}_i &= \dfrac{\left(\frac{1+2s_i}{1+4s_i}\right)^{2/3}}{2\times 2^{1/3}} > e^*, \\ \hat{f}_i &= \dfrac{s_i}{(2+20s_i+64s_i^2+64s_i^3)^{1/3}} < f^*. \end{cases}$$

For the present example we assume that the probability of winning the reduction award is given by the Tullock success function.[18] Under this incentive scheme, the individual problem is to max

$$\alpha e_i^{\frac{1}{2}} + \frac{f_i^r}{f_i^r+f_j^r}\beta(1-\alpha)(e_i^{\frac{1}{2}}+e_j^{\frac{1}{2}}) + \frac{f_j^r}{f_i^r+f_j^r}(1-\beta)(1-\alpha)(e_i^{\frac{1}{2}}+e_j^{\frac{1}{2}}) - s_i(e_i+e_j-f_i-f_j)^2 - (e_i^2+f_i^2)$$

which gives the set of simultaneous focs as

$$16e_i = 8f_i + \frac{1+\alpha}{\sqrt{e_i}},\ 2e_i = 4f_i + \frac{\sqrt{e_i}r(\alpha-1)(2\beta-1)}{2f_i}. \tag{5}$$

Setting $e = e_i = e^*$, $f = f_i = f^*$ and solving for symmetric efforts (under symmetric $s_i = 1/2$) gives the efficiency inducing design parameters

$$\alpha^* = \frac{3}{5},\ \beta^* = \frac{1}{2} + \frac{1}{6r}. \tag{6}$$

Notice that $\beta^*$ only depends on $r$. Thus, the rewards scheme—and in particular the relative size of the prizes paid to the winner and loser—can be designed as seen fit and allowed by equilibrium existence. The mechanism satisfies 'limited liability' if $r \geq 1/3$. There are well-known existence issues of symmetric pure strategy equilibria with $r > 2$ in standard contests (see, eg. Schweinzer

[17] Jesuit missionary Paul Le Jeune testifies to the convexity of damage cost in his 1635 experiences during a winter hunting party on which he accompanied native Canadians. The bitter cold made it necessary to light fires inside the Indians' hunting cabin, "but, as to the smoke, I confess to you that it is martyrdom. It almost killed me, and made me weep continually, although I had neither grief nor sadness in my heart. It sometimes grounded all of us who were in the cabin; that is, it caused us to place our mouths against the earth in order to breathe. For, although the Savages were accustomed to this torment, yet occasionally it became so dense that they, as well as I, were compelled to prostrate themselves, and as it were to eat the earth, so as not to drink the smoke. I have sometimes remained several hours in this position, especially during the most severe cold and when it snowed; for it was then the smoke assailed us with the greatest fury, seizing us by the throat, nose, and eyes."

[18] The particular monitoring technology is not important as we generalise over the set of applicable success functions in section 5.5. What is important is that the success function incorporates enough randomness in its outcome. If the ranking is too precise (as is the case with the all-pay auction) then equilibria in pure strategies typically fail to exist. This would be problematic as our contest strives to implement the efficient pure effort choices.

and Segev (2011)) but, as shown in proposition 2, these do not apply with the same severity to our problem. Figure 1 shows that participating in the contest gives higher utility than staying out and free-riding on the other's effort. It confirms $(\alpha^*, \beta^*, e^*, f^*)$ as unique equilibrium in pure strategies.[19]

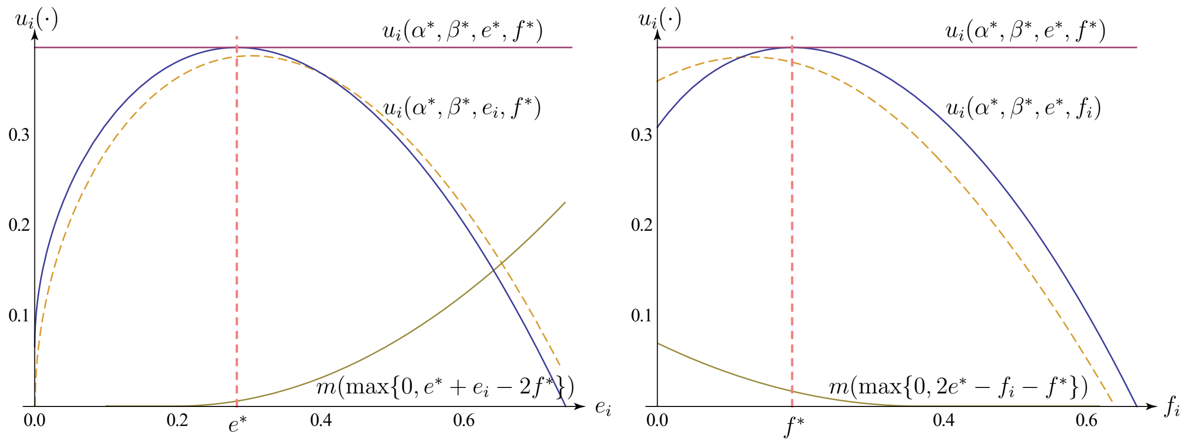


Figure 1: The top, horizontal line is the equilibrium utility from $(\alpha^*, \beta^*, e^*, f^*)$. The curves below show the utility from unilaterally deviating in either effort dimension. Notice the positive utility from free-riding at zero efforts. The dashed lines give the (outside) utility from no agreement formation.

The economics behind this result is simple: An increase in productive efforts $e_i$ causes individual output $y(e_i)$ and global pollution $m(\sum_h e_h - \sum_h f_h)$ to rise. Of these, the player retains shares $\alpha$ and $s_i$, respectively. An increase in reductive efforts $f_i$ enlarges the player's chance to win the prize share $\beta$ in the reduction contest and simultaneously decreases global pollution. Trading off $\alpha$ against $\beta$ allows us to fine-tune efforts to their efficient levels.

# 4 Results

Recall that under the contest scheme, an individual $i = 1, 2$ chooses a pair of efforts $(e_i, f_i)$ to max

$$\alpha y(e_i) + (1-\alpha)\left(\beta p(\mathbf{f}) \sum_j y(e_j) + (1-\beta)(1-p(\mathbf{f})) \sum_j y(e_j)\right) - s_i m\left(e_i + e_j - f_i - f_j\right) - c(e_i, f_i)$$

where $p(\mathbf{f})$ is the probability of coming first in a ranking of reductive efforts $f$. We require that $y' > 0$, $y'' < 0$, $m' > 0$, $m'' > 0$, and $c'_{1,2} > 0$, $c''_{1,2} > 0$. We moreover assume that $m(\cdot)$ only depends on the difference of total productive minus reductive efforts and that the cost function is additively separable in both types of efforts. Taking derivatives wrt both effort types, we obtain the simultaneous pair of focs defining individually optimal efforts $(e_i, f_i)$ as

$$c_e(e_i, f_i) + s_i m'(e_i + e_j - f_i - f_j) = (1 - \beta + \alpha\beta + (1-\alpha)(2\beta - 1)p(f_i, f_j))y'(e_i)$$
$$c_f(e_i, f_i) + (\alpha - 1)(2\beta - 1)(y(e_i) + y(e_j))p'(f_i, f_j) = s_i m'(e_i + e_j - f_i - f_j).$$

[19] Since the objective is fully separable, it is sufficient to investigate optimality of the two-dimensional problem along both effort dimensions separately. As long as this remains true, relaxing our full separability of costs assumption is innocuous.

Assuming that a symmetric equilibrium $e = e_i = e_j$, $f = f_i = f_j$, $s_i = {}^1\!/_2$ exists, this simplifies to

$$\begin{aligned} 2c_e(e,f) + m'(2e-2f) &= (\alpha+1)y'(e) \\ 2c_f(e,f) - m'(2e-2f) &= 4(1-\alpha)(2\beta-1)p'(f,f)y(e). \end{aligned} \tag{7}$$

Equating these efforts to those in (1), we obtain

$$4p'(\mathbf{f}^*)(2\beta-1) = \frac{y'(e^*)}{y(e^*)} \Leftrightarrow \begin{cases} c_e(e^*,f^*) = \alpha y'(e^*), \\ c_f(e^*,f^*) = 4(1-\alpha)(2\beta-1)p'(\mathbf{f}^*)y(e^*) \end{cases} \tag{8}$$

where $\mathbf{f}^* = (f^*, f^*)$. Efficiency can be obtained as we know from (1) that there exists an $\alpha \in [0,1]$ to satisfy the first equation. Substituting this $\alpha$ into the second equation determines $\beta \in [{}^1\!/_2, 1]$ for a suitably chosen ranking $p(\cdot)$. Without further restrictions on the design parameters—and in particular the slope of the ranking technology $p(\cdot)$ in equilibrium—(8) can always be accomplished. Taking equilibrium existence as given (until we verify it in proposition 2), the following proposition establishes the precise criteria on the parameters for both productive and reductive efficiency to obtain simultaneously for any number of players $n \geq 2$. In all following results we employ the simple prize structure $\beta = \left(\beta^1, \frac{1-\beta^1}{n-1}, \ldots, \frac{1-\beta^1}{n-1}\right)$ assigning a single winning prize and another prize to all losers. This is not necessary but considerably simplifies the exposition.

