

# Self-reinforcing feedback mechanism in R&D

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## Abstract

This paper studies the innovation process in knowledge-based industries. It considers a cost-reducing R&D technology, in which cross-firm R&D spillovers occur and the feedback mechanism is self-reinforcing. This technology displays dynamic learning and increasing returns to spillovers. Spillovers boost R&D resulting in greater efficiency enhancement.

**Keywords:** R&D spillovers, process innovation, increasing returns

## 1 Introduction

Innovation in science-based industries is an ongoing process and depends on the technological interactions among the research units. Empirical works, reviewed by Arthur (1994) and Feldman (1999), document that, in bioengineering and microelectronic-based industries such as semiconductors, computers, communications equipment and aircraft, knowledge is transmitted within research networks and the feedback mechanism is self-reinforcing.<sup>1</sup> This paper examines the production of R&D in such high-technology industries and technological parks, such as Silicon Valley and Route 128. It considers a cost-reducing R&D technology, in which cross-firm R&D spillovers occur and there is a ‘cumulative causation’ among firms’ R&D efforts. In particular, a firm’s R&D output spilled over increases the others’ performance. In turn, the increased R&D-output spilled over by the other, being mixed with firm’s R&D effort, enhances its performance further, and so on. This technology displays increasing returns to spillovers. The feedback is positive and ‘*regenerative*’. The theoretical literature based on D’Aspremont and Jacquemin (1988) and Kamien, Muller, and Zang (1992) is weak to explain this ‘dynamic’ learning since it supposes that there are no interactions during the R&D production.

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<sup>1</sup>See Feldman (1999) for a survey of empirical findings on the innovation process and Arthur (1994) for a discussion on the operation of high-technological firms.

One could interpret this technology by considering two engineering students who work on similar projects and sit in adjacent positions. During the R&D process, each student can see behind the back of another and replicate some part of individual R&D outcome. By doing so, both students get smarter and improve their R&D-outputs more as spillovers increase. This happens more strongly at broader levels of aggregation. A positive and significant relationship between spillovers and R&D-outputs has been documented empirically in science-based industries. For instance, pharmaceuticals exist within a network of academic departments, testing labs, hospitals and other organizations. Scientists appointed by these institutions observe some part of others' research outcomes due to lack of information control over their, embodiment of knowledge in chemical compounds and genetic sequences, or some special characteristics of R&D technologies. Thus, researchers learn more and improve their own research outcomes. As knowledge transmission within this network is intensified, the R&D process is reinforced. This paper attempts to shed light into this innovation process which exhibits speed-up of innovation and increasing returns to spillovers resulting in greater efficiency enhancement.

## 2 R&D technology

The market is composed of two profit-seeking firms  $i$  and  $j$  where  $i, j = 1, 2$  and  $i \neq j$ . Each firm carries out R&D projects for achieving efficiency enhancement and its marginal cost becomes  $c_i = \bar{c} - z_i$ ,  $c_i \geq 0$ ;  $\bar{c}$  denotes the initial marginal cost who is constant and identical for both firms and  $z_i$  is the R&D output.  $i$ 's R&D-output depends on own R&D effort,  $x_i$ , and the size of the spillover,  $hz_j$ :

$$z_i = x_i + hz_j, \quad h \in [0, \bar{h}] \quad (1)$$

$h$  stands for the degree of cross-firm spillovers; i.e. the fraction of firm  $j$ 's R&D-output that can be absorbed by firm  $i$ .<sup>2</sup> The spillover rate lies in  $[0, \bar{h}]$  where the bound  $\bar{h}$  is endogenously determined and less than unity,  $\bar{h} < 1$ , implying that the R&D-outputs are imperfectly appropriable. The own-action effect dominates the cross-action effect. Substituting  $z_j = x_j + hz_i$  into (1), we get

$$z_i = \frac{1}{1 - h^2} (x_i + hx_j) \quad (2)$$

The feasibility line  $\bar{x}_i(x_j) = (1 - \bar{h}^2)\bar{c} - \bar{h}x_j$  sets an upper bound on choices  $x_i$ ,  $x_j$  as well as on the value of  $\bar{h}$  so that the post-innovation marginal cost to be positive. Thus, each firm

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<sup>2</sup>The spillover rate is assumed to be identical for all agents regardless of who is in charge to carry out the R&D activities and independent of the actions undertaken (symmetric and exogenous spillovers).

will commit to a level of cost-reducing R&D  $x_i \in X$  where  $X \equiv \left[0, \left(1 - \bar{h}^2\right) \bar{c}\right]$ . Lemma 1 sheds light into the feedback mechanism.