**Proposition 1.** *For appropriately chosen $\langle \alpha, \beta, p(\mathbf{f}) \rangle$, player $i \in \mathcal{N}$ chooses efficient productive as well as reductive efforts $(e^*, f^*)$ in*

$$\max_{(e_i,f_i)} \alpha y(e_i) + (1-\alpha)\sum_h \left(\beta^h p^h(\mathbf{f}) \sum_j y(e_j)\right) - s_i m\left(\sum_h e_h - \sum_h f_h\right) - c(e_i, f_i). \tag{9}$$

All proofs can be found in the appendix. It is straightforward to show that proposing $C = \langle \alpha, \beta, p(\mathbf{f}) \rangle$ at the first stage of the game maximises player 1's expected utility, given that players' efforts are functions of the proposed mechanism $(e(\alpha, \beta, p), f(\alpha, \beta, p))$. This is unsurprising as payoffs are symmetric and what maximises welfare must also maximise the proposer's utility.[20]

A consequence of the previous result is that full efficiency in the symmetric $n$-player model can be obtained with just two different prizes: one for the winner and another for everyone else. As one only needs to check for a winner, such a scheme is easy to monitor. Since the general objective is not necessarily well behaved without further assumptions on $p(\cdot)$, we proceed to show that equilibria exist for the subclass of problems governed by the Tullock success function $p_i(f) = f_i^r / \sum_j f_j^r$.

**Proposition 2.** *The provision of efficient efforts $(e^*, f^*)$ is globally optimal provided that reduction costs are sufficiently convex. In particular, the sufficient threshold (24) assures the existence of a symmetric pure strategy equilibrium for contests governed by the Tullock success function.*

Equilibrium existence implies that free-riding is not attractive once a nation is committed to the agreement. As the number of participants in the mechanism $n$ goes up, the utility from free-riding

[20] The design of the proposal stage is less straightforward in the asymmetric case.

increases as the disutility from pollution $m(\sum_h (e_h - f_h))$ approaches the efficient level. Hence the only leverage left in the efficient contract $C$ is the contest on the pre-committed output share of $(1-\alpha)$—which is generally not sufficient to deter free-riding once an agreement is in place. The alternative contract $C'$ is, however, capable of eradicating all gains from free-riding on the agreement by—in its most extreme form—replicating non-agreement pollution levels.

**Proposition 3.** *Participation in the mechanism specifying the pair of contracts $C = \langle \alpha^*, \beta^*, p^*(\mathbf{f}) \rangle$ determined through (9) and $C' = \langle \alpha' = 1, \beta' = {}^1/_2, \cdot \rangle$ is individually rational in the sense that the utility from free riding efforts $e^s, f^s$ on $C'$ cannot exceed the utility obtained when agreeing to $C$*

$$y(e_i^s) - s_i m(e_i^s + (n-1)e'(\alpha', \beta', \cdot) - f_i^s - (n-1)f'(\alpha', \beta', \cdot)) - c(e_i^s, f_i^s) \leq u_i(e^*, f^*). \quad \text{(IR)}$$

The result is intuitive as the efficient allocation maximises welfare and the agreement will therefore always be formed. As participation in the agreement is individually rational, free-riding is fully deterred. Off the equilibrium path, the second-best contract $C'$—which is implemented if at least one player fails to participate—will generally still allow substantial emissions reductions. An example in the following section shows that the agreement's raison d'être need not necessarily be surrendered to holdup attempts.

Finally, we show that our efficiency result is not an artifact of our symmetry assumptions. The result is presented for any number of players $n \geq 2$ and identity dependent shares $(\alpha_i, \beta_i)$. An illustrative example and further discussions of the asymmetric case can be found in subsection 5.3.

**Proposition 4.** *Let $i \in \mathcal{N}$ and $n \geq 2$. For appropriately chosen $\langle \alpha_i, \beta_i; p(\mathbf{f}) \rangle_i^n$, prize pool $P = \sum_{j=1}^n (1-\alpha_j) y_j(e_j)$, and prize structure $\left(\beta_i, \frac{1-\beta_i}{n-1}, \ldots, \frac{1-\beta_i}{n-1}\right)$, efficient solutions exist to player $i$'s asymmetric problem*

$$\max_{(e_i, f_i)} \alpha_i y_i(e_i) + p_i^1(\mathbf{f}) \beta_i P + \sum_{i \neq j} p_j^1(\mathbf{f}) \left( \frac{1-\beta_j}{n-1} \right) P - s_i m \left( \sum_{i=1}^n e_i - f_i \right) - c_i(e_i, f_i). \quad (10)$$

Although we demonstrate most of our results in a simplified symmetric setup, the previous propositions show that this can be done without loss of generality. The following section confirms this in examples and also shows that our results are robust to the choice of contest success function. Together with the above results, this demonstrates that the proposed contest mechanism indeed provides incentives for efficient provision in a general setup.

# 5 Extensions and robustness

## 5.1 Second-best participation

Our argument in proposition 3 uses the maximal threat $C' = \langle \alpha' = 1, \beta', p(\mathbf{f})' \rangle$ to show that free-riding can always be discouraged. This extreme case, however, renders the agreement wholly ineffective if a punishment becomes necessary. The purpose of this subsection is to show that such severe measures are not generally needed. Typically, the punishment of a deserter leaves enough freedom to increase abatement levels over those realising under no agreement. As outlined in the previous sections, contract $C$ implements efficient efforts. Consider now a deviation by some player which triggers $C' = \langle \alpha', \beta', p(\mathbf{f})' \rangle$. Denote the equilibrium agreement utility attained by adhering to $C'$ by $u'(\cdot)$ and the corresponding equilibrium efforts by $e'(\alpha', \beta', p'), f'(\alpha', \beta', p')$. By inflicting sufficient damage through $m(\cdot)$, we need to ascertain that free riding utility $u_i^s(e_i^s, f_i^s)$—with the agreement members adhering to $C'$—is smaller than what participation in $C$ gives, that is

$$u_i^s(e_i^s, f_i^s|C') = y(e_i^s) - s_i m(e_i^s + (n-1)e'(\cdot) - f_i^s - (n-1)f'(\cdot)) - c(e_i^s, f_i^s) \leq u_i(e^*, f^*|C).$$

Since (4) implies that efforts $e', f'$ are monotonic in $\alpha', \beta'$, payoff $u_i^s(\cdot|C')$ is continuous in $\alpha'$ and $\beta'$. Hence there exists an $\alpha' \in (\alpha, 1]$ which ensures the above inequality for suitable $\beta'$ and $p'(\mathbf{f})$. Consider the case of $n+1$ players in the example setup of section 3.1. Then, full participation efficient efforts are given by

$$e^* = \frac{n+2}{2 \times 2^{1/3} \left((2+n)(3+2n)^2\right)^{1/3}}, \; f^* = \frac{n+1}{2 \times 2^{1/3} \left((2+n)(3+2n)^2\right)^{1/3}}$$

which are implemented by

$$\alpha^* = \frac{4\sqrt{e^*}(2e^* - f^*)(n+1) - 1}{n}, \; \beta^* = \frac{1}{n+1} + \frac{2(e^* - 2f^*)f^*}{\sqrt{e^*} r(\alpha - 1)}$$

for the Tullock success function parameterised by $r$. This determines $u_i(e^*, f^*|C)$. For the deviation utility $u_i^s(e_i^s, f_i^s|C')$, the deviation efforts $e^d, f^d$ are determined by the focs

$$2e^s = \frac{1}{2\sqrt{e^s}} + \frac{2(f^s + n(f'(\cdot) - e'(\cdot)) - e^s)}{n+1}, \; f^s = \frac{e^s + n(e'(\cdot) - f'(\cdot))}{n+2}$$

where $e'(\cdot), f'(\cdot)$ are the agreement equilibrium efforts in the agreement under $C'$. These functions $e'(\cdot), f'(\cdot)$, and therefore the damage they inflict on the deviator through $m(e^s + ne' - f^s - nf')$, are determined by

$$\alpha' = \frac{n\left(4\sqrt{e'}(e' + e^s - f^s + 2e'n - f'n) - 1\right) - 1}{n^2 - 1},$$

$$\beta' = \frac{\left(e'^2(4+8n) - 2f'(n-1)(f' + f^s + 2f'n - e^s) + 2e'(2e^s - 2f^s + f'(n-3)n) - \sqrt{e'}(n+1)\right)}{\left(4e'^2 n(1+2n) - 4e'n(-e^s + f^s + fn) - \sqrt{e'}n(1+n)\right)}.$$

In our example setup, it turns out that participation is individually rational for any number of players. The details for the simplest case of three players (two in the agreement, one outside) are

$$e' = 0.302269, f' = 0.195276, e^s = 0.314075, f^s = 0.132016, (e^* = 0.273276, f^* = 0.204957)$$

for $C' = \langle \alpha' \approx 0.910124, \beta' = 1, r' = 1 \rangle$. If there is no agreement, efforts are $e^d = 0.30286, f^d = 0.15142$, so the 29% increased abatement efforts achieved by the agreement are substantial.