**Lemma 1 (Increasing returns)** *The marginal contribution of a firm's R&D effort on its own R&D output increases with spillovers even progressively; i.e.  $\frac{\partial(\partial z_i / \partial x_i)}{\partial h} > 0$ ,  $\frac{\partial^2(\partial z_i / \partial x_i)}{\partial h^2} > 0$  for all  $x_i \in X$ ,  $h \in (0, \bar{h}]$ .*

In this R&D process,  $i$ 's R&D-output spilled over increases  $j$ 's R&D performance. In turn, the increased R&D-output spilled over by  $j$  enhances  $i$ 's performance further, and so on. Thus, the feedback from R&D efforts is 'regenerative' since spillovers lead to a self-amplifying sequence of R&D reactions. There is a causation causality between R&D efforts that accelerates the R&D activity. This feedback mechanism displays increasing returns to spillovers and represents an "echo chamber" since information and ideas are reinforced by transmission inside an 'enclosed' R&D space. The spillover rate is the feedback factor. Such mechanism also differs from those considered by D'Aspremont and Jacquemin (1988) (hereafter, AJ model) and Kamien, Muller, and Zang (1992) (hereafter, KMZ model).

In AJ model, the final result of individual R&D activity is spilled over; i.e.  $z_i^{AJ} = x_i + hx_j$  where  $x_i \in [0, \bar{c}]$ ,  $h \in [0, 1]$ . Each firm  $i$  conducts autonomous research whose product  $x_i$  enters additively into  $j$ 's R&D technology. Spillovers do not affect the marginal productivity of own R&D since there is no interaction between firms during the R&D process,  $\frac{\partial(\partial z_i^{AJ} / \partial x_i)}{\partial h} = 0$ . Kamien et al (1992) consider the spillovers to be positioned on the R&D expenditures; each firm observes the other's research inputs at the beginning of the R&D process rather than after. To comparatively evaluate the innovation processes employed by these three models, we use a specification of the KMZ model proposed by Amir (2000). The R&D technology is a 'square root' autonomous R&D expenditures function where the inputs are spilled over; i.e.  $z_i^{KMZ} = \left[\frac{2}{k}(y_i + \beta y_j)\right]^{\frac{1}{2}}$  where  $y_i \in \left[0, \frac{k}{2}\sqrt{\bar{c}}\right]$ ,  $\beta \in [0, 1]$ .  $y_i$  is  $i$ 's R&D expenditure and  $\beta$  is the spillover rate. This R&D production function is concave in the spillover rate and exhibits diminishing marginal productivity of R&D,  $\frac{\partial(\partial z_i^{KMZ} / \partial y_i)}{\partial \beta} < 0$ .

The R&D technologies in KMZ and AJ models might describe the innovation processes in material and drug industries before the revolution of genetic engineering. In such industries, innovation stands alone and, once made, does not generate a sequence of discoveries (Levin (1988)). A firm's innovation may even discourage the other firm to conduct R&D. The technology considered in this model attempts to capture the 'dynamic' innovation process displayed in knowledge-based industries. Microelectronics devices, computer software, genetic sequences are produced by exploiting 'dynamic' learning. The "building blocks" creating type of innovation is reinforced as more information is diffused.

### 3 R&D incentives

The market is populated by a continuum of consumers of the same type with mass equal to 1. The representative consumer's preferences are described by the standard quadratic utility function  $V(q_i, q_j) = A(q_i + q_j) - \left[\frac{1}{2}(q_i^2 + q_j^2)\right]$  where  $q_i$  is  $i$ 's output and  $A$  is market demand,  $A > \bar{c} > 0$ .<sup>3</sup> There are no strategic interactions between firms and thus, they act as monopolists. The inverse demand functions are linear of the form  $p_i = A - q_i$  where  $p_i$  is  $i$ 's price,  $p_i : R_+^2 \rightarrow R_+$ .