## 5.2 Exclusive trade agreements and enforcing standards

The purpose of this subsection is to show that a simple way to deter free-riding of individual nations is to grant most favoured 'green' trading terms only to participating nations. Similarly, environmental certification conditional on treaty commitment can be a powerful complementary tool to enforce participation in the IEA. The expansion of equilibrium abatement efforts from their no agreement level $f^0$ to the level within the agreement $f^*$ creates 'green' products; the higher abatement efforts, the greener is productive output. The idea is to label free-riding countries' products and thereby creating (political) incentives to respect commitments to the IEA. Consequently, firms' lobbying against environmental regulation may result in that country's desertion followed by the IEA labelling its goods. We thus propose a negative label which signals a product lacking the 'green' environmental standards enforced by the IEA.[21]

This idea can be formalised in our setup by decreasing the value of a deserting country's output. We assume that—once certified as environmentally unfriendly—a consumer's willingness to pay for labelled products decreases[22]. Therefore the revenue generated from the production of labelled products sinks as these products suffer a decrease in price of $x(\mathbf{f}) \in [0, 1]$. This fraction corresponds to the deserter's deviation from agreed abatement levels. Denoting the outside equilibrium efforts of a single deserter by $(e^d, f^d)$, desertion utility is

$$u_i^d(e_d, f_d) = \underbrace{\left(\frac{f^* - f_i^0}{f^*}\right)}_{=x\in[0,1]} y(e_i^d) - s_i m(e_i^d + (n-1)e^* - f_i^d - (n-1)f^*) - c(e_i^d, f_i^d) \qquad (11)$$

for $i \in \mathcal{N}$ and $n > 2$ (since there must be at least two players left in the agreement after $i$ deserts). A sufficiently large fraction $x$ will successfully deter free riding and can be seen as alternative to the blunt global threat represented by contract $C'$.[23]

A further step in this direction is the formation of an exclusive trade agreement. If a deserter can be excluded from the fraction of trade corresponding to the necessary abatement investments within the agreement, then individual desertion can be again discouraged. As above, green production—generated by the expanded abatement effort—is traded among countries and produces wealth. Our model does not take into account the international trading aspects of production and thus there is no direct way of measuring the involved consequences to individual wealth.[24] A simple (ad-hoc) way of nevertheless capturing the idea of enforcement through an exclusive trading agreement is to restrict trade on (and therefore capitalising on) the fraction of productive output corresponding to

[21] There are many green labelling examples: UK supermarket chain Tesco has recently introduced a promotional campaign on carbon labels. US Walmart and French Casino have similar ambitions. Examples of negative labelling campaigns are the mandatory GMO labelling implemented in Europe, and "we're Greenpeace, and we want a fresh green Apple" targeted at US computer maker Apple. Grankvist, Dahistrand, and Biel (2004) argue that negative labelling may have a higher consumption impact than positive labelling. Engel (2004) underlines the necessity to inform consumers, especially when a firm is found cheating on its environmental claims.

[22] Models on certification and standard settings have been studied intensely; see for instance Lerner and Tirole (2006) or Harbaugh, Maxwell, and Roussillon (2006)

[23] To some extent, the particular choice of $x(\mathbf{f})$ is arbitrary. Any function of abatement efforts implementing sufficient deterrence (such as the function used for the exclusive trade example below) could be used instead.

[24] Modelling both these aspects formally is possible and certainly provides grounds for future research.

the reductive investments within the treaty to agreement members. Then player $i$'s desertion utility in (11) can be reduced by using

$$x(\mathbf{f}) = \left(1 - \frac{f^* - f_i^0}{e^*}\right) \tag{12}$$

where reductive effort $f_i^0$ is the equilibrium abatement level without agreement. As indicated above, to some extent this choice is arbitrary.[25] The intuition for (12) is that an agreement defector $j$ can free ride on the reductive efforts of agreement members (through a cleaner environment) but is punished by restricted access to the agreement market consisting of the tradables $\sum_{h \neq j} y(e_h)$.

Returning to the simple quadratic cost, square-root production example of example section 3.1, this implies that it is individually rational to participate in the agreement if

$$u_i(e^*, f^*) \geq x(\mathbf{f}) e_i^{\frac{1}{2}} - \frac{1}{n}\left(e_i + (n-1)e^* - f_i - (n-1)f^*\right)^2 - e_i^2 - f_i^2.$$

Solving the deserter's maximisation problem (on the rhs) for the exclusive trade agreement under (12) leads to the focs

$$2f^* + \frac{e^* - f^* + f_i^0}{2e^* \sqrt{e_i^d}} = \frac{2(e_i^d + f^* - f_i^d + e^*(n-1) + e_i^d n)}{n}, \; f_i^d = \frac{e_i^d + (e^* - f^*)(n-1)}{n+1}$$

where $(e^*, f^*)$ are the equilibrium effort levels provided inside the agreement. Plotting the utilities from the desertion efforts $(e_i^d, f_i^d)$ solving above focs results in a graph similar to figure 1 for the case of $n = 3$ showing that deviations are not profitable. Thus, the exclusive trade agreement ensures participation. (The analysis is nearly identical for the labelling setup discussed above and therefore not replicated.) As the severity of punishment $(1 - x(\mathbf{f}))$ in (12) is given by

$$\frac{f^* - f_i^0}{e^*} = \frac{6 - 3^{1/3}(-2-4n)^{2/3}(-1-n)^{1/3}}{6(1+n)}$$

which is increasing in $n$, the punishment gets more severe for larger $n$ and participation is easier to obtain in the general case. (The limit as $n \to \infty$ equals $2^{1/3}/3^{2/3} \approx .606$.)

## 5.3 Asymmetries

Following the general strategy outlined in proposition 4 we use identity (class) dependent shares $(\alpha_i, \beta_i)$ and prize structures $\left(\beta_i, \frac{1-\beta_i}{n-1}, \ldots, \frac{1-\beta_i}{n-1}\right)$. Consider the following three players example where we parameterise player $i = 1, 2, 3$'s cost wrt productive and reductive efforts by the pair of scalars

[25] If monitoring of the deserter's abatement efforts is good, then the actual level $f_i^d$ could be used. (In the present example setup, this leads to a rather unintuitive corner solution.) The idea, however, seems important for dealing with competing abatement agreements: As long as they are effective, there is no reason to punish them.

$(\gamma_i, \delta_i)$.[26] The planner's objective in the otherwise unchanged example setup is then to

$$\max_{(e_1,f_1,e_2,f_2,e_3,f_3)} \sum_i e_i^{1/2} - \left(\sum_i (e_i - f_i)\right)^2 - \sum_i \gamma_i e_i^2 - \sum_i \delta_i f_i^2. \tag{14}$$

For total prize pool $P = \sum_i (1-\alpha_i) e_i^{1/2}$, individuals $i = 1, 2, 3$ choose $(e_i, f_i)$ to maximise

$$\begin{aligned}
\alpha_1 e_1^{\frac{1}{2}} + \tfrac{f_1^r}{\sum_h f_h^r}\beta_1 P + \left(1 - \tfrac{f_1^r}{\sum_h f_h^r}\right)\left(\tfrac{f_2^r}{f_2^r+f_3^r}\tfrac{1-\beta_2}{2} + \tfrac{f_3^r}{f_2^r+f_3^r}\tfrac{1-\beta_3}{2}\right) P - s_1\left(\textstyle\sum_h (e_h - f_h)\right)^2 - \gamma_1 e_1^2 - \delta_1 f_1^2,\\
\alpha_2 e_2^{\frac{1}{2}} + \tfrac{f_2^r}{\sum_h f_h^r}\beta_2 P + \left(1 - \tfrac{f_2^r}{\sum_h f_h^r}\right)\left(\tfrac{f_1^r}{f_1^r+f_3^r}\tfrac{1-\beta_1}{2} + \tfrac{f_3^r}{f_1^r+f_3^r}\tfrac{1-\beta_3}{2}\right) P - s_2\left(\textstyle\sum_h (e_h - f_h)\right)^2 - \gamma_2 e_2^2 - \delta_2 f_2^2,\\
\alpha_3 e_3^{\frac{1}{2}} + \tfrac{f_3^r}{\sum_h f_h^r}\beta_3 P + \left(1 - \tfrac{f_3^r}{\sum_h f_h^r}\right)\left(\tfrac{f_1^r}{f_1^r+f_2^r}\tfrac{1-\beta_1}{2} + \tfrac{f_2^r}{f_1^r+f_2^r}\tfrac{1-\beta_2}{2}\right) P - s_3\left(\textstyle\sum_h (e_h - f_h)\right)^2 - \gamma_3 e_3^2 - \delta_3 f_3^2,
\end{aligned}$$

The result for $(\gamma_1 = 1, \delta_1 = 1, \gamma_2 = 2/3, \delta_2 = 3/4, \gamma_3 = 1/3, \delta_3 = 1/2)$ and $s_i = 1/3$ leads to the efficiency inducing asymmetric shares of

$$\begin{aligned}
\alpha_1 = 0.618533, \beta_1 = 0.511359, \alpha_2 = 0.552969, \beta_2 = 0.549507, \alpha_3 = 0.312778, \beta_3 = 0.711638,\\
e_1^* = 0.273795, f_1^* = 0.203984, e_2^* = 0.338534, f_2^* = 0.271979, e_3^* = 0.475588, f_3^* = 0.407969.
\end{aligned}$$

In the remainder of this subsection, we extend the example setup of section 3.1 with unequal relative damage shares $s_i \in (0,1)$, $i = 1, 2$. Since shares sum to 1, both efficient effort types are still given by (4). Player $i's$ problem is unchanged and imposing efficiency (4) we obtain the shares

$$\alpha^* = \frac{1}{5}(1 + 4s_i), \ \beta^* = \frac{1}{2} + \frac{1}{6r}. \tag{15}$$

Notice that only $\alpha^*$ turns out to depend on the player's identity (class), the efficiency-inducing prize structure $\beta$ is identical to the symmetric case. As to be expected, the share $(1-\alpha^*)$ of output which has to be committed to the contest gets arbitrarily small when the public bad problem disappears as $s_i$ approaches 1. On the other extreme, a player who does not suffer from the effects of global warming at all must be asked to commit close to $4/5$ of her output to the contest in order to induce efficient efforts on her behalf. A numerical example taking relative damage shares of $s_1 = 1/4$, $s_2 = 3/4$ requires $\alpha_1 = 0.4$ and $\alpha_2 = 0.8$ in order to implement efficiency. In the general case of $n > 2$ players with damage shares parameterised by $s_i = \frac{2i}{n+n^2}$, $i = 1, 2, \ldots, n$, with $\sum_{i=1}^n s_i = 1$. Efficient efforts are then given by

$$4e(1+n) = \frac{1}{\sqrt{e}} + 4fn, en = f(1+n) \ \Leftrightarrow \left\{ \begin{aligned} e^* &= \frac{1+n}{2 \times 2^{1/3}\left((1+n)(1+2n)^2\right)^{1/3}},\\ f^* &= \frac{n}{2 \times 2^{1/3}\left((1+n)(1+2n)^2\right)^{1/3}}. \end{aligned} \right.$$

[26] The two players asymmetric case in the example setup is rather special. There is no problem in solving it but the symmetry induced by the small number of players requires success function slopes of precisely

$$\frac{\partial}{\partial f_i} p_i(\mathbf{f}) = \frac{(1-s_i) c_{f_i}(e_i, f_i)}{(\beta_1 + \beta_2 - 1)P}. \tag{13}$$

Cases with higher numbers of players than two induce no such complications.

Solving the $n$-player individual asymmetric problem in the example setup under the two-part price structure $\left(\beta, \frac{1-\beta}{n-1}, \ldots, \frac{1-\beta}{n-1}\right)$ employed previously

$$\alpha e_i^{1/2} + \frac{f_i^r}{f_i^r + (n-1)(f^*)^r}\beta P + \left(1 - \frac{f_i^r}{f_i^r + (n-1)(f^*)^r}\right)\left(\frac{1-\beta}{n-1}\right)P - \\ s_i(e_i + (n-1)e^* - f_i - (n-1)f^*)^2 - (e_i^2 + f_i^2)$$

for $P = (1-\alpha)(e_i^{1/2} + (n-1)(e^*)^{1/2})$ results in the intimidating but straightforward

$$\alpha^* = \frac{\sqrt{\frac{1+n}{((1+n)(1+2n)^2)^{1/3}}}(1+n+ns_i)n - ((1+n)(1+2n)^2)^{1/3}}{(n-1)\left((1+n)(1+2n)^2\right)^{1/3}},$$

$$\beta^* = \frac{(n-1)n(1+2n)^2\left(\frac{1+n}{((1+n)(1+2n)^2)^{1/3}}\right)^{3/2} + n\left(n^2-1\right)(1+n+ns_i) + 2(1+n)^2 r(2+n(3+s_i))}{2n(1+n)^2 r(2+n(3+s_i))}.$$

A numerical example for $n = 187$, $r = 3$ gives $\alpha^* = 0.50133$, implying for 'type' $s_i = 1/n$ an efficiency-inducing redistribution vector of $\left(\beta^* = 0.17024, \frac{1-\beta^*}{n-1} = 0.00446, \ldots, \frac{1-\beta^*}{n-1} = 0.00446\right)$ which compares to the flat $1/n = 0.00534$. Under the contest, type $s_i = 1/n$ gives up roughly 50% of her output but gets back 41.6% even if losing the contest. She gets almost 16 times her output if she wins. Notice that if equilibrium existence allows this can be further equalised by employing a more precise ranking and thereby increasing $r$.

## 5.4 Comparative statics

In this subsection we compare the wealth distribution without an international environmental agreement (IEA) to the distribution under the proposed contest scheme. The three players example of the previous subsection suggests that the most productive countries (with the lowest $\gamma, \delta$ in (14)) will exert higher efforts of both types in equilibrium—as would be the case without an IEA. As is the case for taxes achieving emissions reduction, effectiveness is ensured at the smallest possible cost.

The output share $\alpha$ and the prize pool share $\beta$ are mainly determined by the relative efficiency of the different countries' reductive efforts. The country with the lowest reductive effort cost (lowest $\delta$) is willing to contribute the most to the prize pool—smallest $\alpha$—accepting unequal prize pool shares—high $\beta$—in order to achieve efficient efforts in the output and reductive dimensions. Indeed, improved reductive efforts increase the probability of winning the contest, thus augmenting the country's willingness to participate in the contest and also the desired prize pool share.

The relative efficiency of productive efforts influences the proposed scheme in a smooth fashion. Low output costs induce higher prize pool contributions—lower $\alpha$—as the country is getting increasingly more productive. However, low efficiency of reductive effort partially offsets this, as the probability of winning is the driving force. Hence, even if efficiency in both dimensions seems to work in the same direction, the relative efficiency of reductive efforts is decisive.

The efficiency of productive efforts can also play an indirect role in the determination of the proposed scheme. Actually, for the most efficient country in the reductive dimension, competing