Firms interact and play a two-period game. In period 1, firms simultaneously make their R&D choices and in period 2, they offer their output in the product market. Then, profits are realized. We solve the two-period game by using the backward induction approach and restrict our attention to subgame-perfect Nash equilibria.

In period 2, each firm  $i$  chooses the level of output that maximizes  $\Pi_i = [A - q_i - \bar{c} + z_i] q_i$ . In equilibrium, firm  $i$ 's output, price and production profits have respectively as  $q_i^*(z_i) = \frac{1}{2}(A - \bar{c} + z_i)$ ,  $p_i^* = c_i^* + q_i^*$  and  $\Pi_i^* = q_i^{*2}$ . In period 1, individual net profits,  $\pi_{net,i}$ , are given by  $\Pi_i$  net the cost of R&D. In the (say) "regenerative feedback" model (hereafter, RF model), firm  $i$ 's net profits are given by:

$$\pi_{net,i}^{RF} = \frac{1}{4} \left[ A - \bar{c} + \frac{1}{1-h^2} (x_i + hx_j) \right]^2 - \frac{k}{2} x_i^2 \quad (\text{RF})$$

$k$  denotes the efficiency or productivity of the R&D technology,  $k > 1$ . Higher  $k$  represent lower efficiency. Firm  $i$ 's R&D reaction function,  $RF_i(x_j) = \frac{1}{2(1-h^2)^2 k-1} [(A - \bar{c})(1 - h^2) + hx_j]$ , is continuous and strictly increasing in  $x_j$  in  $[0, \bar{x}_j]$  implying that R&D actions are strategic complements. In  $[\bar{x}_j, \bar{c}(1 - \bar{h})]$ ,  $RF_i$  coincide with the feasibility line  $\bar{x}_i(x_j)$  and decreases. It is assumed that firm  $i$ 's net profits function is decreasing in  $x_i$  at the point where the feasibility lines intersect; i.e.  $\frac{\partial \pi_{net,i}}{\partial x_i} |_{(\bar{x}_i, \bar{x}_j)} < 0$ . This Inada-type assumption guarantees positive post-innovation marginal costs. The equilibrium will lie in the jointly undominated effective action space  $\Gamma \triangleq \left\{ (x_1, x_2) : x_i \in \left[ 0, \bar{c} \left( 1 - \bar{h}^2 \right) - \bar{h} x_j \right] ; i, j = 1, 2, i \neq j \right\}$  where  $\Gamma$  is a subset of  $X^2$ .

In AJ and KMZ models, firm  $i$ 's net profits have respectively as:

$$\pi_{net,i}^{AJ} = \frac{1}{4} (A - \bar{c} + x_i + hx_j)^2 - \frac{k}{2} x_i^2 \quad (\text{AJ})$$

$$\pi_{net,i}^{KMZ} = \frac{1}{4} \left[ A - \bar{c} + \left( \frac{2}{k} (y_i + \theta y_j) \right)^{\frac{1}{2}} \right]^2 - y_i \quad (\text{KMZ})$$

Note that, if no spillovers occur,  $h = \theta = 0$ , these three R&D models turn out to be

<sup>3</sup>Each consumer has a utility function separable and linear in the numeraire good. There are no income effects on the monopolistic sector and thus, we can perform partial equilibrium analysis.

equivalent. Following Amir (2000), the monotone transformation  $x_i = \left(\frac{2}{k}y_i\right)^{\frac{1}{2}}$  and thus,  $y_i = \frac{k}{2}x_i^2$  allows either objective function to be recovered from the other. The form  $\left(\frac{2}{k}y_i\right)^{\frac{1}{2}}$  is the inverse mapping of the quadratic R&D cost function used in AJ and RF model. Firms end up in a symmetric equilibrium where

$$x^{RF} = \frac{(A - \bar{c})(1 - h)}{2(1 - h^2)(1 - h)k - 1}, \quad x^{AJ} = \frac{A - \bar{c}}{2k - (1 + h)}, \quad y^{KMZ} = \frac{(A - \bar{c})^2 k}{2(2k - 1)^2(1 + \theta)}$$

**Proposition 2 (Amplification of R&D)** *The optimal R&D level  $x^{RF}$  increases with spillovers at an increasing rate; i.e.  $\frac{\partial x^{RF}}{\partial h} > 0$ ,  $\frac{\partial^2 x^{RF}}{\partial h^2} > 0$  for all  $h \in (0, \bar{h}]$ .*