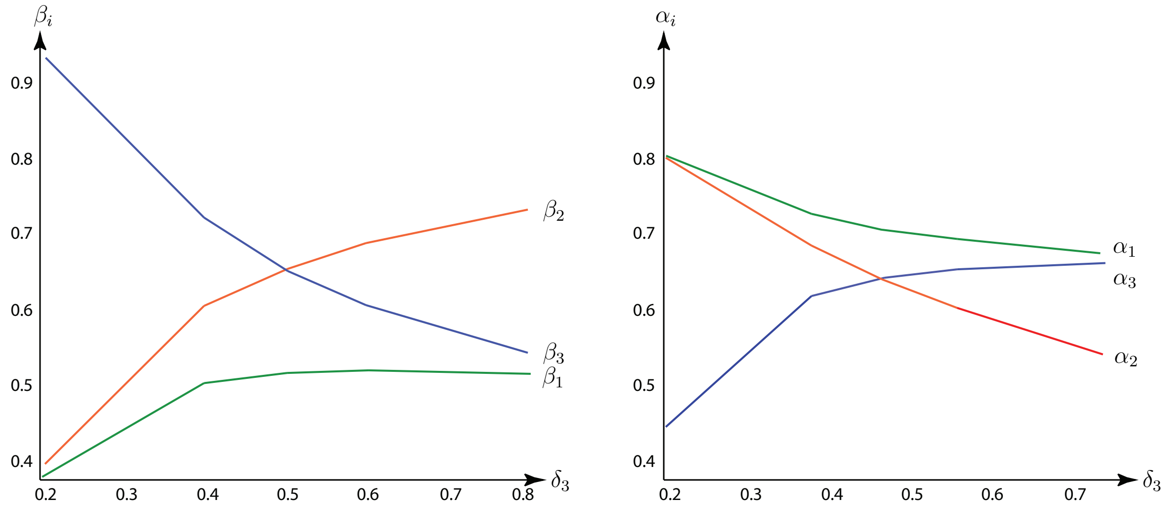


Figure 2: The three curves in the left panel represent the variation of $\beta_1, \beta_2, \beta_3$ and those on the right show $\alpha_1, \alpha_2, \alpha_3$ according to different levels of country 3's reductive efficiency $\delta_3$. The horizontal axis represents the different values that $\delta_3$ can take, the other parameters of the cost function are fixed at $\gamma_1 = \gamma_2 = \gamma_3 = 1$ and $\delta_1 = 1, \delta_2 = 0.5$. Country 3 becomes less efficient in comparison with country 2 and 3 as $\delta_3$ increases. Hence, as country 3 becomes less efficient, her probability of winning the prize decreases and therefore output share $\alpha_3$ increases while the prize pool share $\beta_3$ decreases (blue line). Similarly, as $\delta_3$ increases, countries 1 and 2 become relatively more efficient and the output share $\alpha_1, \alpha_2$ decrease while $\beta_1, \beta_2$ increase (green and red lines).

with a nation which is more efficient than herself in the output dimension can even increase the share that she is willing to commit to participate in the contest and augments the inequality of the prize pool shares.

The resulting wealth distribution depends on the type of asymmetry countries face. Our results suggest that when output productivity is positively correlated with reductive productivity (as is the case in the previous example) the spread of expected utilities narrows with respect to the situation without an agreement and the resulting wealth distribution is more equitable. Similarly, if the costs of reductive and productive efforts are negatively correlated, the proposed mechanism generates a more equitable wealth distribution.[27] Finally, if the costs in both dimensions are uncorrelated, both a spread and reduction of the wealth distribution are possible. Notice that if the asymmetry only concerns the cost of reductive efforts, then the proposed mechanism will increase the range of the wealth distribution.

The remainder of this subsection tentatively applies the proposed mechanism to the debate on international cooperation on climate change. We consider three different groups of countries suggestively labelled EU, US and developing poor (DP). We postulate that EU and US are more efficient in productive output than DP. For the reductive efforts, we assume that US and DP have

[27] Participation may be problematic in this context, as a highly productive country with low productivity in reductive efforts may profit from staying out of the IEA. As global utility increases, however, compensating schemes can be introduced to ensure participation.

low costs but EU has not. In particular, we assume[28]

$$\delta_{EU} = 1, \delta_{US} = \delta_{DP} = 0.5, \text{ and } \gamma_{EU} = \gamma_{US} = 0.5, \gamma_{DP} = 1.$$

This seems to coincide with reality as rich countries are arguably more efficient in producing output than poor states. Metaphorically speaking, US and DP have lots of low hanging fruit to take advantage of since they are quite inefficient in their energy use, whereas EU is relatively energy efficient and subsequent improvements are costly.

Our results suggest that the proposed scheme will improve the welfare distribution as the range of expected utility narrows. Under the proposed mechanism, we obtain expected utilities of

$$EU^*_{EU} = 0.44, EU^*_{US} = 0.46, EU^*_{DP} = 0.45$$

whereas without an IEA, the corresponding wealth distribution is $U_{EU} = 0.46, U_{US} = 0.46, U_{DP} = 0.37$. We thus find that global welfare increases by 7% thanks to the introduction of the IEA. EU, however, could loose in comparison with the status quo and therefore needs to be compensated. Other considerations, such as her historical responsibility, may also be underlined to motivate her participation in the contest.

Finally, the question whether and in how far the efficient asymmetric abatement levels can be used directly as inputs into the contest success function or not is, of course, politically charged. We therefore restrict ourselves to pointing out that any standard normalisation of input efforts is feasible. For instance, it is perfectly possible to normalise inputs such that each country which exerts it's efficient abatement level has the same chance of winning the first prize $^1/_n$. A strength of the contest mechanism is its ability to adopt different abatement-cost distribution rules. Our concept of reductive effort can encompass population size, GDP, areal expansion or incorporate fairness considerations.

## 5.5 The choice of success function

Consider a $n$-player extension of the problem of subsection 3.1 with prize structure $\left(\beta, \frac{1-\beta}{n-1}, \ldots, \frac{1-\beta}{n-1}\right)$. The present example shows that efficiency can be obtained in proposition 2 for a 'difference-form' success function. Difference-form success functions have been widely used in the literature, for instance by Che and Gale (2000), but suffer from the lack of a generally accepted, simple extension to more than two players. We define player $i$'s probability of winning as[29]

$$p_i(\Delta) = \frac{\exp^{\Delta_i^r}}{\sum_{j=1}^n \exp^{\Delta_j^r}}, \text{ where } \Delta = (\Delta_1, \ldots, \Delta_n),\ \Delta_i = f_i - \frac{\sum_{j \neq i} f_j}{n-1}, \text{ and } r > 0. \qquad (16)$$

[28] The numbers we use are inspired by Ellerman and Decaux (1998). The debate on marginal abatement cost of greenhouse gases is not settled, however. For a recent discussion see Morris, Paltsev, and Reilly (2008) and the references therein.

[29] This formulation is due to Schweinzer and Segev (2010) who also provide the equilibrium analysis for a broad class of difference-form success functions and the corresponding existence results.

Setting $P = (1-\alpha)(e_i^y + (n-1)e_j^y)$, $y \in (0,1)$, $m, b > 1$ and all $j \neq i$ equal, player $i$'s individual problem is to

$$\max_{(e_i,f_i)} \alpha e_i^y + p_i(\Delta)\beta P + (1-p_i(\Delta))\frac{1-\beta}{n-1}P - s_i(e_i + (n-1)e_j - f_i - (n-1)f_j)^m - (e_i^b + f_i^b)$$

which, in symmetric equilibrium $e = e_i = e_j$, $f = f_i = f_j$ gives for any $p_i(\Delta)$

$$\alpha = \frac{e^{-y}\left((e-f)\left(be^b n - e^y y\right) + em((e-f)n)^m s_i\right)}{(e-f)(n-1)y},$$
$$\beta = \frac{e^{-y}\left(-b(e-f)f^b(n-1)n + f\left(m(n-1)((e-f)n)^m s_i + e^y(e-f)n^2(\alpha-1)p_i'(0)\right)\right)}{(e-f)fn^3(\alpha-1)p'(0)}$$

where $\Delta = 0$ is the equilibrium vector of deviations. Plugging in the efficient efforts from (4), employing (16), and returning to the example setup from section 3.1: $n = 2$, $y = 1/2$, $b = m = 2$, and $s_i = 1/2$, this results in a very similar efficient mechanism as under the Tullock success function

$$\alpha^* = \frac{3}{5},\ \beta^* = \frac{1}{2} + \frac{r + (5/6)^{2/3}}{2r}$$

where $\beta^* \in (.5, 1]$ is ensured for $r \geq (5/6)^{2/3}$. A picture nearly identical to figure 1 confirms, for instance, $(\alpha^*, \beta^*, r = 2)$ as equilibrium. The precise form of ranking technology employed is thus immaterial to our results.

# 6 Concluding remarks

We show that a simple contest organised among nations can implement both efficient productive and reductive efforts. Our model provides a benchmark for the cost of achieving efficiency. Many desirable generalisations of the model are left for future work: Which share of global (per capita) GDP would have to be redistributed—in reality—to the country with the highest emissions reduction in order to implement our results? Is the resulting wealth redistribution one we would like to see? These questions are to a large extent empirical and all have huge policy implications. At any rate we do not feel qualified to answer these questions now. What we do provide, however, are firm results showing that an incentive mechanism along the lines we indicate can *in principle* solve the world's emission problems.

# Appendix

**Proof of proposition 1**. Efficient efforts are extending (1) as the pair $(e^*, f^*)$ solving

$$y'(e) = m'(ne - nf) + c_e(e,f),\ m'(ne - nf) = c_f(e,f). \tag{17}$$

Let $P = (1-\alpha)\sum_{h=1}^{n} y(e_h)$. Since we are only interested in deviations from symmetric equilibrium, we set $e_j = e_{-i}$. Rewriting (9) for the 2-prize structure $\left(\beta^1, \frac{1-\beta^1}{n-1}, \ldots, \frac{1-\beta^1}{n-1}\right)$ results in

$$\alpha y(e_i) + \beta^1 p_i^1(\mathbf{f})P + \sum_{h=2}^{n} \frac{1-\beta^1}{n-1} p_i^h(\mathbf{f})P - s_i m\left(e_i + (n-1)e_j - f_i - (n-1)f_j\right) - c(e_i, f_i)$$

which simplifies to

$$\alpha y(e_i) + \beta^1 p_i^1(\mathbf{f})P + \frac{1-\beta^1}{n-1}(1 - p_i^1(\mathbf{f}))P - s_i m\left(e_i + (n-1)e_j - f_i - (n-1)f_j\right) - c(e_i, f_i).$$

The symmetric $e = e_i = e_j$, $f = f_i = f_j$, focs for this problem are

$$\begin{aligned} c_e(e,f) + s_i m'(ne - nf) &= \frac{1-\beta+\alpha(n+\beta^1-2)+(1-\alpha)(n\beta-1)}{n-1} p(f) y'(e), \\ c_f(e,f) &= s_i m'((e-f)n) + \frac{n(1-\alpha)(n\beta^1-1)}{n-1} p'(f) y(e). \end{aligned} \tag{18}$$

Plugging in (17) and imposing $s_i = 1/n$, one obtains

$$\alpha^* = 1 - \frac{y'(e^*) - c_e(e^*, f^*)}{y'(e^*)} \text{ and } \beta^* = \frac{1}{n} + \frac{(n-1)^2 y'(e^*)}{n^3 y(e^*) p'(\mathbf{f}^*)} \tag{19}$$

which can always be achieved by picking a suitably steep ranking technology $p(\mathbf{f}^*)$. □

**Proof of proposition 2**. Since under our assumptions (9) is fully separable we can split the problem into two independent problems along the respective effort dimensions. Setting $P = (1-\alpha)\sum_{h=1}^{n} y(e_h)$, the two separate problems are

$$\begin{aligned} \alpha y(e_i) + \beta^1 p_i^1(\mathbf{f}^*)P + \frac{1-\beta^1}{n-1}(1 - p_i^1(\mathbf{f}^*))P - s_i m\left(e_i + (n-1)e^* - n f^*\right) - c(e_i, f^*), \\ \alpha y(e_i^*) + \beta^1 p_i^1(\mathbf{f})P + \frac{1-\beta^1}{n-1}(1 - p_i^1(\mathbf{f}))P - s_i m\left(ne^* - f_i - (n-1)f^*\right) - c(e^*, f_i). \end{aligned} \tag{20}$$

**1**) We show that exerting productive effort $e_i = e^*$ gives a global maximum. As players are symmetric and we are looking for a profitable deviation from the efficient level we set $\mathbf{f}^* = (f_1 = f^*, \ldots, f_n = f^*)$ implying that the probability of wining is $p_i^1(\mathbf{f}^*) = 1/n$. Thus the problem simplifies to

$$\alpha y(e_i) + \frac{1}{n} P - s_i m\left(e_i + (n-1)e^* - (n)f^*\right) - c(e_i, f^*) \tag{21}$$

giving the foc for productive effort $e_i$ as[30]

$$\underbrace{y'(e_i)\left(\alpha + \frac{1}{n}(1-\alpha)\right)}_{\searrow} = \underbrace{s_i m'\left(\max\{0, e_i + (n-1)e_j^* - (n)f^*\}\right)}_{\nearrow} + \underbrace{c_{e_i}(e_i, f^*)}_{\nearrow}.$$

Notice that output is strictly increasing in $e_i$ and is strictly concave. Thus $y''(e_i) < 0$ and $y'(e_i)$ is

[30] It is routine to verify that both focs identify a maximum.

decreasing. Both cost functions are increasing and convex, therefore $s_i m''(\cdot) + c''(e_i) > 0$ and the rhs is increasing. As $y'(0) > s_i m'(\max\{0, (n-1)e^* - nf^*\}) + c'(0)$,[31] this confirms single crossing of rhs and lhs and ensures the existence of a unique equilibrium.

**2**) We now demonstrate global optimality of $f_i = f^*$. Assuming efficient productive effort provision, the foc for reductive effort is

$$\underbrace{n\, y\,(e^*)\,(1-\alpha)\,(\beta\, n-1)\, p'(f_i, f^*)}_{=B} = \underbrace{c_{f_i}(f_i, e^*)}_{=C\nearrow} - \underbrace{s_i m'(\max\{0, ne^* - (n-1)f^* - f_i\})}_{=A\searrow}. \quad (22)$$

Notice that the rhs is strictly increasing as we know that wrt $f_i$, $s_i m''(\cdot) \leq 0$ and thus that $A$ is decreasing and the cost function is convex. Without further assumptions on the monitoring technology $p(\cdot)$ we cannot sign the slope of $B$. Notice, however, that increasing the slope of the (convex) cost function $c_{f_i}(f_i, e^*)$ sufficiently guarantees single crossing and thus equilibrium uniqueness whatever the precise specification of $p(\cdot)$.

**3**) We now show that (22) identifies a global maximum for the Tullock success function.[32] Again, $s_i m(\max\{0, ne^* - (n-1)f^* - f_i\}) > 0$ for $f_i = 0$ while $p'(f_i, f^*) = 0$ and thus the lhs of (22) is zero at $f_i = 0$ while the rhs is negative. Single crossing is immediate for the case of $r \in (0, 1]$ as $B$ is (weakly) decreasing. In the general case of

$$p_i(\mathbf{f}) = \frac{f_i^r}{\sum_{j=1}^n f_j^r}, \; r > 1, \quad (23)$$

the function $B$ has a single critical point and is decreasing when $f_i \geq f^* \left(\frac{(n-1)(r-1)}{r+1}\right)^{1/r}$.

To get single crossing if the two curves are increasing we need to ensure either strict concavity or convexity for the lhs and strict convexity for the rhs and prove that if $f_i = 0$, lhs is larger than the rhs. As we have not specified anything about our functions regarding the third derivative we illustrate this point using the specific $c_{f_i}(f_i, e^*) = bf^{b-1}$ and $s_i m(\max\{0, ne^* - (n-1)f^* - f_i\}) = s_i(\max\{0, ne^* - (n-1)f^* - f_i\})^b$. We also set $s_i = \frac{1}{n}$. We find that both curves have an inflection point, thus we need to find a condition to ensure single crossing.

We first show that the rhs starts out negative and eventually becomes positive as for $f_i = 0$ we have $C - A = -s_i m'(\max\{0, ne^* - (n-1)f^*\}) < 0$. Therefore, as long as the lhs is positive and the rhs negative, the two curves cannot cross. We find that $C - A < 0$ for $f_i < f^* \frac{2}{n^{\frac{1}{b-1}} + 1}$ because

$$C - A = f_i^{b-1} b - \frac{(\overbrace{ne^* - (n-1)f^*}^{=2f^*} - f_i)^{b-1} b}{n} = 0 \;\Leftrightarrow (2f^* - f_i)^{b-1} = n f_i^{b-1}.$$

Moreover, for the rhs, the inflection point occurs when the curve is negative, and it is first concave and then convex. Thus we can conclude than when the curve is above zero, it is strictly convex.

[31] Since output is concave and the sum of cost functions is convex in $e_i$, the above inequality holds.

[32] A nearly identical argument can be made for any other ratio-based success function. In that more general case, however, we cannot derive an explicit existence threshold.

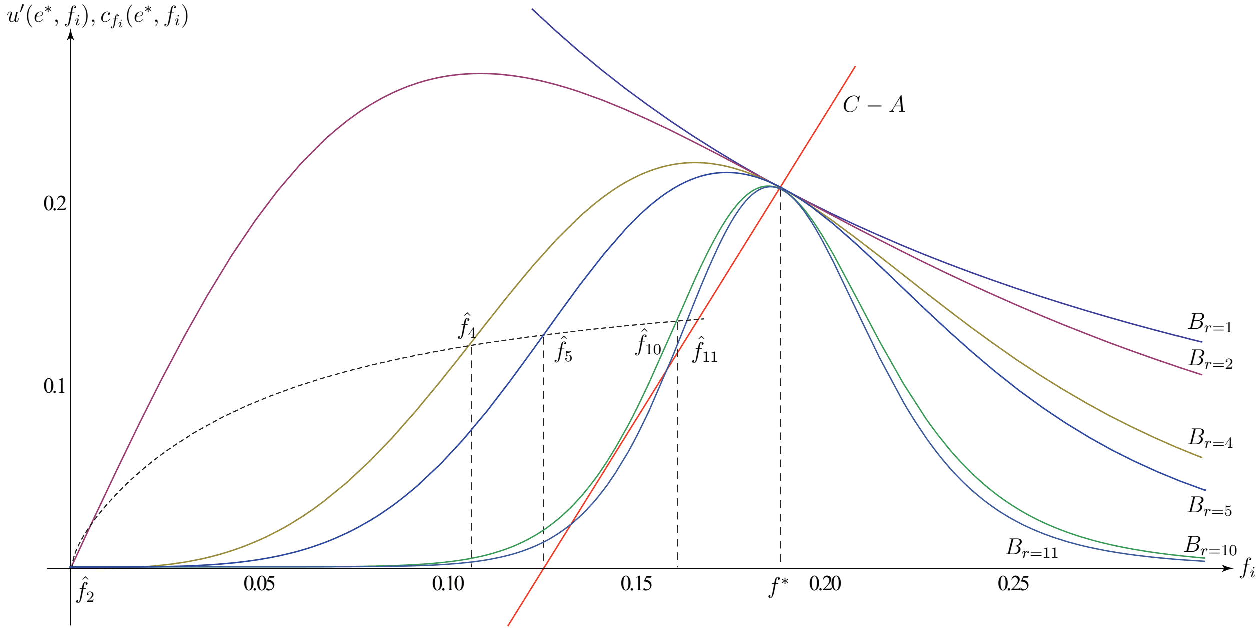


Figure 3: Single crossing in equation (22) ensures a unique global maximum at $f_i = f^*$ for the example setup of section 3.1. The dotted line gives the location of inflection points $\hat{f}$ for different $r$.