In the absence of spillovers,  $h = \theta = 0$ , the same level of R&D is undertaken in each model; i.e.  $x^{RF} = x^{AJ} = x^{KMZ}$  where  $x^{KMZ} = \left(\frac{2}{k}y^{KMZ}\right)^{\frac{1}{2}}$ . Spillovers though affect R&D choices in these models differently. To better understand proposition 2, note that R&D expenditures  $y^{KMZ}$  decrease with  $\theta$  in the KMZ model,  $\frac{\partial y^{KMZ}}{\partial \theta} < 0$ , indicating the existence of the "free-rider" problem. Provided that the other firm does not change its R&D choices, an increase in  $\theta$  induces a firm to free ride on other's and lowers its own effort. Such incentives result in under-provision of R&D. Firms face the free-rider problem even under asymmetric initial marginal costs. For high spillovers, the lower-cost firm has incentives to donate the extra 'funds' to the other, let it innovate and then appropriate the results through spillovers.

In the RF model, as spillovers increase, a firm with cost-advantage finds it best to intensify its own R&D activity. It is so because, for a given increase in  $h$ , the extra 'funds' decrease the marginal cost of a low-cost firm (say)  $i$  to a greater extent compared to the marginal cost reduction of the higher-cost firm  $j$ . A firm or a technology that gains an advantage stands to gain a further advantage. In the symmetric equilibrium where both firms choose  $x^{RF}$ , the derivative  $\frac{\partial(\partial z^{RF}/\partial x^{RF})}{\partial h} = \frac{1}{1-h}$  exceeds unity implying that spillovers drive all firms to innovate more. Higher  $h$  speeds up the R&D activity. Efforts increase progressively with spillovers due to the amplification effect. By producing all firms at a more efficient scale, the R&D market also appears unlikely to be monopolized and thus, locked-in (at least not forever).

**Proposition 3 (Efficiency enhancement)** *Greater marginal cost reduction is realized in the 'regenerative feedback' model; i.e.  $z^{RF} \geq z^{AJ} \geq z^{KMZ}$  for all  $h \in [0, \bar{h}]$  and  $\beta \in [0, 1]$ .*

There is also a bias towards a greater marginal cost reduction in the RF model compared to the other two. Monopolists can exploit the cumulative causation between agents' actions for efficiency enhancing reasons stimulating the R&D activity. Proposition 3 also demonstrates that firms have greater incentives to collude in the RF model so as to internalize the knowledge flows and eliminate the duplication of effort. The cross-profit effect,  $\frac{\partial \pi_{net,j}}{\partial x_i}$ , is

substantial and firms can take advantage of it by coordinating their R&D actions. By doing so, given that there are no strategic interactions in the product market, firms enjoy a higher price-cost margin and increased monopoly power.

## 4 Conclusion

We discuss an R&D technology where the feedback mechanism is self-reinforcing and spillovers intensify the R&D incentives. This innovation process might be used to interpret some empirical evidence on the R&D performance of modern corporations. Knowledge-based firms that operate in rapidly changing R&D markets differ in culture, behavior, management techniques and strategies. The literature supports the idea that technical advance and, in a sense, the development of knowledge-driven industries go hand in hand with feedback mechanisms based on increasing returns to spillovers. It is so since high-technology outputs depend heavy on know-how and light on resources. Each agent can exploit the cross-firm mutual feedbacks in industries or ‘learning’ regions where innovation is rushed and profitable. The government’s policies and the codes of regulation have to be adopted to these mechanisms. Future research on firms strategies and R&D policies have to make use of the special economics of positive and ‘regenerative’ feedback.

### Acknowledgements

I am indebted to Lambros Pechlivanos and Spyridon Vasilakis for the very useful discussions on this topic. I would like to thank Ana Rodrigues, Anastasios Xepapadeas, Chrysovalantou Milliou, Eric Maskin, Gianluca Femminis, Johannes Hörner, Katharine Rockett, Mathias Dewatripont, Nicolaos Vettas, Raymond De Bondt, Tymofiy Mylovanov, Vijay Krishna and the participants at the 20th Jerusalem school in economic theory, EARIE ’09 and ASSET ’09 for their comments and suggestions. Financial support from the State Scholarships Foundation in Greece is also acknowledged.

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