We find that $(C-A)'' < 0$ for $f_i < f^* \frac{2}{n^{\frac{1}{b-3}}+1}$ and $f_i < f^* \frac{2}{n^{\frac{1}{b-3}}+1} < f^* \frac{2}{n^{\frac{1}{b-1}}+1}$ because[33]

$$(C-A)'' = f_i^{b-3}(b-2)(b-1)b - \frac{(2f^*-f_i)^{b-3}(b-2)(b-1)b}{n} = 0 \Leftrightarrow (2f^*-f_i)^{b-3} = nf_i^{b-3},$$

$$\Leftrightarrow f^* \frac{2}{n^{\frac{1}{b-1}}+1} - f^* \frac{2}{n^{\frac{1}{b-3}}+1} = 2\frac{f^*\left(n^{\frac{1}{b-3}} - n^{\frac{1}{b-1}}\right)}{\left(n^{\frac{1}{b-1}}+1\right)\left(n^{\frac{1}{b-3}}+1\right)} \geq 0.$$

We conclude that the rhs is strictly increasing and convex when it is positive.

For the lhs, there are two inflection points: one in the increasing part and the other in the decreasing part. In the increasing part we find a condition which implies that the inflection occurs if the rhs is negative.[34] A sufficient condition for equilibrium uniqueness is therefore that

$$\frac{2^r(f^*)^r}{\left(n^{\frac{1}{b-1}}+1\right)^r} \geq \underbrace{\frac{(n-1)\left(2(f^*)^r(r^2-1) - \sqrt{3}\sqrt{(f^*)^{2r}r^2(r^2-1)}\right)}{2+3r+r^2}}_{=:\hat{f}}. \tag{24}$$

Thus if the rhs of (22) is positive, it is also strictly convex. If (24) is respected, the lhs is strictly concave or convex. Notice also that at the inflection point, the rhs is positive and the lhs is negative and therefore the lhs is larger than the rhs. The geometric intuition of (24) is shown in figure 3 for the setup of the example section 3.1. The figure shows a family of curves $B$ for $r \in \{1, 2, 4, 10, 11\}$ with inflection points labelled $\hat{f}_2$, $\hat{f}_4$, $\hat{f}_{10}$, and $\hat{f}_{11}$, respectively. Condition (24) is fulfilled as long as the red cost curve $C-A$ is negative at the respective inflection point. This is true for $r = 2$ and

[33] This is true for any $b \geq 3$.

[34] The inflection point in the decreasing part does not matter. As long as one curve is increasing and the other is decreasing they can only cross once.

$r = 4$ soon after which (24) starts failing. Uniqueness, however, is actually only lost for $r > 10$. $\square$

**Proof of proposition 3**. Player $i$'s equilibrium participation utility for $P = (1 - \alpha)ny(e^*)$ under the efficiency-inducing contract $C = \langle \alpha^*, \beta^*, p^*(\mathbf{f}) \rangle$ defined in (9) with prizes $\left(\beta, \frac{1-\beta}{n-1}, \ldots, \frac{1-\beta}{n-1}\right)$ is

$$\begin{aligned} u_i(e^*, f^*) &= \alpha^* y(e^*) + \tfrac{1}{n}\beta^* P + (n-1)\tfrac{1}{n}\tfrac{1-\beta^*}{n-1} P - s_i m\left(ne^* - nf^*\right) - c(e^*, f^*) \\ &= \alpha y(e^*) + \tfrac{1}{n}(1-\alpha^*)ny(e^*) - s_i m\left(ne^* - nf^*\right) - c(e^*, f^*) \\ &= y(e^*) - s_i m\left(ne^* - nf^*\right) - c(e^*, f^*). \end{aligned} \tag{25}$$

Now consider the contract $C' = \langle \alpha', \beta', p' \rangle$. Since second-stage efforts $e(\alpha', \beta', p'), f(\alpha', \beta', p')$ are continuous in $\alpha'$, it is sufficient to consider the extreme case of $\alpha' = 1$ which implements no contest at all. A deserter's utility when subjected to $C'$ can therefore be driven down to[35]

$$u_i^s(e_i^s, f_i^s) = y(e_i^s) - s_i m(e_i^s + (n-1)e^a - f_i^s - (n-1)f^a) - c(e_i^s, f_i^s) \tag{26}$$

where $e^a$ and $f^a$ are the equilibrium inside agreement efforts prescribed by $C'$. For $\alpha' = 1$, these equal the equilibrium efforts without agreement $e_i^s, f_i^s$. Hence (25) and (26) are identical but implement different efforts. Since both $e^s > e^*$ and $f^s < f^*$, it is therefore individually rational to join the agreement implementing $C$ if the alternative is $C'$ because the cost differential

$$s_i(m(ne^s - nf^s) - m\left(ne^* - nf^*\right)) \tag{27}$$

is increasing in $n$ and convex. As the efficient allocation is welfare maximising, this outweighs any productivity gains from free riding $y(e^s) - y(e^*) + c(e^*, f^*) - c(e^s, f^s)$. Thus, every player finds it individually rational to join the reductive contest if threatened by the alternative $C'$. $\square$

**Proof of proposition 4**. Analogous to (1), let player $i$'s asymmetric efficient efforts be given by

$$y_i'(e_i^*) = m'(A) + c_{e_i}(e_i^*, f_i) \text{ and } m'(A) = c_{f_i}(e_i, f_i^*) \tag{28}$$

where $A = \sum_{j=1}^n (e_j - f_j)$. Let the payments' shares $\alpha_i$ and winning shares be identity-dependent, ie. a winning player $i$ gets share $\beta_i$ and a winning $j$ gets share $\beta_j$ of the total prize pool $P = \sum_{j=1}^n (1 - \alpha_j) y_j(e_j)$. Thus, taking all $e_j^*, f_j^*$, $j \neq i$, as given, player $i$ maximises (10). Taking derivatives wrt $e_i, f_i$ and inserting (28), determines player $i$'s best response through[36]

$$\begin{aligned} \alpha_i &= \frac{y_i'(e_i)(1-B) - (1-s_i)m'(A)}{y_i'(e_i)(1-B)}, \text{ where } B = \beta_i p_i^1(\mathbf{f}) + \frac{\sum_{j\neq i}(1-\beta_j)p_j^1(\mathbf{f})}{(n-1)}, \\ \beta_i &= \frac{(n-1)\left((1-s_i)m'(A)\right) - \sum_{j\neq i}(1-\beta_j)p_{j(f_i)}^1(\mathbf{f})P}{p_{i(f_i)}^1(\mathbf{f})(n-1)P} \end{aligned} \tag{29}$$

and $p_{i(f_i)}^1(\mathbf{f})$ denotes the $\frac{\partial}{\partial f_i} p_i^1(\mathbf{f})$. (29) corresponds to (19) and elicits asymmetric efficient efforts

[35] Notice that the latter formulation requires $n > 2$ as the contest can only produce incentives if at least two players participate in the contest.

[36] The expressions (29) can be simplified further but then get excessively lengthy.

$(e_i^*, f_i^*)$. Solving for the best responses of player $i$'s opponents to $e_i^*, f_i^*$ gives $2(n-1)$ more equations complementing (29). Thus for $4n$ unknowns, we obtain two equations per player and a further $2n$ equations through the planner's problem. As illustrated in the three-players example of section 5.3, solving for the complete system $(\alpha_1, \ldots, \alpha_n; \beta_1, \ldots, \beta_n)$ is unproblematic. □

# References


Allison, I., N. L. Bindoff, R. Bindschadler, P. Cox, N. de Noblet, M. England, J. Francis, N. Gruber, A. Haywood, D. Karoly, G. Kaser, C. L. Quéré, T. Lenton, M. Mann, B. McNeil, A. Pitman, S. Rahmstorf, E. Rignot, H. Schellnhuber, S. Schneider, S. Sherwood, R. Somerville, K.Steffen, E. Steig, M. Visbeck, and A. Weaver (2009): *The Copenhagen Diagnosis, 2009: Updating the world on the Latest Climate Science*. The University of New South Wales, Climate Change Research Centre (CCRC), Sydney, Australia.

Arbatskaya, M., and H. Mialon (2010): "Multi-activity contests," *Economic Theory*, 43(1), 23–43.

Barrett, S. (1994): "Self-Enforcing International Environmental Agreements," *Oxford Economic Papers*, 46, 878–94.

——— (1998): "Political Economy of the Kyoto Protocol," *Oxford Review of Economic Policy*, 14(4), 20–39.

——— (2003): *Environment and statecraft: The strategy of environmental treaty-making*. Oxford University Press, New York.

——— (2006): "Climate Treaties and 'Breakthrough' Technologies," *American Economic Review*, 96, 22–5.

Beccherle, J., and J. Tirole (2010): "Regional Initiatives and the Cost of Delaying Binding Climate Change Agreements," *Toulouse School of Economics*, Discussion paper.

Biermann, F., P. Pattberg, H. van Asselt, and F. Zelli (2009): "The Fragmentation of Global Governance Architectures: A Framework for Analysis," *Global Environmental Politics*, 9(4), 14–40.

Buchholz, W., and K. Konrad (1994): "Global Environmental Problems and the Strategic Choice of Technology," *Journal of Economcs*, 60(3), 299–321.

Carbone, J. C., C. Helm, and T. F. Rutherford (2006): "The case for international emission trade in the absence of cooperative climate policy," *Journal of Environmental Economics and Management*, 58, 266–80.

Carraro, C., C. Marchiori, and S. Oreffice (2009): "Endogenous Minimum Participation in International Environmental Treaties," *Environmental & Resource Economics*, 42(3), 411–425.

Chander, P. (2007): "The gamma-core and coalition formation," *International Journal of Game Theory*, 35(4), 539–56.

Chander, P., and H. Tulkens (2006): "Cooperation, stability and self-enforcement in international environmental agreements: A conceptual discussion," *Université Catholique de Louvain, CORE*, Discussion Paper 2006003.

Che, Y.-K. C., and I. Gale (2000): "Difference-form contests and the robustness of all-pay auctions," *Games and Economic Behavior*, 30, 22–43.

Dasgupta, S., B. Laplante, C. Meisner, D. Wheeler, and J. Yan (2009): "The impact of sea level rise on developing countries: a comparative analysis," *Climatic Change*, 93, 379–88.

Diamantoudi, E., and E. S. Sartzetakis (2006): "Stable International Environmental Agreements: An Analytical Approach," *Journal of Public Economic Theory*, 8, 247–63.

Dijkstra, B. R. (2007): "An investment contest to influence environmental policy," *Resource and Energy Economics*, 29(4), 300–24.

Dixit, A. (1987): "Strategic Behavior in Contests," *American Economic Review*, 77, 891–98.

Ellerman, A. D., and A. Decaux (1998): "Analysis of Post-Kyoto CO2 Emissions Trading Using Marginal Abatement Curves," *MIT Joint Program on the Science and Policy of Global Change*, Report 40.

Engel, S. (2004): "Achieving environmental goals in a world of trade and hidden action: the role of trade policies and eco-labeling," *Journal of Environmental Economics and Management*, 48(3), 1122–45.

Engers, M., and B. McManus (2007): "Charity auctions," *International Economic Review*, 48, 953–94.

Finus, M. (2008): "Game Theoretic Research on the Design of International Environmental Agreements: Insights, Critical Remarks, and Future Challenges," *International Review of Environmental and Resource Economics*, 2(1), 29 – 67.

Gershkov, A., J. Li, and P. Schweinzer (2009): "Efficient Tournaments within Teams," *Rand Journal of Economics*, 40(1), 103–19.

Goeree, J. K., E. Maasland, S. Onderstal, and J. L. Turner (2005): "How (Not) to Raise Money," *Journal of Political Economy*, 113, 897–926.

Grankvist, G., U. Dahistrand, and A. Biel (2004): "The Impact of Environmental Labelling on Consumer Preference: Negative vs Positive Labels," *Journal of Consumer Policy*, 27(2), 213–30.

Green, J., and N. Stokey (1983): "A Comparison of Tournaments and Contracts," *Journal of Political Economy*, 91, 349–64.

Guesnerie, R., and H. Tulkens (eds.) (2009): *The Design of Climate Policy*, CESifo Seminar Series. MIT Press, Cambridge, Mass.

Harbaugh, R., J. W. Maxwell, and B. Roussillon (2006): "The Groucho Effect of Uncertain Standards," *Indiana University Working Paper*, # 2006-09.

Harstad, B. (2010): "The Dynamics of Climate Agreements," *Northwestern University, Center for Mathematical Studies in Economics and Management Science*, Discussion paper #1474.

HEAYES, A. G. (1997): "Environmental Regulation by Private Contest," *Journal of Public Economics*, 63, 407–28.

HOLMSTRÖM, B. (1982): "Moral Hazard in Teams," *Bell Journal of Economics*, 13, 324–40.

HURLEY, T. M., AND J. F. SHOGREN (1997): "Environmental Conflicts and the SLAPP," *Journal of Environmental Economics and Management*, 33(3), 253–73.

KOLSTAD, C. D. (2007): "Systematic uncertainty in self-enforcing international environmental agreements," *Journal of Environmental Economics and Management*, 53(1), 68 – 79.

KONRAD, K. (2008): *Strategy and Dynamics in Contests*. Oxford University Press, Oxford.

LAZEAR, E., AND S. ROSEN (1981): "Rank Order Tournaments as Optimal Labor Contracts," *Journal of Political Economy*, 89, 841–64.

LERNER, J., AND J. TIROLE (2006): "A Model of Forum Shopping, with Special Reference to Standard Setting Organizations," *American Economic Review*, 96, 1091–13.

LIVERMAN, D. M. (2009): "Conventions of climate change: constructions of danger and the dispossession of the atmosphere," *Journal of Historical Geography*, 35(2), 279–96.

MARYLAND DEPARTMENT OF NATURAL RESOURCES (2007): "Environmental Review of Proposed Air Pollution Control Project at Brandon Shores," *PPRP*, PSC Case # 9075(JS-3).

MITROVICA, J. X., N. GOMEZ, AND P. U. CLARK (2009): "The Sea-Level Fingerprint of West Antarctic Collapse," *Science*, 323(5915), 753 [DOI: 10.1126/science.1166510].

MOLDOVANU, B., AND A. SELA (2001): "The Optimal Allocation of Prizes in Contests," *American Economic Review*, 91(3), 542–58.

MORGAN, P. (2000): "Financing Public Goods by Means of Lotteries," *Review of Economic Studies*, 67, 761–84.

MORRIS, J., S. PALTSEV, AND J. REILLY (2008): "Marginal Abatement Costs and Marginal Welfare Costs for Greenhouse Gas Emissions Reductions: Results from the EPPA Model," *MIT Joint Program on the Science and Policy of Global Change*, Report 164.

MÜNSTER, J. (2007): "Selection Tournaments, Sabotage, and Participation," *Journal of Economics & Management Strategy*, 16, 943–70.

NALEBUFF, B. J., AND J. E. STIGLITZ (1983): "Prizes and Incentives: Towards a General Theory of Compensation and Competition," *Bell Journal of Economics*, 14, 21–43.

NORDHAUS, W. D. (2006): "After Kyoto: Alternative Mechanisms to Control Global Warming," *American Economic Review*, 96, 31–4.

OGAWA, H., AND D. E. WILDASIN (2009): "Think Locally, Act Locally: Spillovers, Spillbacks, and Efficient Decentralized Policymaking," *American Economic Review*, 99(4), 1206–17.

RUBIO, S. J., AND A. ULPH (2007): "An infinite-horizon model of dynamic membership of international environmental agreements," *Journal of Environmental Economics and Management*, 54(3), 296–310.

SCHWEINZER, P., AND E. SEGEV (2010): "Difference-form contests with multiple players," *University of Manchester Discussion Paper*.

——— (2011): "The optimal prize structure of symmetric Tullock contests," *Public Choice*, forthcoming.

Siegel, R. (2009): "Asymmetric Contests with Conditional Investments," *American Economic Review*, forthcoming.

Wagner, U. J. (2002): "The Design of Stable International Environmental Agreements: Economic Theory and Political Economy," *Journal of Economic Surveys*, 15, 377–411.